$\qquad$ Class: $\qquad$ Date: $\qquad$

## circle

## Multiple Choice

Identify the choice that best completes the statement or answers the question.
$\qquad$ 1. $O$ is the centre of this circle.

Which line is a tangent?

a. OQ
b. ST
c. PR
d. SU
2. O is the centre of this circle and point T is a point of tangency.

Determine the value of $x^{\circ}$.

a. $90^{\circ}$
b. $50^{\circ}$
c. $130^{\circ}$
d. $40^{\circ}$
$\qquad$ 3. O is the centre of this circle and point Q is a point of tangency.

Determine the value of $x^{\circ}$.

a. $139^{\circ}$
b. $49^{\circ}$
c. $41^{\circ}$
d. $90^{\circ}$
$\qquad$ 4. $O$ is the centre of this circle and point $M$ is a point of tangency.

Determine the value of $x^{\circ}$.

a. $37^{\circ}$
b. $53^{\circ}$
c. $143^{\circ}$
d. $90^{\circ}$
$\qquad$ 5. O is the centre of this circle and point G is a point of tangency.

Determine the value of $a$. If necessary, give your answer to the nearest tenth.

a. 11.3
b. 22.5
c. 4.6
d. 14.9
$\qquad$ 6. O is the centre of this circle and point A is a point of tangency.

Determine the value of $b$. If necessary, give your answer to the nearest tenth.

a. 5.5
b. 11
c. 23.2
d. 35.5
7. O is the centre of this circle and point Q is a point of tangency.

Determine the value of $c$. If necessary, give your answer to the nearest tenth.

a. 48
b. 27.1
c. 11
d. 5.5
$\qquad$ 8. O is the centre of this circle and point A is a point of tangency.

Determine the value of $m$. If necessary, give your answer to the nearest tenth.

a. 38
b. 7.2
c. 67.8
d. 57.8
$\qquad$ 9. O is the centre of this circle and point T is a point of tangency.

Determine the value of $n$. If necessary, give your answer to the nearest tenth.

a. $\quad 5.7$
b. 51
c. 24
d. 40.4
10. O is the centre of this circle and point Q is a point of tangency.

Determine the value of $t$. If necessary, give your answer to the nearest tenth.

a. 61.3
b. 55.7
c. 55
d. 82.2
11. Which of the following constructions would enable you to determine the centre of this circle?
i) Draw the perpendicular bisectors of PS and PQ.
ii) Join PR and QS.
iii) Join the mid-points of PS and QR and the mid-points of PQ and SR.

a. i and iii
b. iii
c. i
d. ii
12. A circle has radius 7 cm . Which of the following measures could NOT be the length of a chord in the circle: $2 \mathrm{~cm}, 11 \mathrm{~cm}, 14 \mathrm{~cm}$, or 17 cm ?
a. $\quad 17 \mathrm{~cm}$
b. $\quad 11 \mathrm{~cm}$
c. 2 cm
d. $\quad 14 \mathrm{~cm}$
13. O is the centre of the circle.

Determine the value of $v^{\circ}$.

a. $19^{\circ}$
b. $71^{\circ}$
c. $52^{\circ}$
d. $38^{\circ}$
14. $O$ is the centre of the circle.

Determine the value of $a^{\circ}$.

a. $49^{\circ}$
b. $20.5^{\circ}$
c. $41^{\circ}$
d. $69.5^{\circ}$
$\qquad$ 15. O is the centre of the circle.

Determine the value of $b^{\circ}$.

a. $144^{\circ}$
b. $81^{\circ}$
c. $72^{\circ}$
d. $18^{\circ}$
$\qquad$ 16. $O$ is the centre of the circle.

Determine the value of $n$ to the nearest tenth, if necessary.

a. 13.5
b. 4.4
c. 19
d. 1
17. $O$ is the centre of the circle.

Determine the value of $z$ to the nearest tenth, if necessary.

a. 4.5
b. 3.6
c. 5
d. 1
$\qquad$ 18. O is the centre of the circle.

Determine the value of $f$ to the nearest tenth, if necessary.

a. 4
b. 8
c. 64
d. 11.7
$\qquad$ 19. O is the centre of the circle.

Determine the value of $s$ to the nearest tenth, if necessary.

a. 3
b. 7.1
c. $\quad 12.2$
d. 51
20. O is the centre of the circle.

Determine the value of $x$ to the nearest tenth, if necessary.

a. 5.7
b. 19.6
c. 288
d. 17
21. $O$ is the centre of this circle.

Identify all the inscribed angles subtended by the minor arc QS.

a. $\angle \mathrm{QOS}$
b. $\angle \mathrm{PQT}$ and $\angle \mathrm{PST}$
c. $\angle \mathrm{QPS}$ and $\angle \mathrm{QTS}$
d. $\angle \mathrm{QPS}$
22. $O$ is the centre of this circle.

Determine the value of $x^{\circ}$.

a. $44^{\circ}$
b. $90^{\circ}$
c. $180^{\circ}$
d. $88^{\circ}$
23. $O$ is the centre of this circle. Determine the value of $m^{\circ}$.

a. $90^{\circ}$
b. $80^{\circ}$
c. $180^{\circ}$
d. $40^{\circ}$
$\qquad$ 24. $O$ is the centre of this circle.

Determine the value of $q^{\circ}$.

a. $60^{\circ}$
b. $90^{\circ}$
c. $180^{\circ}$
d. $45^{\circ}$
25. $O$ is the centre of this circle. Determine the value of $n^{\circ}$.

a. $108^{\circ}$
b. $54^{\circ}$
c. $90^{\circ}$
d. $36^{\circ}$
26. O is the centre of this circle.

Determine the value of $c^{\circ}$.

a. $90^{\circ}$
b. $44^{\circ}$
c. $180^{\circ}$
d. $88^{\circ}$
$\qquad$ 27. $O$ is the centre of this circle.

Determine the value of $a^{\circ}$.

a. $47^{\circ}$
b. $86^{\circ}$
c. $94^{\circ}$
d. $90^{\circ}$
28. O is the centre of this circle.

Determine the value of $v^{\circ}$.

a. $118^{\circ}$
b. $59^{\circ}$
c. $90^{\circ}$
d. $31^{\circ}$
29. O is the centre of this circle.

Determine the value of $z^{\circ}$.

a. $55^{\circ}$
b. $110^{\circ}$
c. $90^{\circ}$
d. $70^{\circ}$
30. O is the centre of this circle.

Determine the value of $g^{\circ}$.

a. $70^{\circ}$
b. $55^{\circ}$
c. $110^{\circ}$
d. $90^{\circ}$

## Short Answer

31. $O$ is the centre of this circle.

Which line is a tangent?

32. O is the centre of this circle. Point T is a point of tangency. What is the value of $e^{\circ}$ ?

33. Draw a line through point $P$ that is NOT a tangent to the circle.

34. Draw a line through point $P$ that is a tangent to the circle.

Label the point of tangency Q .

$\stackrel{\rightharpoonup}{P}$
35. Is the line that passes through points U and V a tangent to the circle?

36. $O$ is the centre of this circle and point $B$ is a point of tangency. Determine the values of $v^{\circ}$ and $w^{\circ}$.

37. O is the centre of this circle and point V is a point of tangency. Determine the values of $a^{\circ}$ and $b^{\circ}$.

38. O is the centre of this circle and point Q is a point of tangency.

Determine the values of $s$ and $t$. If necessary, give your answers to the nearest tenth.

39. $O$ is the centre of this circle and point $S$ is a point of tangency.

Determine the values of $m$ and $n^{\circ}$. If necessary, give your answers to the nearest tenth.

40. $O$ is the centre of this circle and point $Q$ is a point of tangency.

Determine the values of $d$ and $e^{0}$. If necessary, give your answers to the nearest tenth.

41. $O$ is the centre of this circle.

Which line segment is a diameter?

42. O is the centre of the circle.

What can you say about the lengths of WX and XY?

43. O is the centre of the circle.

What can you say about the measure of $\angle \mathrm{OBC}$ ?

44. Point O is the centre of this circle.

Determine the values of $x^{\circ}$ and $y^{\circ}$.

45. Point O is the centre of this circle.

Determine the values of $c^{\circ}$ and $d^{\circ}$.

46. Point O is the centre of this circle. Without solving for $a$, sketch and label the length of any extra line segments you need to draw to determine the value of $a$.

47. Point O is the centre of this circle. Without solving for $s$, sketch and label the lengths of any extra line segments you need to draw to determine the value of $s$.

48. Point O is the centre of this circle.

Determine the value of $n$ to the nearest tenth, if necessary.

49. Point O is the centre of this circle.

Determine the value of $m$ to the nearest tenth, if necessary.

50. Point O is the centre of this circle.

Determine the value of $a$ to the nearest whole number.

51. Label the major arc CD and the minor arc CD of this circle.

52. O is the centre of this circle. Is $\angle \mathrm{ACB}$ a central angle or an inscribed angle?

53. O is the centre of this circle.

What is the relationship between the measures of $\angle \mathrm{DOE}$ and $\angle \mathrm{DFE}$ ?

54. O is the centre of this circle.

In this circle, identify the inscribed angle and the central angle subtended by the same minor arc.

55. What is the relationship between the measures of $\angle \mathrm{WYX}$ and $\angle \mathrm{WZX}$ ?

56. Point O is the centre of the circle. Arc AB is a semicircle.

What is the measure of $\angle A O B$ ?

57. Point O is the centre of this circle.

Determine the values of $y^{\circ}$ and $z^{\circ}$.

58. Point O is the centre of the circle.

Determine the values of $b^{\circ}$ and $c^{\circ}$.

59. Point O is the centre of the circle.

Determine the values of $a^{\circ}$ and $c^{\circ}$.

60. Point O is the centre of the circle.

Determine the values of $y^{\circ}$ and $z^{\circ}$.


## Problem

61. A Ruppell's Griffon Vulture holds the record for the bird with the highest documented flight altitude. It was spotted at a height of about 11 km above the Earth's surface. The radius of Earth is approximately 6400 km . How far was the vulture from the horizon, H? Calculate this distance to the nearest kilometre.

62. A circular mirror with radius 27 cm hangs from a hook.

The wire is 46 cm long and is a tangent to the circle at points A and B .
How far, to the nearest tenth, above the top of the mirror is the hook?

63. When are two tangent lines to a circle parallel?

Draw a sketch to support your answer.
64. AC, AE, and CE are tangents to this circle. The points of tangency are: B, F, and D The circle has radius 11 . The distance from the centre of the circle to each vertex of the triangle is: $\mathrm{OC}=34, \mathrm{OA}=\mathrm{OE}=19$
Determine the side lengths of $\triangle \mathrm{ACE}$, to the nearest tenth.

65. AQ is a tangent to the circle with centre B and to the circle with centre C .

The points of tangency are P and Q .
Determine the value of $y$ to the nearest tenth.

66. Draw a point at the centre of this circle. Label the point O . How do you know your answer is correct?

67. a) In a circle, can a chord be longer than a diameter of the circle? Explain.
b) In a circle, can a chord be shorter than a radius of the circle? Explain.
68. This arc is part of a circle.

Explain how you could locate the centre of the original circle.

69. A circle has diameter 32 cm . How far from the centre of the circle, to the nearest centimetre, is a chord 20 cm long?
70. A pedestrian underpass is constructed using a cylindrical pipe of radius 2.6 m . The bottom of the pipe will be filled and paved. The headroom at the centre of the path is 3.9 m .
How wide is the path to the nearest tenth of a metre?

71. Determine the measure of each interior angle of quadrilateral ABCD .

72. Point O is the centre of the circle.

Determine the radius of the circle to the nearest tenth.
What circle property did you use?

73. Determine the values of $x^{\circ}$ and $y^{\circ}$.

What can you say about line segment AD?

74. Sheila is planning a shooting drill for a soccer team. She wants the soccer players to practice shooting on a net with a shooting angle of $20^{\circ}$. She has sketched this diagram.
Complete Sheila's sketch to show the curve or line along which she should have the players stand so their shooting angle is $20^{\circ}$.

75. Point O is the centre of the circle.

Determine the values of $x^{\circ}, y^{\circ}$, and $z^{\circ}$.


## circle

Answer Section

## MULTIPLE CHOICE

1. ANS: C PTS: 1 DIF: Easy

REF: 8.1 Properties of Tangents to a Circle LOC: 9.SS1
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
2. ANS: B PTS: 1 DIF: Easy

REF: 8.1 Properties of Tangents to a Circle LOC: 9.SS1
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
3. ANS: B PTS: 1 DIF: Easy

REF: 8.1 Properties of Tangents to a Circle LOC: 9.SS1
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
4. ANS: D PTS: 1 DIF: Easy

REF: 8.1 Properties of Tangents to a Circle LOC: 9.SS1
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
5. ANS: D PTS: 1 DIF: Easy

REF: 8.1 Properties of Tangents to a Circle LOC: 9.SS1
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
6. ANS: C PTS: 1 DIF: Easy

REF: 8.1 Properties of Tangents to a Circle LOC: 9.SS1
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
7. ANS: B PTS: 1 DIF: Easy

REF: 8.1 Properties of Tangents to a Circle LOC: 9.SS1
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
8. ANS: D PTS: 1 DIF: Moderate

REF: 8.1 Properties of Tangents to a Circle LOC: 9.SS1
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
9. ANS: D PTS: 1 DIF: Moderate

REF: 8.1 Properties of Tangents to a Circle LOC: 9.SS1
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
10. ANS: B PTS: 1 DIF: Moderate

REF: 8.1 Properties of Tangents to a Circle LOC: 9.SS1
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
11. ANS: CTS: 1 DIF: Easy REF: 8.2 Properties of Chords in a Circle

LOC: 9.SS1
12. ANS: A

LOC: 9.SS1
13. ANS: D

LOC: 9.SS1
14. ANS: C

LOC: 9.SS1
15. ANS: A

LOC: 9.SS1
16. ANS: B

LOC: 9.SS1

TOP: Shape and Space (Measurement)
PTS: 1 DIF: Easy
TOP: Shape and Space (Measurement)
DIF: Easy
TOP: Shape and Space (Measurement)
PTS: 1 DIF: Easy
TOP: Shape and Space (Measurement)
DIF: Easy
TOP: Shape and Space (Measurement)
PTS: 1 DIF: Easy
TOP: Shape and Space (Measurement)

KEY: Conceptual Understanding
REF: 8.2 Properties of Chords in a Circle
KEY: Conceptual Understanding
REF: 8.2 Properties of Chords in a Circle
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REF: 8.2 Properties of Chords in a Circle
KEY: Conceptual Understanding
REF: 8.2 Properties of Chords in a Circle
KEY: Conceptual Understanding
REF: 8.2 Properties of Chords in a Circle
KEY: Conceptual Understanding
17. ANS: A

LOC: 9.SS1
18. ANS: B

LOC: 9.SS1
19. ANS: B

LOC: 9.SS1
20. ANS: D

LOC: 9.SS1
21. ANS: C

LOC: 9.SS1
22. ANS: A

LOC: 9.SS1
23. ANS: D

LOC: 9.SS1
24. ANS: B

LOC: 9.SS1
25. ANS: B

LOC: 9.SS1
26. ANS: B

LOC: 9.SS1
27. ANS: A

LOC: 9.SS1
28. ANS: B

LOC: 9.SS1
29. ANS: D

LOC: 9.SS1
30. ANS: C

LOC: 9.SS1

## SHORT ANSWER

31. ANS:

AC
PTS: 1
LOC: 9.SS1
32. ANS:
$90^{\circ}$
PTS: 1
LOC: 9.SS1

PTS: 1 DIF: Easy REF: 8.2 Properties of Chords in a Circle
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
PTS: 1 DIF: Moderate
TOP: Shape and Space (Measurement)
REF: 8.2 Properties of Chords in a Circle
KEY: Conceptual Understanding
REF: 8.2 Properties of Chords in a Circle
KEY: Conceptual Understanding
REF: 8.2 Properties of Chords in a Circle
KEY: Conceptual Understanding
REF: 8.3 Properties of Angles in a Circle
KEY: Conceptual Understanding
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KEY: Conceptual Understanding
REF: 8.3 Properties of Angles in a Circle
KEY: Conceptual Understanding

DIF: Easy REF: 8.1 Properties of Tangents to a Circle
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding

DIF: Easy REF: 8.1 Properties of Tangents to a Circle
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
33. ANS:

Answers will vary. For example:

or


PTS: 1
DIF: Easy
REF: 8.1 Properties of Tangents to a Circle
LOC: 9.SS1
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
34. ANS:

or


PTS: 1
DIF: Moderate
REF: 8.1 Properties of Tangents to a Circle
LOC: 9.SS1
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
35. ANS:

Yes.


PTS: 1 DIF: Moderate REF: 8.1 Properties of Tangents to a Circle
LOC: 9.SS1
TOP: Shape and Space (Measurement)
KEY: Conceptual Understanding
36. ANS:
$v^{\circ}=65^{\circ}, w^{\circ}=35^{\circ}$
PTS: 1
DIF: Moderate REF: 8.1 Properties of Tangents to a Circle
LOC: 9.SS1
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
37. ANS:
$a^{\circ}=63^{\circ}, b^{\circ}=56^{\circ}$

PTS: 1
LOC: 9.SS1
DIF: Moderate REF: 8.1 Properties of Tangents to a Circle TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
38. ANS:
$s=20.6, t=37.2$

PTS: 1
LOC: 9.SS1
DIF: Moderate REF: 8.1 Properties of Tangents to a Circle
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
39. ANS:
$m=63.4, n^{\circ}=60^{\circ}$

PTS: 1
DIF: Moderate REF: 8.1 Properties of Tangents to a Circle
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
40. ANS:
$d=24.1, e^{\circ}=28^{\circ}$

PTS: 1
LOC: 9.SS1
41. ANS:

DE
PTS: 1
LOC: 9.SS1
42. ANS:

WX = XY

PTS: 1
LOC: 9.SS1
43. ANS:
$\angle \mathrm{OBC}=90^{\circ}$
PTS: 1
LOC: 9.SS1
44. ANS:
$x^{\circ}=61^{\circ}, y^{\circ}=29^{\circ}$
PTS: 1
LOC: 9.SS1
DIF: Easy
REF: 8.2 Properties of Chords in a Circle
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
45. ANS:
$c^{\circ}=34^{\circ}, d^{\circ}=112^{\circ}$
PTS: 1
LOC: 9.SS1

DIF: Easy
REF: 8.2 Properties of Chords in a Circle
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding

DIF: Easy
REF: 8.2 Properties of Chords in a Circle
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
DIF: Moderate REF: 8.1 Properties of Tangents to a Circle
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding

DIF: Easy REF: 8.2 Properties of Chords in a Circle
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding

TOP: Shape and (
46. ANS:

Answers may vary. For example:


PTS: 1 DIF: Easy REF: 8.2 Properties of Chords in a Circle
LOC: 9.SS1 TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
47. ANS:

Answers may vary. For example:


PTS: 1
LOC: 9.SS1
DIF: Easy
REF: 8.2 Properties of Chords in a Circle
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
48. ANS:
$n=4$
PTS: 1
LOC: 9.SS1
DIF: Moderate
REF: 8.2 Properties of Chords in a Circle
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
49. ANS:
$m=10.6$
PTS: 1
LOC: 9.SS1
DIF: Moderate REF: 8.2 Properties of Chords in a Circle
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
50. ANS:
$a=3$
PTS: 1
DIF: Moderate
REF: 8.2 Properties of Chords in a Circle
LOC: 9.SS1 TOP: Shape and Space (Measurement)

KEY: Conceptual Understanding
51. ANS:


PTS: 1 DIF: Easy REF: 8.3 Properties of Angles in a Circle
LOC: 9.SS1 TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
52. ANS:

Inscribed angle
PTS: 1 DIF: Easy REF: 8.3 Properties of Angles in a Circle
LOC: 9.SS1 TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
53. ANS:
$\angle \mathrm{DOE}=2 \angle \mathrm{DFE}$, or $\angle \mathrm{DFE}=\frac{1}{2} \angle \mathrm{DOE}$
PTS: 1 DIF: Easy REF: 8.3 Properties of Angles in a Circle
LOC: 9.SS1 TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
54. ANS:

Inscribed angle: $\angle \mathrm{PRQ}$
Central angle: $\angle \mathrm{POQ}$
PTS: 1 DIF: Easy REF: 8.3 Properties of Angles in a Circle
LOC: 9.SS1 TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
55. ANS:
$\angle \mathrm{WYX}=\angle \mathrm{WZX}$
PTS: 1
LOC: 9.SS1
DIF: Easy
REF: 8.3 Properties of Angles in a Circle
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
56. ANS:
$180^{\circ}$
PTS: 1 DIF: Easy REF: 8.3 Properties of Angles in a Circle
LOC: 9.SS1
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
57. ANS:
$y^{\circ}=68^{\circ}, z^{\circ}=136^{\circ}$
PTS: 1
LOC: 9.SS1
DIF: Easy
REF: 8.3 Properties of Angles in a Circle
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
58. ANS:
$c^{\circ}=43^{\circ}, b^{\circ}=44^{\circ}$
PTS: 1 DIF: Easy REF: 8.3 Properties of Angles in a Circle
LOC: 9.SS1 TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
59. ANS:
$a^{\circ}=39^{\circ}, c^{\circ}=51^{\circ}$
PTS: 1 DIF: Moderate REF: 8.3 Properties of Angles in a Circle
LOC: 9.SS1 TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
60. ANS:
$y^{\circ}=38^{\circ}, z^{\circ}=52^{\circ}$
PTS: 1
DIF: Moderate REF: 8.3 Properties of Angles in a Circle
LOC: 9.SS1
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding

## PROBLEM

61. ANS:

$$
\begin{aligned}
\mathrm{OV} & =11 \mathrm{~km}+6400 \mathrm{~km} \\
& =6411 \mathrm{~km} \\
\mathrm{OH} & =6400 \mathrm{~km}
\end{aligned}
$$

Use the Pythagorean Theorem in $\triangle \mathrm{OHV}$ to solve for HV.

$$
\begin{aligned}
\mathrm{HV}^{2} & =\mathrm{OV}^{2}-\mathrm{OH}^{2} \\
\mathrm{HV}^{2} & =6411^{2}-6400^{2} \\
\mathrm{HV}^{2} & =140921 \\
\mathrm{HV} & =\sqrt{140921} \\
\mathrm{HV} & \doteq 375.3944 \ldots
\end{aligned}
$$



The vulture was about 375 kilometres from the horizon.
PTS: 1 DIF: Moderate REF: 8.1 Properties of Tangents to a Circle
LOC: 9.SS1
62. ANS:

The distance from the centre of the mirror to the hook is: OT
So, the distance from the top of the mirror to the hook is: OT -27 cm
Solve for OT.
$\mathrm{OT}^{2}=27^{2}+23^{2}$
$\mathrm{OT}^{2}=1258$
$\mathrm{OT}=\sqrt{1258}$
$\mathrm{OT} \doteq 35.4682 \ldots$
So,
OT - 27 cm
$=35.4682 \ldots \mathrm{~cm}-27 \mathrm{~cm}$
$=8.4682 \ldots \mathrm{~cm}$
So, the hook is about 8.5 cm above the mirror.
PTS: 1 DIF: Moderate REF: 8.1 Properties of Tangents to a Circle
LOC: 9.SS1 TOP: Shape and Space (Measurement) KEY: Problem-Solving Skills
63. ANS:

Two tangent lines are parallel when they intersect a circle at opposite endpoints of a diameter.

For example, in this sketch, the tangent through Q is perpendicular to the diameter $P Q$, and the tangent through $P$ is also perpendicular to the diameter PQ , so the tangent lines are parallel.


PTS: 1 DIF: Moderate REF: 8.1 Properties of Tangents to a Circle
LOC: 9.SS1 TOP: Shape and Space (Measurement)
KEY: Conceptual Understanding | Communication
64. ANS:
$\mathrm{AC}=\mathrm{AB}+\mathrm{BC}$
Use the Pythagorean Theorem in $\triangle \mathrm{OAB}$ and $\triangle \mathrm{OBC}$ :

$$
\begin{array}{rlrl}
\mathrm{AB}^{2} & =\mathrm{OA}^{2}-\mathrm{OB}^{2} & \text { and } & \mathrm{BC}^{2}=\mathrm{OC}^{2}-\mathrm{OB}^{2} \\
\mathrm{AB}^{2} & =19^{2}-11^{2} & \mathrm{BC}^{2}=34^{2}-11^{2} \\
\mathrm{AB} & =\sqrt{19^{2}-11^{2}} & \mathrm{BC}=\sqrt{34^{2}-11^{2}} \\
\mathrm{AB} & =15.4919 \ldots & \mathrm{BC} & \doteq 32.1714 \ldots \\
\mathrm{So}, \mathrm{AC} & \doteq 15.4919 \ldots+32.1714 \ldots \\
& \doteq 47.6633 \ldots & &
\end{array}
$$

$\mathrm{AE}=\mathrm{AF}+\mathrm{FE}$
Use the Pythagorean Theorem in $\triangle \mathrm{OAF}$ and $\triangle \mathrm{OEF}$ :

$$
\begin{array}{clr}
\mathrm{AF}^{2}=\mathrm{OA}^{2}-\mathrm{OF}^{2} & \text { and } & \mathrm{FE}^{2}=\mathrm{OE}^{2}-\mathrm{OF}^{2} \\
\mathrm{AF}^{2}=19^{2}-11^{2} & & \mathrm{FE}^{2}=19^{2}-11^{2} \\
\mathrm{AF}=\sqrt{19^{2}-11^{2}} & \mathrm{FE}=\sqrt{19^{2}-11^{2}} \\
\mathrm{AF} \doteq 15.4919 \ldots & \mathrm{FE} \doteq 15.4919 \ldots
\end{array}
$$

$$
\mathrm{So}, \mathrm{AE} \doteq 15.4919 \ldots+15.4919 \ldots
$$

$$
\doteq 30.9838 \ldots
$$

$\mathrm{CE}=\mathrm{CD}+\mathrm{DE}$
Use the Pythagorean Theorem in $\triangle \mathrm{OCD}$ and $\triangle \mathrm{ODE}$ :

$$
\begin{array}{rlrl}
\mathrm{CD}^{2} & =\mathrm{OC}^{2}-\mathrm{OD}^{2} & \text { and } & \mathrm{DE}^{2}=\mathrm{OE}^{2}-\mathrm{OD}^{2} \\
\mathrm{CD}^{2} & =34^{2}-11^{2} & & \mathrm{DE}^{2}=19^{2}-11^{2} \\
\mathrm{CD} & =\sqrt{34^{2}-11^{2}} & \mathrm{DE}=\sqrt{19^{2}-11^{2}} \\
\mathrm{CD} & =32.1714 \ldots & \mathrm{DE} & =15.4919 \ldots \\
\mathrm{So}, \mathrm{CE} & \doteq 32.1714 \ldots+15.4919 \ldots \\
& \doteq 47.6633 \ldots & &
\end{array}
$$

The triangle has side lengths of about 47.7, 47.7, and 31.
PTS: 1 DIF: Moderate REF: 8.1 Properties of Tangents to a Circle LOC: 9.SS1 TOP: Shape and Space (Measurement) KEY: Problem-Solving Skills
65. ANS:

Use the Pythagorean Theorem in $\triangle \mathrm{ABP}$ to solve for AP .
$\mathrm{AP}^{2}=18^{2}-6^{2}$

$$
\begin{aligned}
& \mathrm{AP}=\sqrt{18^{2}-6^{2}} \\
& \mathrm{AP} \doteq 16.9706 \ldots
\end{aligned}
$$

$\triangle \mathrm{ABP} \cong \triangle \mathrm{ACQ}$
Consider $\triangle A C Q$ as an enlargement of $\triangle A B P$.
The scale ratio is:

$$
\begin{aligned}
\frac{\mathrm{CQ}}{\mathrm{BP}} & =\frac{12}{6} \\
& =2
\end{aligned}
$$

So, $\mathrm{AQ}=2(\mathrm{AP})$
Then,

$$
\begin{aligned}
y & =\mathrm{AQ}-\mathrm{AP} \\
& =2(\mathrm{AP})-\mathrm{AP} \\
& =\mathrm{AP}
\end{aligned}
$$

So, $y \doteq 17.0$
PTS: 1 DIF: Difficult REF: 8.1 Properties of Tangents to a Circle
LOC: 9.SS1

> TOP: Shape and Space (Measurement) KEY: Problem-Solving Skills
66. ANS:


I know that the centre of the circle lies along the perpendicular bisector of a chord. So, when two different perpendicular bisectors are drawn, the centre of the circle is the point where they intersect.

PTS: 1
DIF: Easy
REF: 8.2 Properties of Chords in a Circle
LOC: 9.SS1
TOP: Shape and Space (Measurement)
KEY: Problem-Solving Skills | Communication
67. ANS:
a) No. A chord joins two points on a circle. Given one point on a circle, the point farthest from that point is on the opposite side of the circle. The line connecting these two points passes through the centre of the circle, so it is a diameter.
b) Yes. For example, in this circle, chord AB is shorter than radius OC.


PTS: 1 DIF: Moderate REF: 8.2 Properties of Chords in a Circle
LOC: 9.SS1 TOP: Shape and Space (Measurement)
KEY: Problem-Solving Skills | Communication
68. ANS:

Draw two chords.
Construct the perpendicular bisectors of the chords.
The intersection of the perpendicular bisectors is the centre of the circle.
PTS: 1 DIF: Moderate REF: 8.2 Properties of Chords in a Circle
LOC: 9.SS1 TOP: Shape and Space (Measurement)
KEY: Problem-Solving Skills | Communication
69. ANS:

Sketch a diagram.
Let $d$ represent the distance from the chord to the centre of the circle.

Draw a radius from the centre to one end of the chord.

Label the known lengths.


PR is a chord of the circle, and OQ is perpendicular to the chord, passing through the centre of the circle, so $\mathrm{PQ}=\mathrm{QR}$ and QR is $\frac{1}{2}$ of PR :
$\mathrm{QR}=\frac{1}{2}(20 \mathrm{~cm})$

$$
=10 \mathrm{~cm}
$$

ST is a diameter of the circle, and OR is a radius of the circle, so OR is $\frac{1}{2}$ of ST:

$$
\begin{aligned}
\mathrm{ST} & =\frac{1}{2}(32 \mathrm{~cm}) \\
& =16 \mathrm{~cm}
\end{aligned}
$$

Use the Pythagorean Theorem in $\triangle O Q R$.

$$
\begin{aligned}
d^{2}+10^{2} & =16^{2} \\
d^{2} & =16^{2}-10^{2} \\
d^{2} & =156 \\
d & =\sqrt{156} \\
d & \doteq 12.4899 \ldots
\end{aligned}
$$



So, the chord is approximately 12 cm from the centre of the circle.
PTS: 1
DIF: Moderate REF: 8.2 Properties of Chords in a Circle
LOC: 9.SS1
TOP: Shape and Space (Measurement) KEY: Conceptual Understanding
70. ANS:

Draw a radius from the centre of the pipe, O , to an edge of the path, E.
Label the midpoint of the path F .
OE is a radius, so: $\mathrm{OE}=2.6 \mathrm{~m}$
$\mathrm{OF}=3.9 \mathrm{~m}-2.6 \mathrm{~m}$
$=1.3 \mathrm{~m}$
Use the Pythagorean Theorem in $\triangle \mathrm{OEF}$ to solve for EF .

$E F^{2}+1.3^{2}=2.6^{2}$

$$
\begin{aligned}
\mathrm{EF}^{2} & =2.6^{2}-1.3^{2} \\
\mathrm{EF}^{2} & =5.07 \\
\mathrm{EF} & =\sqrt{5.07} \\
\mathrm{EF} & =2.2516 \ldots
\end{aligned}
$$

The width of the path is twice the length of EF.
$2(2.2516 \ldots)=4.5033 \ldots$
So, the width of the path is about 4.5 m .
PTS: 1
DIF: Difficult REF: 8.2 Properties of Chords in a Circle
LOC: 9.SS1
TOP: Shape and Space (Measurement) KEY: Problem-Solving Skills
71. ANS:

AC is a diameter of the circle, so $\angle \mathrm{ABC}=90^{\circ}$ and $\angle \mathrm{ADC}=90^{\circ}$.
The sum of the interior angles of a triangle is $180^{\circ}$. So, in $\triangle \mathrm{ABC}$ : $43^{\circ}+90^{\circ}+\angle \mathrm{ACB}=180^{\circ}$

$$
\begin{aligned}
133^{\circ}+\angle \mathrm{ACB} & =180^{\circ} \\
\angle \mathrm{ACB} & =180^{\circ}-133^{\circ} \\
\angle \mathrm{ACB} & =47^{\circ}
\end{aligned}
$$

So, $\angle \mathrm{BCD}=47^{\circ}+24^{\circ}$

$$
=71^{\circ}
$$

The sum of the interior angles of a triangle is $180^{\circ}$. So, in $\triangle \mathrm{ACD}$ :
$24^{\circ}+90^{\circ}+\angle \mathrm{CAD}=180^{\circ}$

$$
114^{\circ}+\angle \mathrm{CAD}=180^{\circ}
$$

$\angle \mathrm{CAD}=180^{\circ}-114^{\circ}$
$\angle \mathrm{CAD}=66^{\circ}$
So, $\angle \mathrm{BAD}=66^{\circ}+43^{\circ}$

$$
=109^{\circ}
$$

So, the interior angles of quadrilateral ABCD have these measures: $\angle \mathrm{ABC}=90^{\circ}, \angle \mathrm{BCD}=71^{\circ}, \angle \mathrm{ADC}=90^{\circ}, \angle \mathrm{BAD}=109^{\circ}$

PTS: 1
DIF: Moderate REF: 8.3 Properties of Angles in a Circle
LOC: 9.SS1
72. ANS:

Use the Angles in a Semicircle Property: FG is a diameter of the circle, so $\angle \mathrm{FHG}=90^{\circ}$.
Use the Pythagorean Theorem in right $\triangle \mathrm{FGH}$ to determine the length of the hypotenuse, or diameter, $d$.
$d^{2}=8^{2}+7^{2}$
$d^{2}=113$
$d=\sqrt{113}$
$d \doteq 10.6301 \ldots$
So, the diameter of the circle is approximately 10.6.
The radius of the circle is $\frac{1}{2}$ the diameter.
$\frac{1}{2}(10.6301 \ldots) \doteq 5.3150 \ldots$
So, the radius of the circle is approximately 5.3.
PTS: 1 DIF: Moderate REF: 8.3 Properties of Angles in a Circle
LOC: 9.SS1 TOP: Shape and Space (Measurement)
KEY: Problem-Solving Skills | Communication
73. ANS:

Since $\angle \mathrm{ACB}$ and $\angle \mathrm{ADB}$ are inscribed angles subtended by the same arc AB , they are congruent. $x^{\circ}=37^{\circ}$

The sum of the interior angles of a triangle is $180^{\circ}$. So, in $\triangle \mathrm{ABD}$ :
$53^{\circ}+37^{\circ}+y^{\circ}=180^{\circ}$

$$
\begin{aligned}
90^{\circ}+y^{\circ} & =180^{\circ} \\
y^{\circ} & =180^{\circ}-90^{\circ} \\
y^{\circ} & =90^{\circ}
\end{aligned}
$$

Since arc AD subtends right $\angle \mathrm{ABD}$, arc AD is a semicircle and line segment AD is a diameter of the circle.
PTS: 1 DIF: Moderate REF: 8.3 Properties of Angles in a Circle
LOC: 9.SS1 TOP: Shape and Space (Measurement)
KEY: Problem-Solving Skills | Communication
74. ANS:

Label the endpoints of the goal, R and S , and the third point O .
Construct a circle with radius OR about point O.
Sheila should have the players stand along the major arc RS.


PTS: 1 DIF: Moderate REF: 8.3 Properties of Angles in a Circle LOC: 9.SS1 TOP: Shape and Space (Measurement)
KEY: Problem-Solving Skills | Communication
75. ANS:

The sum of the central angles in a circle is $360^{\circ}$.
$133^{\circ}+107^{\circ}+x^{\circ}=360^{\circ}$

$$
\begin{aligned}
240^{\circ}+x^{\circ} & =360^{\circ} \\
x^{\circ} & =360^{\circ}-240^{\circ} \\
x^{\circ} & =120^{\circ}
\end{aligned}
$$

$\angle A C B$ is an inscribed angle and $\angle A O B$ is a central angle subtended by the same arc.
So, $\angle \mathrm{ACB}=\frac{1}{2} \angle \mathrm{AOB}$

$$
\begin{aligned}
& y^{\circ}=\frac{1}{2} \times 120^{\circ} \\
& y^{\circ}=60^{\circ}
\end{aligned}
$$

OA and OB are radii, so $\triangle \mathrm{AOB}$ is isosceles with
 $\angle \mathrm{OAB}=\angle \mathrm{OBA}=z^{\circ}$.
The sum of the angles in a triangle is $180^{\circ}$, so in $\triangle \mathrm{AOB}$ :
$z^{\circ}+z^{\circ}+120^{\circ}=180^{\circ}$

$$
\begin{aligned}
2 z^{\circ}+120^{\circ} & =180^{\circ} \\
2 z^{\circ} & =180^{\circ}-120^{\circ} \\
2 z^{\circ} & =60^{\circ} \\
z^{\circ} & =\frac{60^{\circ}}{2} \\
z^{\circ} & =30^{\circ}
\end{aligned}
$$

PTS: 1
LOC: 9.SS1
DIF: Difficult REF: 8.3 Properties of Angles in a Circle
TOP: Shape and Space (Measurement) KEY: Problem-Solving Skills

