Teaching Through Problems Worth Solving
- Grade 8 (Version 1.0) -

Inquiry-based, Curriculum-linked, Differentiated Math Problems for Grade 8

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<u>A 21st Century Learning Promise</u>: I promise to do all I can to keep the spark of curiosity, creativity, and learning alive in every child; to help all children discover their talents, develop their passions, deepen their understanding, and apply all this to helping others, and to creating a better world for us all.
-author unknown

The inspiration for this compilation of problems has come from many sources.
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Read This First

This resource is the result of a year-long collaborative project to identify and compile problems that align with the grade 8 curriculum outlined by the Alberta Mathematics Program of Studies (2007). It is an initial attempt to answer our essential question, "**How can teaching through problem solving engage every student and drive learning forward?**"

This resource is not meant as a bank of worksheets to be given arbitrarily to students. Rather, it is designed to be a journey through problem solving for the entire math classroom. Problems worth solving take time. Some problems may take only one block, others will take longer. Use your professional judgment to choose your problems, guide your teaching, and facilitate student learning. The focus is meant to be on the experience of the problem solving process - the thinking, the connections, and the understanding. Sample solutions are provided as a single example of many possible problem solving strategies. Our intent is for you the teacher to be deeply involved in the problem solving process with your students and hopefully with your colleagues. Take risks, make mistakes, and don't worry as much about the destination as about the journey. Complement these problems with mini-lessons, games, and projects to teach the Grade 8 Program of Studies.

This is our first draft. Some outcomes have more problems linked to them than others. This project is ongoing; it will continue to be tested with students and improved. We also have plans to translate the problems into French in the near future.

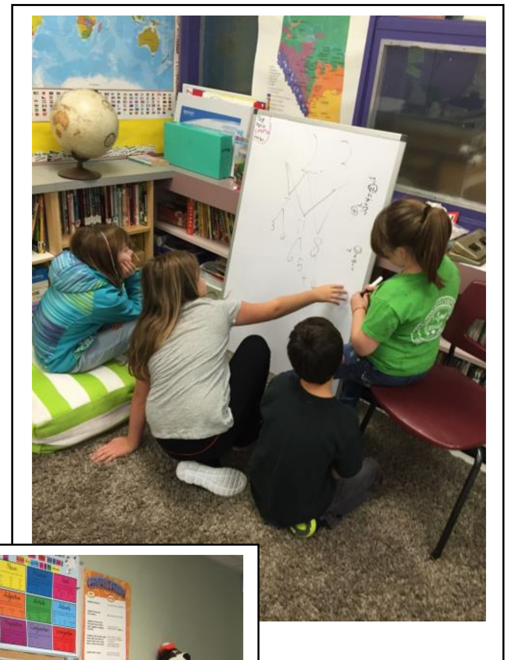
A Problem Solving Classroom

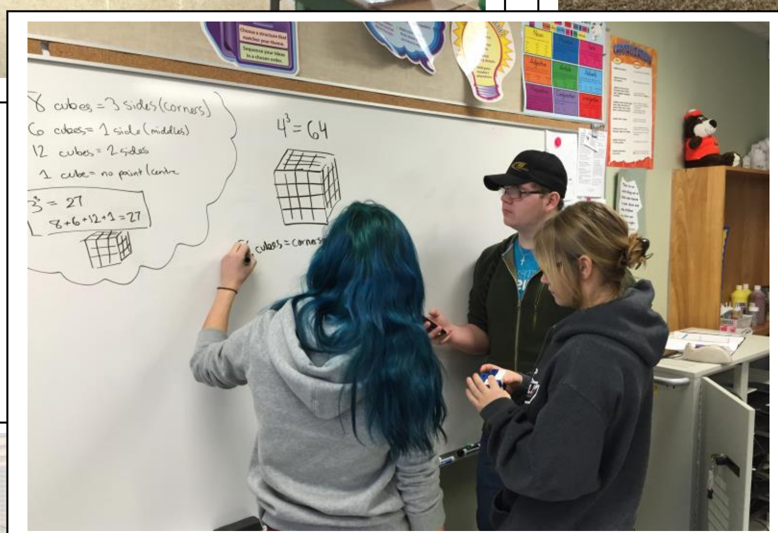
"Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk takers. Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type How would you …? or How could you …?, the problem-solving approach is being modeled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies" (Alberta Mathematics **Program of Studies, 2007).**

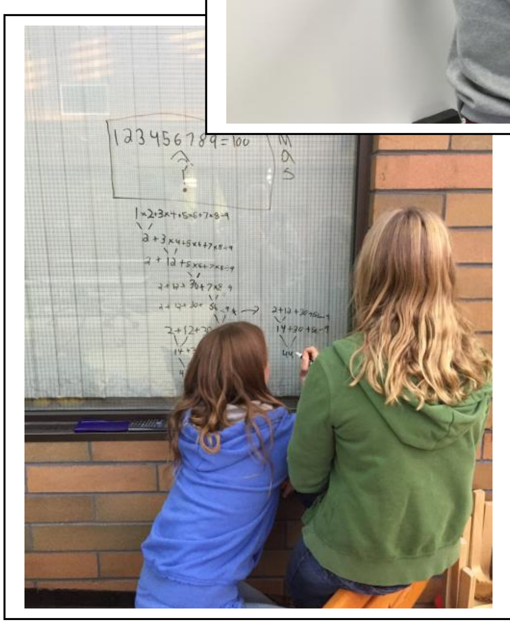
Teaching through problem solving is about inviting students to think about mathematics, to take risks, and to persevere. Collaboration is the key component of problem solving. Students need to be working together, sharing strategies, and learning from one another. The role of the teacher is to inspire, facilitate, and regulate. No telling, no showing, no giving answers. Your job is to motivate, question, and direct attention to big ideas!

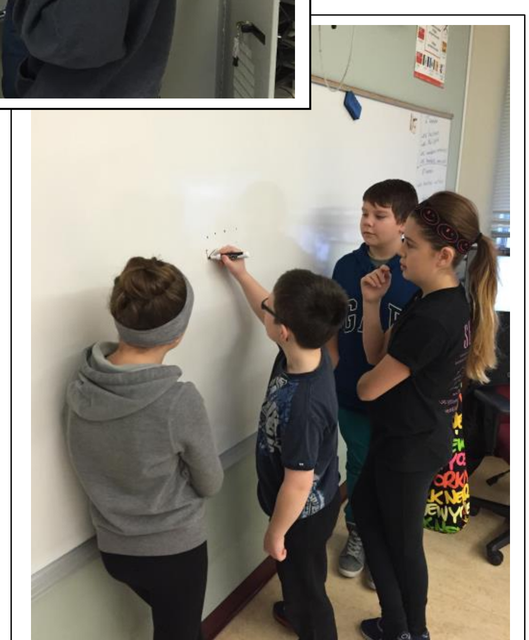
Problem solving is our focus and problem solving is our lesson. This collection includes low- floor, high-ceiling problems with multiple entry points enabling all students to access and experience success with the problems. In our experience, teaching through problem solving levels the playing field. Students will struggle; this struggle will help them deepen their understanding and expand their skills. Problem solving gives the chance for all learners to be creative, think outside the box, and have a voice.

"Coming to know something is not a spectator sport although numerous textbooks, especially in mathematics, and traditional modes of instruction may give that impression" (Brown and Walter, The Art of Problem Posing).





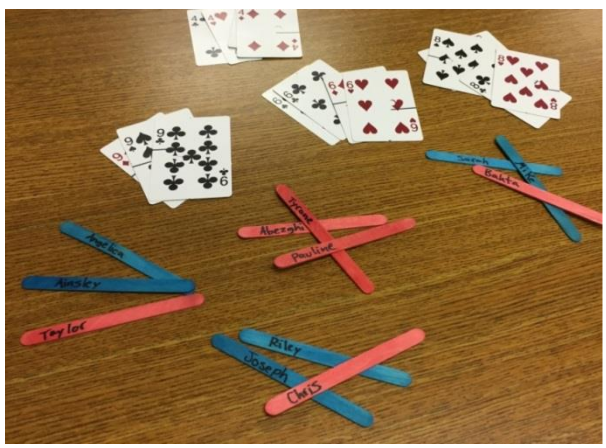




<u>Getting Started With Students</u>
Random Groups + Non-Permanent Pen + Vertical Surfaces + Group Work, Collaboration, Communication + Different Skills and Strategies = A Thinking Classroom

Students should work in **random groups**. This can be done using Popsicle sticks, a deck of cards, the random group generator on the Smart Board, etc. This will teach students how to work with everybody and anybody. This helps break down social barriers and nurtures a learning community in which students feel safe to take risks and make mistakes. This also helps prevent students from being labeled and grouped based on their “pre-conceived” mathematical abilities. If it is always random, it is always fair; the students know that the groups will always change and that they are expected to be able to work with everybody.

(For more information, please read THE AFFORDANCES OF USING VISIBLY RANDOM GROUPS IN A MATHEMATICS CLASSROOM by Peter Liljedahl, Simon Fraser University, Canada – In press)



Students should work at **vertical surfaces**. This allows everyone to have access to the workspace. It also allows for the teacher to easily see how each group is working, and who needs some direction, motivation, or extra help. Vertical surfaces are easily accessible by teachers for formative assessment. By standing in the middle of the room, it is possible to see where everybody is at. It allows both students and teachers to see at-a-glance the problem solving process, identify misconceptions, direct questioning, redirect the students, motivate group work, plan for discussions, mini-lessons and future lessons. Students’ initial work should be on a **non-permanent** surface which encourages the risk-taking necessary for true problem solving. The non-permanence of the surface allows students to make mistakes without any long term consequences.

Whiteboards, windows, lockers, filing cabinets, shower curtains, shelf liner, writeable paint, table and desktops, and interactive whiteboards are a few examples of non-permanent vertical surfaces. Be sure to check the surface to ensure that the dry erase marker comes off prior to students writing on it.

Students need to develop and practice **group work, collaboration, and communication skills**. They need to learn how to listen to each other, to share their ideas, to question, and to trust their abilities and the abilities of others. **Different skills and strategies** need to be embraced while helping each other to create a safe learning environment.

(For more information, please read BUILDING THINKING CLASSROOMS: CONDITIONS FOR PROBLEM SOLVING by Peter Liljedahl, Simon Fraser University, Canada – In press)

Suggestions for Teaching Through Problem Solving

Group sizes depend on the teacher, the students, and the specific problem. We like 3 as a rule, but often have groups of 4 and occasionally students work in partners.

Students solve their problems in random groups at a vertical surface. There is only **one pen per group and it must be shared**. The person with the pen is not allowed to write down his or her ideas. Remind them not to hog the pen! This helps keep the groups working together.

Gallery Walks / Mobilization of knowledge: encourage the students to walk around the classroom to see other groups for ideas, to see different strategies, to get unstuck. This is also a great way to provide feedback instigate new discussions, and direct your teaching.

When you want to utilize a specific group's work to discuss a strategy, some specific math, misconceptions, etc, first **move all of the students to the center of the room, away from the work** in order to remove ownership of it (alleviate fear, embarrassment, etc.). Then move students back to the work to discuss it.

Encourage students to work together to work through the problem and get an answer. Tell them to work with their answer to see if they can find a more elegant way, to use a different strategy, to explain their ideas, and to **present their solution**.

Use non-traditional assessments such as observations, checklists, posters, videos, photos of work, written solutions that tell the story of how the problem was solved, etc. This can be done individually or in partners or groups. This allows students to show their problem solving process, to explain their thinking, and to showcase their understanding. Students can "present" their solutions as a group, in partners, or individually depending on what the teacher is assessing or needs to see.

Remember that problem solving takes practice. The more "traditional" learners may struggle to communicate and collaborate. It may take practice listening to other people's ideas and strategies. It can be frustrating working in groups and some students may find it difficult to explain their ideas. Many students lack confidence in math as well as in problem solving. **Students need to be taught how to think, how to collaborate, how to communicate, how to problem solve, and how to persevere.**

What is a Problem Worth Solving?

A problem worth solving is accessible to all students. It has multiple entry points, has a low floor, wide walls, and a high ceiling. These problems lend themselves to natural differentiation where all students are able to address the problem at their level and experience success. A problem worth solving allows the use of multiple strategies and varying facets of mathematics.

“A problem-solving activity must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement”.

(Alberta Mathematics Program of Studies, 2007).

The **Painted Cube Problem is an exemplary problem** and it is our favourite example of a problem worth solving.

The Painted Cube N1,PR2,SS3,SS4

Problem:

Picture a Rubik's Cube. Now drop it into paint so that it is completely covered. When the paint is dry, imagine smashing it on the floor and it breaking it apart into the smaller cubes.

How many of the cubes have one face covered in paint? How many cubes have two faces covered in paint? How many have three faces covered in paint? How many have zero faces covered in paint?

How could you predict the above for any size Rubik's cube?

What about a 4 x 4 x 4? 5 x 5 x 5? 6 x 6 x 6? N x N x N?

Extension:

What if it wasn't a cube?
Why is the "one by one by one" cube a special case?

The Painted Cube N1,PR2,SS3,SS4

Outcome Objectives:

Number 1- Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers).

Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

- $ax = b$
- $\frac{x}{a}=b$, $a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c$, $a \neq 0$
- $a(x + b) = c$

where a, b and c are integers. [C, CN, PS, V]

Shape and Space (3-D Objects and 2-D Shapes) 3- Determine the surface area of: • right rectangular prisms• right triangular prisms• right cylinders to solve problems. [C, CN, PS, R, V]

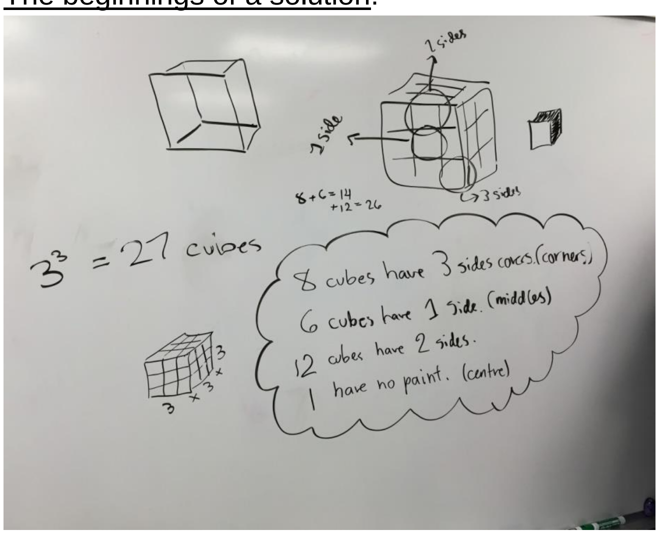
Shape and Space (3-D Objects and 2-D Shapes) 4- Develop and apply formulas for determining the volume of right rectangular prisms, right triangular prisms and right cylinders.

Material Suggestions:

- Snap cubes
- Vertical surfaces
- Dry-erase markers

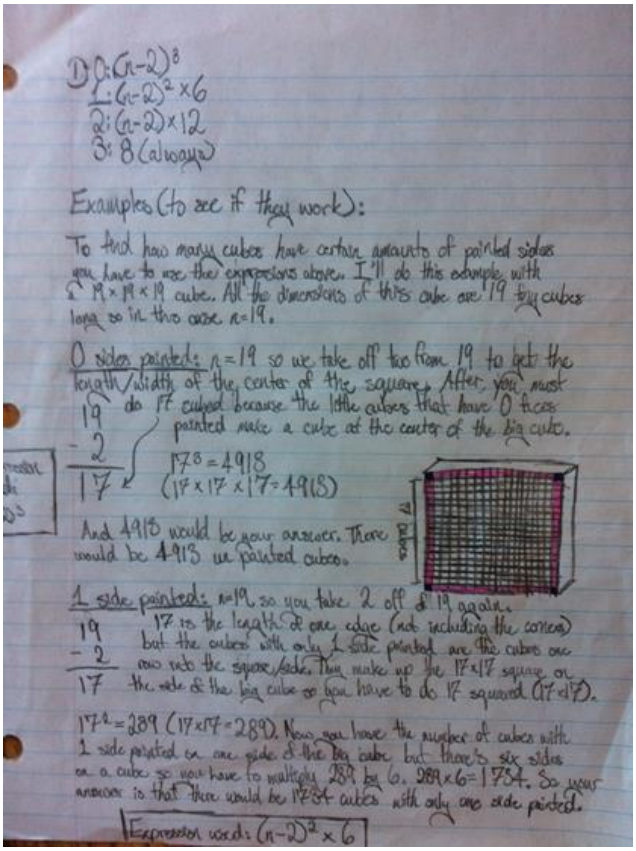
Sample Solutions:

<u>The beginnings of a solution</u>:



The Painted Cube N1,PR2,SS3,SS4

Student's Written Solution:


1) 0: $(n-2)^3$
1: $(n-2)^2 \times 6$
2: $(n-2) \times 12$
3: 8 (always)

Examples (to see if they work):

To find how many cubes have certain amounts of painted sides you have to use the expressions above. I'll do this example with a 19 × 19 × 19 cube. All the dimensions of this cube are 19 tiny cubes long so in this case n=19.

0 sides painted: n=19 so we take off two from 19 to get the length/width of the center of the square. After, you must do 17 cubed because the little cubes that have 0 faces painted make a cube at the center of the big cube.

19
− 2
17

$17^3 = 4913$
(17 × 17 × 17 = 4913)

And 4913 would be your answer. There would be 4913 un painted cubes.

17 cubes

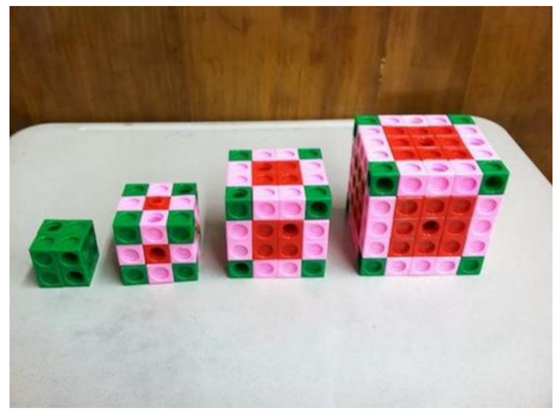
1 side painted: n=19 so you take 2 off of 19 again. 17 is the length of one edge (not including the corners) but the cubes with only 1 side painted are the cubes one row into the square/side. They make up the 17×17 square on the side of the big cube so you have to do 17 squared (17×17).

19
− 2
17

$17^2 = 289$ (17×17=289). Now you have the number of cubes with 1 side painted on one side of the big cube but there's six sides on a cube so you have to multiply 289 by 6. 289×6=1734. So your answer is that there would be 1734 cubes with only one side painted.

Expression used: $(n-2)^2 \times 6$

A Visual Representation of a Solution:



Mathematics related to the coloring of the cubes will emerge:

- Cubes with 3 faces painted: 8

These are always in the corners and there are always 8 (except on a size 1 cube).

- Cubes with 2 faces painted: $12(n - 2)$

These are always along the edges but not on the corners, so on each edge, there are 2 less than the size of the cube. There are 12 edges, so the number needs to be multiplied by 12.

- Cubes with 1 face painted: $6(n-2)^2$

These are in the middle of each face. They are in the shape of a square, two sizes smaller than the face of the original cube. There are 6 faces, so the number needs to be multiplied by 6.

- Cubes with no faces painted: $(n-2)^3$

These are always in the middle. They form a cube shape that is two sizes smaller than the original cube.

The Painted Cube N1,PR2,SS3,SS4

Notes:

This problem not only illustrates linear relationships, but also introduces and reinforces the idea that algebraic relations come from real situations and that can and should be visualized. Students can also graph the different relations.

Assessment idea: Have students initially solve the problem within a group, but individually write up their own solution which explain the process, tells the story of how the problem was solved, and explains the mathematics!

**Credit to David Pimm*

How to Use This Resource

Section 1: Pages 15-61
Problems to Create a Thinking Classroom (Problems to Target the Front Matter)

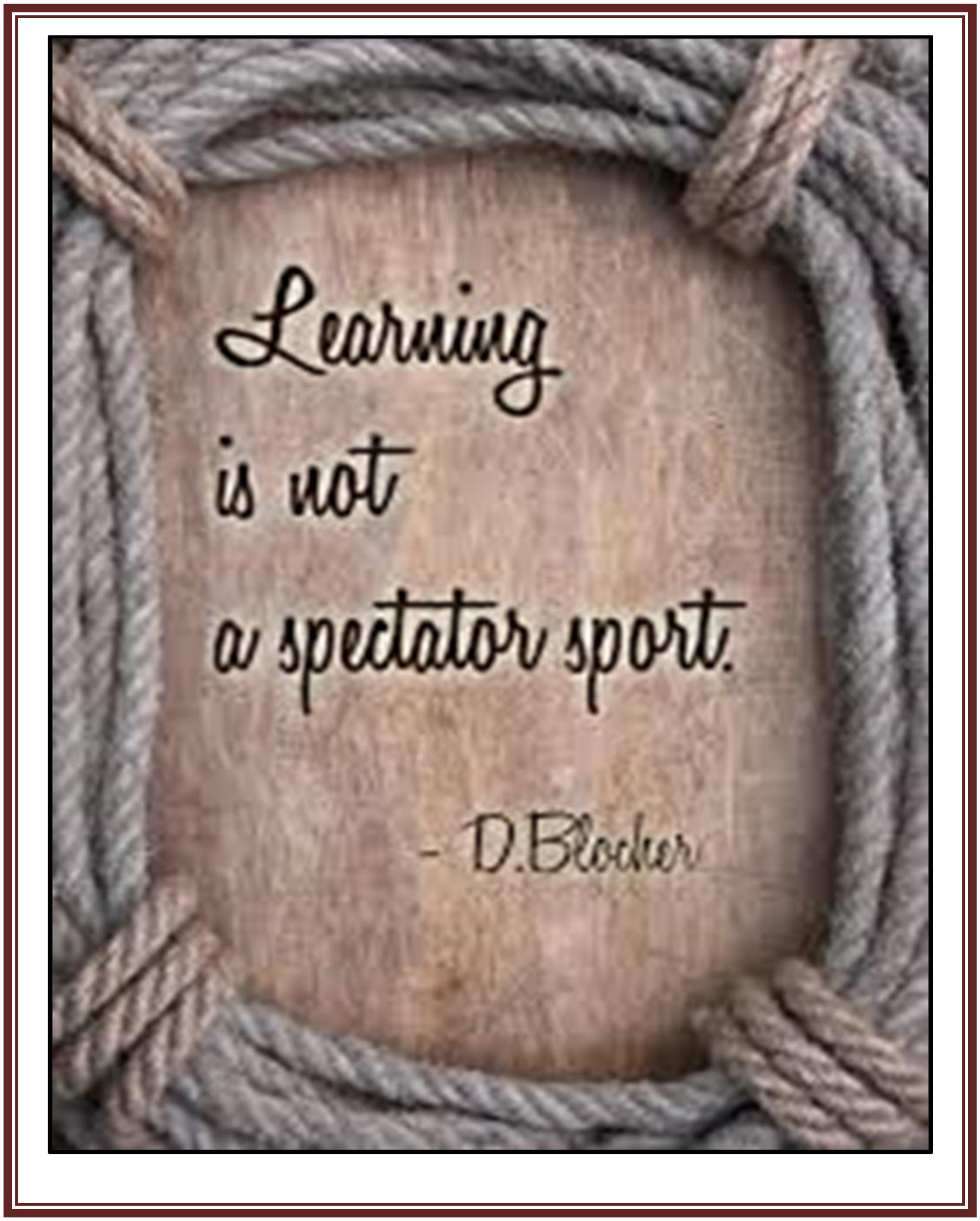
The ***first section*** of problems is meant to be used to create a **thinking classroom.** Use these problems to teach students how to solve problems. These problems are included to address the Front-Matter of the curriculum as well as previous math concepts and outcomes. Students will go through the processes of learning to communicate, collaborate, reason, visualize, take risks, and persevere. The math classroom should become a culture of respect, responsibility, and thinking. Continue using these problems throughout the year! ***No sample solutions or curriculum links are provided for these problems. We want the teachers to learn with the students, take risks, make mistakes, and persevere! Feel free to google answers as a last resort if you really get stuck.**

Section 2: Pages 62-156
Problems to Target Curricular Outcomes (as Well as the Front Matter)

The ***second section*** of problems continues to build on these skills. These problems are **linked to the specific outcomes** in the grade 8 curriculum. As you are planning your lessons, select the problems in section two to best support your practice and to satisfy the needs of your students.

As you work your way through the junior high math curriculum by teaching through problem solving, please contact us with feedback, new ideas, exemplary problems, sample student solutions, etc. at stone_alicia44@hotmail.com.

Problems to Create a Thinking Classroom



(Problems to Target the Front Matter)

Problems to Create a Thinking Classroom

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38	Same Sum Problem	61	Ball of Yarn

3 Questions

Problem:

In a group, solve these 3 problems:

(1) A bat and a ball cost $1.10 in total. The bat costs $1.00 more than the ball. How much does the ball cost?

(2) If it takes 5 machines 5 minutes to make 5 widgets, how long would it take 100 machines to make 100 widgets?

(3) In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half of the lake?

Extension:

Can you create another problem in which your intuition may give you a wrong answer?

Notes:

**Credit to The Journal of Economic Perspectives, Vol. 19, No. 4 (Autumn, 2005), pp. 25-42*

The Bell Boy and the Missing Dollar (150 year old puzzle)

Problem:

Three men check into a hotel. The cost of the room is $30. They pay $10 each and check in. The manager discovers that he overcharged them because the cost of the room is actually only $25. The manager gives the bellboy 5 loonies and tells him to return the money to the men. The Bell boy puts two loonies in his pocket and gives the men back one dollar each. Now each man has paid $9 and the bellboy has two. That adds up to $29. Where is the missing dollar?

Extension:

Notes:

Make 100

Problem:

Given the digits 1-9, make 100 using standard arithmetical symbols.

Extension:

Notes:

http://www.quora.com/Mathematical-Puzzles/Can-you-make-100-out-of-the-digits-1-2-3-4-5-6-7-8-9-in-order

**Credit to Ian Stewart*

One to One Hundred

Problem:

Johann Friederich Carl Gauss was a mathematician born in Germany, on April 30, 1777. His parents were poor, and his father expected him to become a bricklayer or a gardener in the family tradition. Were it not for his strong mother and a persistent uncle, Gauss might not have received an education. In college, Gauss studied many subjects, but committed himself to mathematics as a lifelong pursuit. He was not interested in fame or wealth, but studied and explored primarily for personal satisfaction. For nearly 50 years, Gauss was professor of mathematics and astronomy and director of the observatory at the University of Göttingen. The "Prince of Mathematicians," as Gauss was sometimes called, died February 23, 1855, at age 78. With Archimedes and Newton, he is considered one of the three greatest mathematicians who ever lived.

When Gauss was about 10, his teacher angrily assigned the class of boys a long problem: they should add the first one hundred numbers. Gauss was finished with the problem in a matter of seconds. The teacher was outraged. When the slates were checked, Gauss's sum was correct.

How did Gauss solve this problem?

Extension:

Could you find the sum of the 50 first odd numbers?

How about the average of the first two even numbers? The first three even numbers? The first four even numbers? The first 50 even numbers?

The average of *n* even numbers?

Notes:

**Credit to Wilbert Reimer, Historical Connections in Mathematics, Vol 1*

Bees in the Trees

Problem:

The family tree of a male bee is both unusual and interesting. The male bee is created through a process known as parthenogenesis, whereby he has a single parent, only a mother: The female bee, however; has both a mother and a father: What patterns can you find by tracing the ancestry of a male bee?

Extension:

What if…you examined the ancestry of a species in which both males and females have two parents? How would the patterns be different from those you found with the bumblebees?

Notes:

**Credit to The Super Source: Patterns and Functions, grades 7-8*

Palindromes

Problem:

A palindrome is a number, word, phrase, or sequence that reads the same backward as forward, e.g., madam or 363

Consider a two-digit number – for example 84. 84 is not a palindrome. So, reverse the digits and add it to the original number – 84 + 48 = 132. Repeat this process until the sum becomes a palindrome. 132 + 231 = 363. The number of times the process is repeated determines the depth of the palindrome. For 84, the depth is two. Find the depth of all two digit numbers.

Extension:

What about a three digit number?

Notes:

http://www.magic-squares.net/palindromes.htm

**Credit to Peter Liljedahl*

Marching Band

Problem:

Students in a marching band want to line up for their performance. The problem is that when they line up in twos there is 1 left over. When they line up in threes there are 2 left over. When they line up in fours there are 3 left over. When they line up in fives there are 4 left over. When they line up in sixes there are 5 left over. When they line up in sevens there are no students left over. How many students are there?

Extension:

Notes:

**Credit to John Grant McLoughlin*

Tic-Tac-Toe to 15

Problem:

Using the numbers 1 2 3 4 5 6 7 8 9

Alternate between partners to pick one number at a time.

Once a number is picked, it is gone.

The goal is to have 3 numbers that add to 15.

Extension:

Notes:

**Credit to Peter Liljedahl*

4 Fours

Problem:

Can you make the numbers 1 through 10 by using only 4 fours and any operations?

Extension:

Notes:

**Credit to Jo Boaler*

How to Win at 21

Problem:

Play with a partner. You need 21 snap cubes or other objects. The goal is to make your partner take the last object.
Snap the 21 cubes together in a chain. The first player takes off 1, 2, or 3 cubes off the chain. Then your partner can take off 1, 2, or 3. The person who has to take the last cube loses.
Play multiple games. Figure out how to win the game!

Extension:

Play again but this time you can take 2, 3, or 4 cubes each turn.

Notes:
See You Tube Video:
https://youtu.be/XD_HRBugh34

Frame the Cards

Problem:

Arrange the cards from the ace to the ten into a picture frame so that each the top, bottom, and sides add to the same total of spots (hearts/diamonds…) Right now the top row adds to 23, the bottom adds to 12, the left side is 22 and the right side is 22. Apparently there are 10 solutions to this problem.

Extension:

Notes:

**Credit to Ian Stewart*

30 Scratch

Problem:

Roll a die to choose 4 digits from 2-9 e.g. 3 5 7 9
Use these digits in combination with any operation to make the numbers 1-30.

There are three basic "not allowed rules":

- Digits cannot be used twice. So 3 x 5 =15 and then take away 3 would not be acceptable for 12.
- Digits cannot be put together to make two-digit numbers, so 3 and 5 cannot be used to write 35
- Only basic operations are allowed (no square roots, exponents, etc.)….**brackets are allowed**.

Extension:

What if we did allow other operations?

Notes:

**Credit to John Grant McLoughlin*

Free Throw Problem

Problem:

Bill, Sam and Rob all play for the school basketball team and are very consistent free throw shooters (although not necessarily consistently good).

Bill always misses one shot then makes one shot.
Sam always misses two shots then makes one shot.
Rob always misses three shots then makes one shot.

The three boys are practicing their free throws in rounds where each gets one shot per round. Predictably, all three boys miss in the first round and each of them stays on their usual pattern for the rest of the rounds.
After **16 rounds** they stop to compare the baskets they have made in each round.

What if they played more than 6 rounds? What about 57 rounds?

Extension:

What questions do you want to answer? Answer them!

Examples of Questions from Students:

In how many rounds did Bill make his shot?
How many times did both Sam and Rob make their shot in the same round?
How many times did all three players make shots in the same round?
How many times did all three players miss their shot in the same round?

Notes:
**Credit to Geri Lorway*

Seed Numbers

Problem:

Consider the following pattern of 5 **whole** numbers, where each number is the sum of the previous two numbers:

3, 12, 15, 27, 42

I want the 5th number to be 100.

Find all the **whole seed numbers that will make this so (the seed numbers are the first two whole numbers).**

Extension:

Notes:

**Credit to Peter Liljedahl*

Egg Timer

Problem:

You have 2 egg timers, a 7 min and a 4 min timer and you want to boil a 9 minute egg. How do you time it exactly?

Extension:

Notes:

**Credit to Rina Zazkis*

1001 Pennies

Problem:

On a table, there are 1001 pennies lined up in a row. I then come along and replace every second coin with a nickel. After this, I replace every third coin with a dime. Finally, I replace every fourth coin with a quarter. After all this, how much money is on the table?

Extension:

Notes:

Crossing the Bridge

Problem:

Four people are being pursued by a menacing beast. It is nighttime, and they need to cross a bridge to reach safety. It is pitch black, and only two can cross at once. They need to carry a lamp to light their way. The first person can cross the bridge in no less than 10 minutes, the second in 5 minutes, the third in 2 minutes, and the fourth in 1 minute. If two cross together, the couple is only as fast as the slowest person. (That is, a fast person can't carry a slower person to save time, for example. If the 10-minute person and the 1-minute person cross the bridge together, it will take them 10 minutes.) The person or couple crossing the bridge needs the lamp for the entire crossing and the lamp must be carried back and forth across the bridge (no throwing, etc.) How long will it take them to get across? Is this the shortest time possible?

Extension:

What is the best possible solution?

Notes:

**Credit to http://puzzles.nigelcoldwell.co.uk/twentyfive.htm*

Spell the Cards

Problem:

Rig the cards (Ace to King) so that you spell the cards (see YouTube video).

http://www.youtube.com/watch?v=qDgcLAGgX2A

Order of cards	Spelling
1.	Ace
2.	Two
3.	Three
4.	Four
5.	Five
6.	Six
7.	Seven
8.	Eight
9.	Nine
10.	Ten
11.	Jack
12.	Queen
13.	King

Extension: Could you rig the whole deck? What about in another language?

Notes: *only for the teacher*

Rig the deck first and perform the card trick for your students. Ask "how".

Queen, Four, Ace, Eight, King, Two, Seven, Five, Ten, Jack, Three, Six, Nine

**Credit to Peter Liljedahl*

The Silver Bar

Problem:

A silver prospector was unable to pay his March rent in advance. He owned a bar of pure silver, 31 inches long, so he made the following arrangement with his landlady. He would cut the bar, he said, into smaller pieces and pay her in silver (one inch per day). On the first day of March he would give the lady an inch of the bar, and on each succeeding day he would add another inch to her amount of silver. He doesn't want to cut the bar into 31 pieces because it required considerable labor – he wished to carry out his agreement with the fewest possible number of pieces. Assuming that portions of the bar can be traded back and forth, what is the smallest number of pieces in which the prospector needs to cut his silver bar?

Extension:

Notes:

**Credit to Martin Gardner*

Hoax (Apparently this was a joke)

Problem:

29
AUG

Samsung pays Apple $1 Billion sending 30 trucks full of 5 cent coins

PaperBlog – This morning more than 30 trucks filled with 5-cent coins arrived at Apple's headquarters in California. Initially, the security company that protects the facility said the trucks were in the wrong place, but minutes later, Tim Cook (Apple CEO) received a call from Samsung CEO explaining that they will pay $1 billion dollars for the fine recently ruled against the South Korean company in this way.

The funny part is that the signed document does not specify a single payment method, so Samsung is entitled to send the creators of the iPhone their billion dollars in the way they deem best.

This dirty but genius geek troll play is a new headache to Apple executives as they will need to put in long hours counting all that money, to check if it is all there and to try to deposit it crossing fingers to hope a bank will accept all the coins.

Lee Kun-hee, Chairman of Samsung Electronics, told the media that his company is not going to be intimidated by a group of "geeks with style" and that if they want to play dirty, they also know how to do it.

> You can use your coins to buy refreshments at the little machine for life or melt the coins to make computers, that's not my problem, I already paid them and fulfilled the law.
>
> A total of 20 billion coins, delivery hope to finish this week.

Let's see how Apple will respond to this.

Is this possible? How would you respond if you were Apple?
How many trucks would it take to deliver 1 Billion in nickels? Is 30 trucks enough?

Extension:
What about if they delivered 1 Billion in dimes?

Notes:

The Gold Chain

Problem:

A wealthy man needed to pay the mason building his house. He was running low on cash so he decided to pay the mason with a gold chain made of 7 links. The mason's fee was equal to one gold link each day. The wealthy man needed to pay the mason each day or he would stop working. But, at the same time he didn't want to pay the mason any more than one link in a day because he might run off and not return.

Cutting the chain was very difficult. What is the minimum number of cuts that the wealthy man could make in the chain and still pay the mason each day for seven days?

Extension:

Notes:

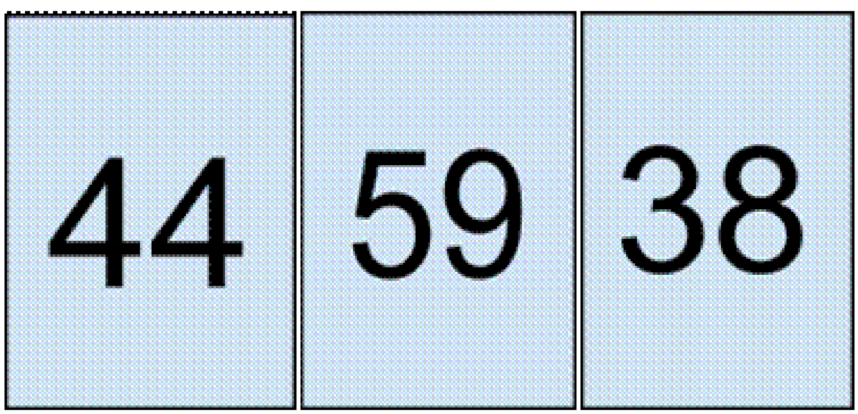
**Credit to TestFunda vol. 1 (p. 1)*

Same Sum Problem

Problem:

You have three cards in front of you. On the back of each of the cards is a different prime number. The sum of the number on the front and the number on the back is the same for each card.

-What are the prime numbers on the back of the cards?



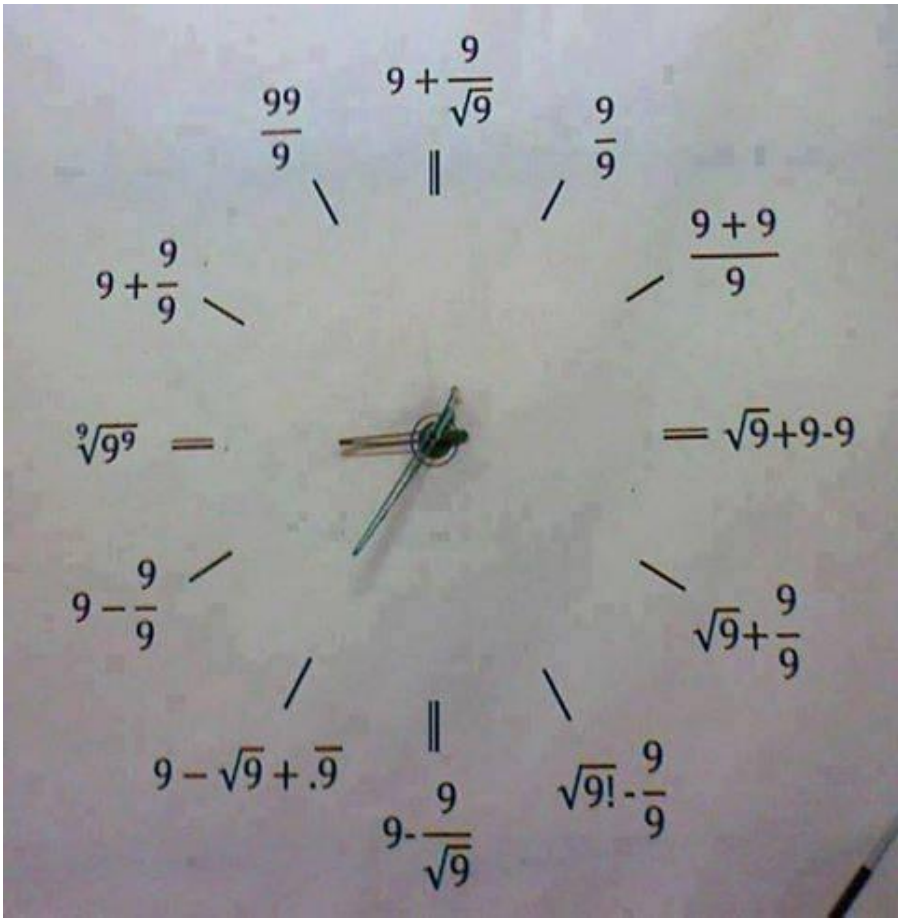
Extension:

Notes:

**Credit to Rina Zazkis*

Square Root Clock

Problem:



Will this clock tell the right time? Prove it mathematically!

Extension:

Notes:

Triangular Numbers

Problem:

The numbers 1,3,6,10,15,… are known as triangular numbers. What is the largest triangular number less than 500?

Extension:

Notes:

Pirate Diamond

Problem:

A band of pirates are going to disband. They have divided up all of their gold, but there remains one GIANT diamond that cannot be divided. To decide who gets it the captain puts all of the pirates (including himself) in a circle. Then he points at one person to begin. This person steps out of the circle, takes his gold, and leaves. The person on his left stays in the circle, but the next person steps out. This continues in a counter-clockwise manner with every second pirate leaving until there is only one left – who gets the diamond. Who should the captain point at if he wants to make sure he gets to keep the diamond for himself?

Extension:

Notes:

**Credit to Peter Liljedahl*

Mother-Daughter Tea Party

Problem:

There is a party for mothers and daughters. All the mothers shake hands among themselves (but not the daughters). Every daughter shakes hands with all the mothers. How many handshakes are there, if it is known that the party involved 17 mother-daughter pairs?

Extension:

What if there were more mother-daughter pairs?

What if every mother had two daughters?

Notes:

**Credit to Rina Zazkis*

Rabbit Problem

Problem:

Suppose a one month old pair of rabbits (one male and one female) are too young to reproduce, but are mature enough to reproduce when they are two months old. Also assume that every month, starting from the second month, they produce a new pair of rabbits (one male, one female).

If each pair of rabbits reproduces in the same way as the above, how many pairs of rabbits will there be at the beginning of each month?

How many pairs will there be after one year?

Extension:

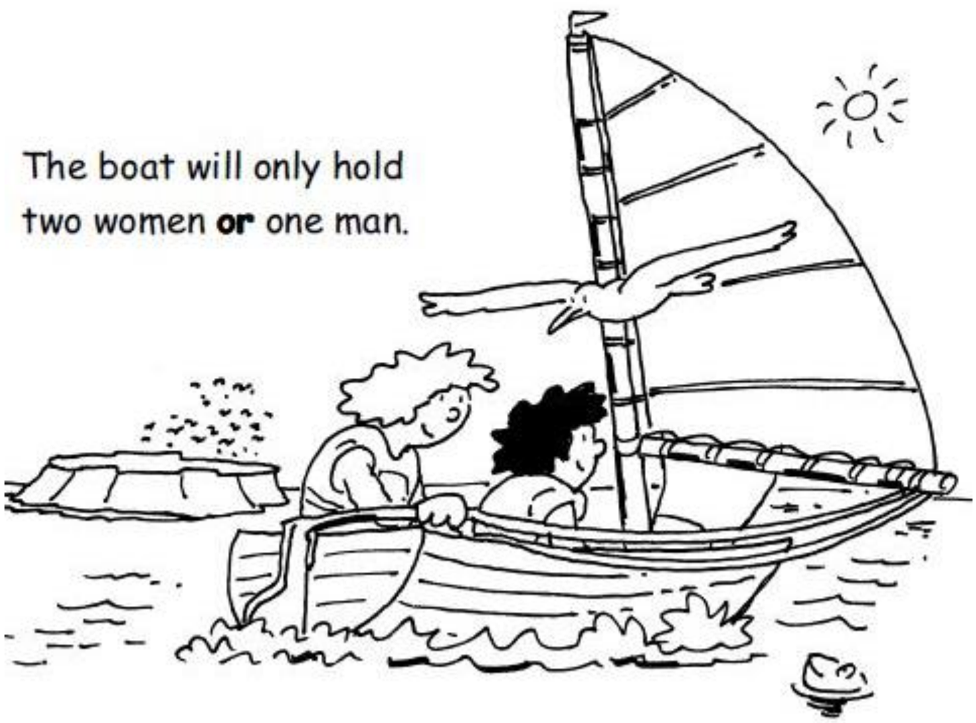
Notes:

Sail Away

Problem:

Two men and two women want to sail to an island.

The boat will only hold
two women **or** one man.



How can all four of them get to the island?

How many trips will it take?

Extension:

Notes:

*****Credit to Mathematical Challenges for Able Pupils*

Jugs – Die Hard 3

Problem:

You have a 3 litre jug and a 5 litre jug. You need to measure out exactly 4 litres. How can you do this? Is there more than one way?

Extension:

Notes:

Search for a You Tube Clip of the scene in the movie (be careful of language).

How Many Heartbeats?

Problem:

How long does it take for your heart to beat 1000 times? If you started counting at midnight tonight, when would you count the millionth beat? What about the billionth beat?

Extension:

Notes:

Paper to the Moon

Problem:

Imagine you have a really long piece of paper and you folded it in half (doubling its thickness, and then in half again (doubling it again), and then in half again (and so forth), how many folds would it take so that the total thickness of your paper could reach the moon?

Extension:

Notes:

21 Casks

Problem:

The sheik addressed the three of them: "Here are my three friends. They are sheep rearers from Damascus. They are facing one of the strangest problems I have come across. It is this: as payment for a small flock of sheep, they received, here in Baghdad, a quantity of excellent wine, in 21 identical casks: 7 full, 7 half full, 7 empty. They now want to divide these casks so that each receives the same number of casks and the same quantity of wine. Dividing up the casks is easy – each would receive 7. The difficulty, as I understand it, is in dividing the wine without opening them, leaving them just as they are. Now, calculator, is it possible to find a satisfactory answer to this problem?

-**The Man Who Counted**

Extension:

Notes:

**Credit to Malba Tahan, The Man Who Counted*

Diagonals in a Polygon

Problem:

If given the number of sides in a polygon, can you determine the number of diagonals?

Extension:

Notes:

More Fractions

Problems:

Pizza Problem
At Pizza Hut, 14 girls equally shared 6 large pizzas and 6 boys equally shared 2 large pizzas. Who got to eat more pizza, a boy or a girl?

Bottles of Pop
At a party 5 girls equally shared 3 bottles of Pop and 6 boys equally shared 2 bottles of Pop. The bottles were the same size. Who would get to drink more pop? The boys or the girls?

I am a proper fraction.
The sum of my numerator and denominator is 1 less than a perfect square. Their difference is 1 more than a perfect square. Their product is 1 less than a perfect square. Who am I?

A half is a third of a fourth. What is it?

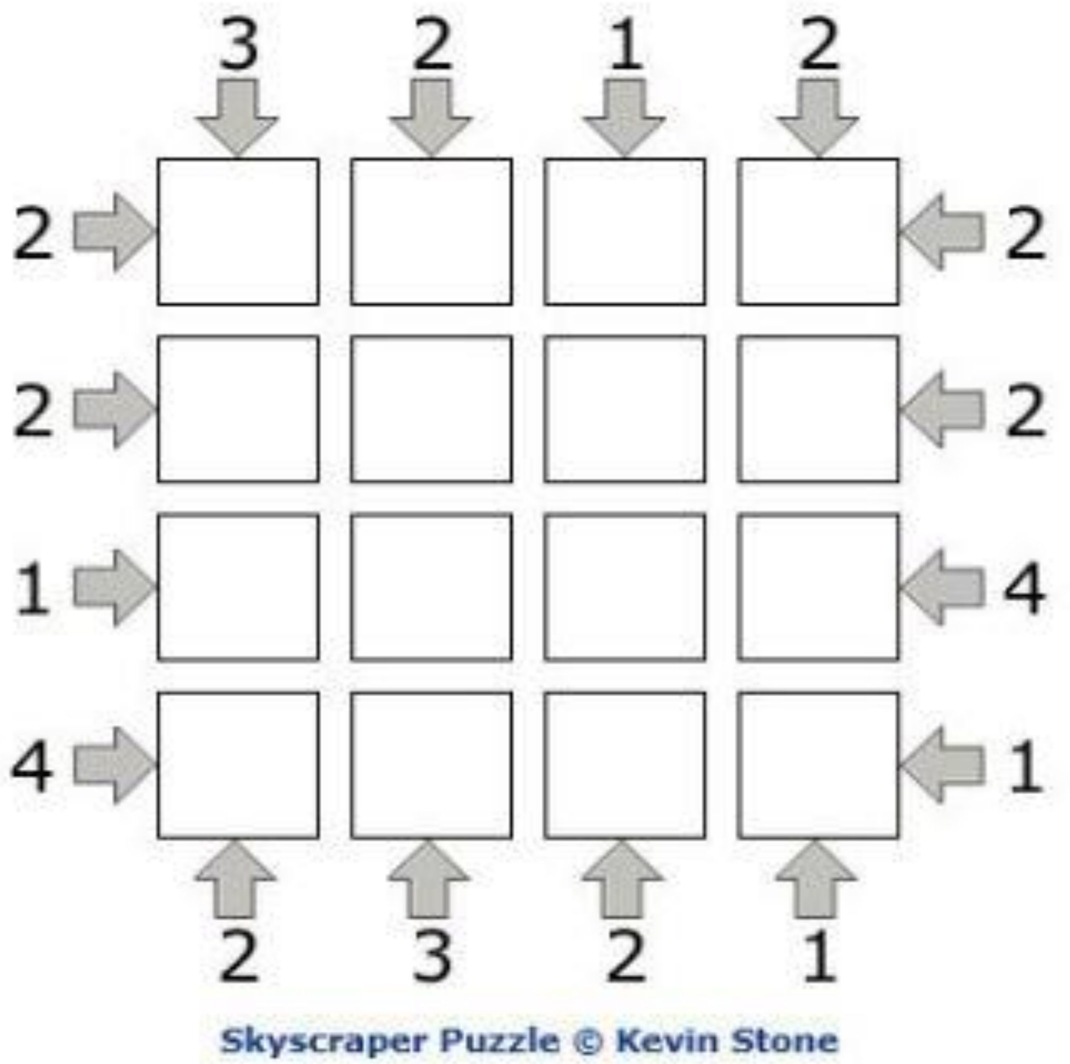
Extension:

Notes:

**First two credit to Geri Lorway, second two credit to NCTM*

Skyscrapers

Problem:



Complete the grid such that every row and column contains the numbers one to four.

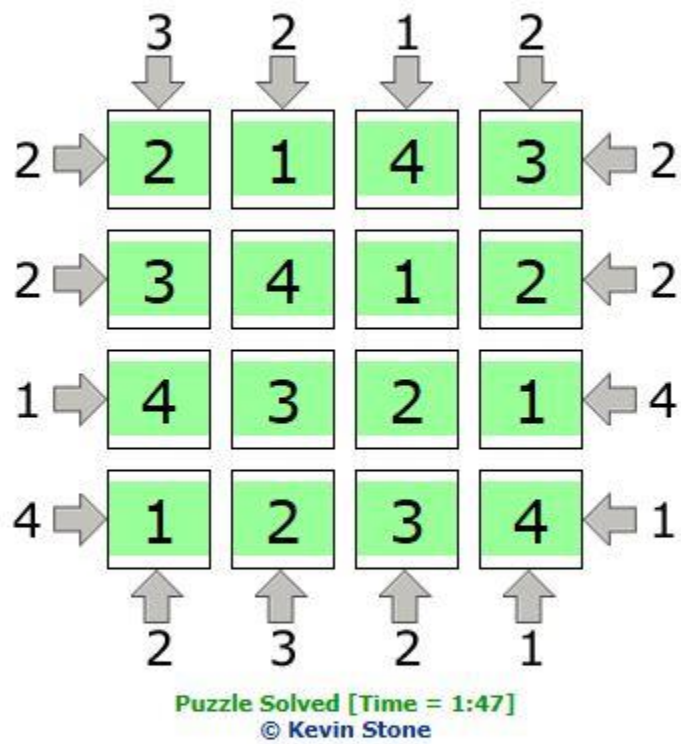
Each row and column contains each number only once.

The numbers around the outside tell you how many skyscrapers you can see from that view.

You can’t see a shorter skyscraper behind a taller one.

Skyscrapers continued

Notes:



https://www.brainbashers.com/showskyscraper.asp?date=0508&size=9&diff=1

- Print out puzzle for students (enlarged so cubes fit)
- Snap cubes to build towers

**Credit to Kevin Stone at www.brainbashers.com*

Rope Around The World

Problem:

If we wanted to hold a rope around the world, how much rope would we need? If we held the rope 1m higher, how much more rope would we need?

Variation: If my rope was 10m longer than the circumference of the world, could an ant fit under it? What about a mouse, a dog, or a human?

Extension:

What if the world was a cube instead of a sphere?

Notes:

**Credit to Rina Zazkis*

Ten Divisors

Problem:

What is the smallest positive number with exactly ten positive integer divisors?

And what is the next one after that?

Extension:

Notes:

**Credit to Derrick Niederman.*

Committees

Problem:

How many committees of 3 people can be formed from a group of 12 people?

Extension:

What if there were more total people? What if there were more people on the committees?

Notes:

Dollar

Problem:

How many different ways are there to make a dollar?

Extension:

How about two dollars?

Notes:

Line Them Up

Problem:

A preschool class has 6 students. Every day they line up to go to lunch. They like to line up in a different order each time. After how many days will it no longer be possible to choose and order that they have not used before?

Extension:

What if there were more students?

Notes:

Desk Calendar

Problem:

In a doctor's office, there is a desk calendar made of 2 cubes to show the day of the month (1-31). What digits have to be on each cube so that every day of any month can be represented properly.

Extension:

Notes:

**credit to Peter Liljedahl*

Lacrosse Tournament

Problem:

The Parks and Recreation Department is planning a tournament for club lacrosse teams in the area. Sixty-four teams have entered to play. Teams will be placed in a single elimination bracket at random. Four fields are available for tournament use. Games will only be played on Saturdays from 8:00 am until 4:00 pm. Games are 40 minutes in length. Ten minutes is allotted for half time and 10 minutes for teams to warm up. A team can play only one round each Saturday.

Your job is to report how many fields are being used each Saturday during the tournament. The parks department also needs to know how much available field space they have each weekend for other activities.

Extension:

Notes:

**Credit to Howard Couty Common Core Math Resources (https://secondarymathcommoncore.wikispaces.gcpss.org/)*

All of The Prime

Problem:

Find all of the prime numbers between 1-100.

Extension:
What about between 100 and 200?

Notes:

Ball of Yarn

Problem:

How do we pass a ball of yard around a circle so that everyone touches the ball? Start with a circle of an odd number of people.

Pass a ball of yarn around the circle skipping every 2nd person. Everyone has to touch the ball of yarn. How many times is the ball of yarn touched? What shape is made? How many intersections of yarn will be made?

What happens when the number of people in the circle changes? How can you make sure that everyone in the circle can touch the ball of yarn?

Extension:

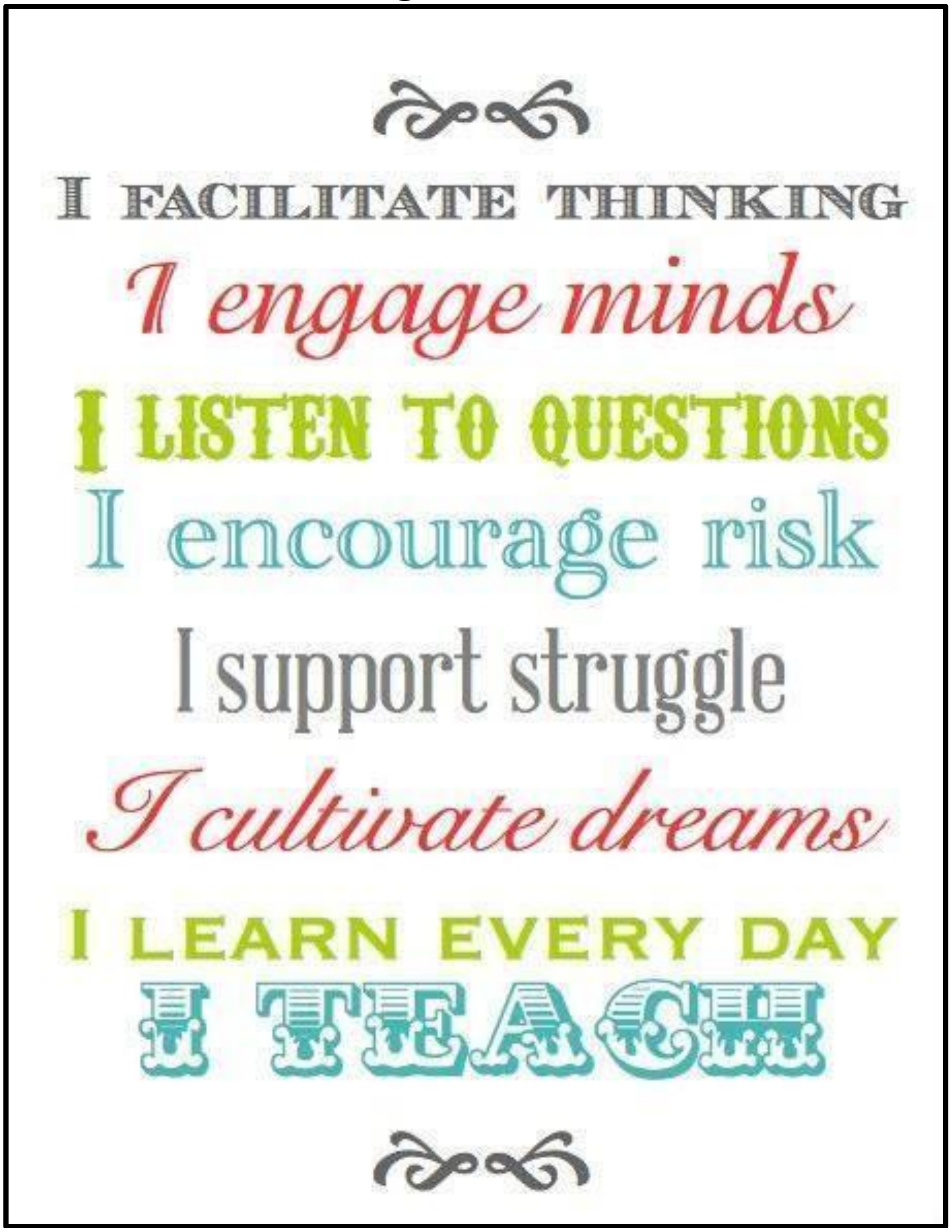
What happens when the passing interval changes? How can you be sure that everyone in the circle can touch the ball of yarn?

Notes:

Use a ball of yard. Everybody holds on to a section as they pass the ball to the next person. Take time with this one!!!

**Credit to David Pimm*

Problems to Target Curricular Outcomes



As Well as the Front Matter

Problem	Pg	N1	N2	N3	N4	N5	N6	N7	PR 1	PR 2
Painted Cube	65-68	x								x
Checkerboard	69	x								
Open Lockers	71	x								
How Many Dots?	73	x							x	x
Discount Machines	75			x						
Percent Benchmarks	77			x						
Amazing Offer	79				x	x				
Cell Phone Plans	81				x	x				
The Jeweler	83				x	x				
12 Cookies	85				x	x				
Bathtub	87				x	x	x			
Retirement	89				x	x	x			
Bungee Barbie	91				x	x			x	x
Grannie Stitchwork's Teddy Bears	93				x	x				
Pennies	95					x	x			
Traffic Lights	97					x				
Bears and Apples	99						x			
Birds on a Lake	101						x			
Cookie Jar	103						x			
Fractions/Decimals/Percents	105			x			x			
Sharing Bread	107						x			
Giving Out Bonuses	109						x			
Sharing Sub Sandwiches	111						x			
Integers- Teaching Progression	113-121							x		
Algebra Sort Problems	122-125							x		x
Moving Squares	126								x	x
Growing Pattern	128								x	x
Mountain Range	130								x	x
Picture Frames	132								x	x
Pool Border Problem	134								x	x
Staircases	136								x	x
Toothpick Pattern	138								x	x
Skyscraper Windows	140-142								x	x
51-Foot Ladder	143									
Pringles	145									
How much Popcorn?	147									
Pentominoes	149									
Eyewitness	151									
Hat and Rabbit	153									
The Monte Hall Problem	155									

SS 1	SS 2	SS 3	SS 4	SS 5	SS 6	SP 1	SP 2	Problem
		x	x					Painted Cube
								Checkerboard
								Open Lockers
								How Many Dots?
								Discount Machines
								Percent Benchmarks
								Amazing Offer
								Cell Phone Plans
								The Jeweler
								12 Cookies
								Bathtub
								Retirement
								Bungee Barbie
x	x	x	x					Grannie Stitchwork's Teddy Bears
								Pennies
							x	Traffic Lights
								Bears and Apples
								Birds on a Lake
								Cookie Jar
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								Moving Squares
								Growing Pattern
								Mountain Range
								Picture Frames
								Pool Border Problem
								Staircases
								Toothpick Pattern
	x							Skyscraper Windows
x								51-Foot Ladder
x		x						Pringles
	x	x	x					How much Popcorn?
				x	x			Pentominoes
							x	Eyewitness
							x	Hat and Rabbit
							x	The Monte Hall Problem

The Painted Cube N1,PR2,SS3,SS4

Problem:

Picture a Rubik's Cube. Now drop it into paint so that it is completely covered. When the paint is dry, imagine smashing it on the floor and it breaking it apart into the smaller cubes.

How many of the cubes have one face covered in paint? How many cubes have two faces covered in paint? How many have three faces covered in paint? How many have zero faces covered in paint?

How could you predict the above for any size Rubik's cube?

What about a 4 x 4 x 4? 5 x 5 x 5? 6 x 6 x 6? N x N x N?

Extension:

What if it wasn't a cube?
Why is the "one by one by one" cube a special case?

The Painted Cube N1,PR2,SS3,SS4

Outcome Objectives:
Number 1- Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers).

Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

- $ax = b$
- $\frac{x}{a}=b,\ a \neq 0$
- $ax + b = c$
- $\frac{x}{a}+b=c,\ a \neq 0$
- $a(x + b) = c$

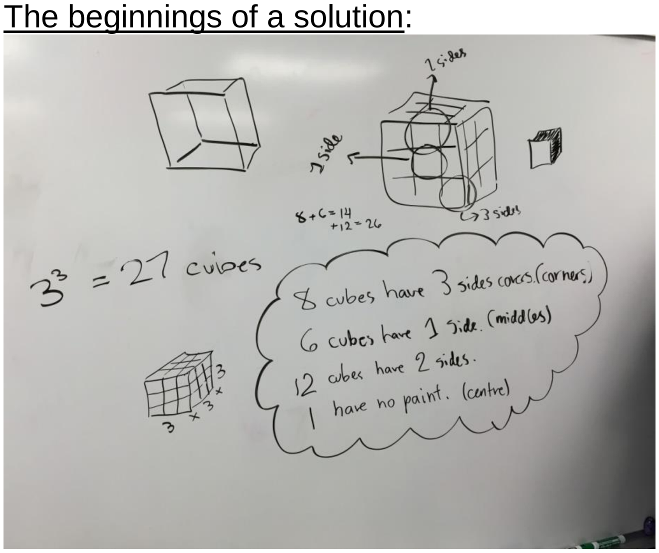
where *a*, *b* and *c* are integers. [C, CN, PS, V]

Shape and Space (3-D Objects and 2-D Shapes) 3- Determine the surface area of:
• right rectangular prisms• right triangular prisms• right cylinders to solve problems. [C, CN, PS, R, V]

Shape and Space (3-D Objects and 2-D Shapes) 4- Develop and apply formulas for determining the volume of right rectangular prisms, right triangular prisms and right cylinders.

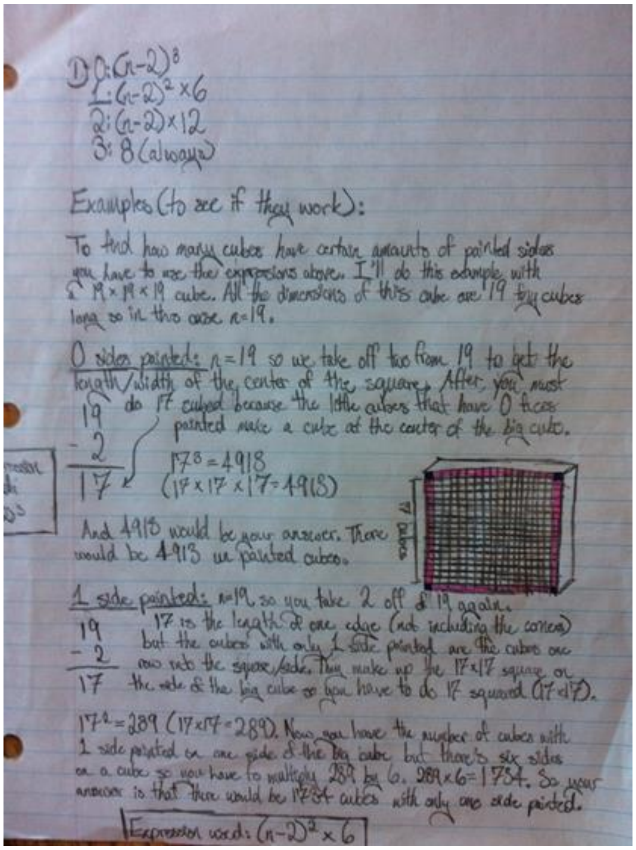
Material Suggestions:
- Snap cubes
- Vertical surfaces
- Dry-erase markers

Sample Solutions:
The beginnings of a solution:



The Painted Cube N1,PR2,SS3,SS4

Student's Written Solution:



1) 0: (n-2)³
1: (n-2)² × 6
2: (n-2) × 12
3: 8 (always)

Examples (to see if they work):

To find how many cubes have certain amounts of painted sides you have to use the expressions above. I'll do this example with a 19 × 19 × 19 cube. All the dimensions of this cube are 19 tiny cubes long so in this case n=19.

0 sides painted: n=19 so we take off two from 19 to get the length/width of the center of the square. After, you must do 17 cubed because the little cubes that have 0 faces painted make a cube at the center of the big cube.

19 − 2 = 17

17³ = 4918
(17 × 17 × 17 = 4918)

And 4913 would be your answer. There would be 4913 un painted cubes.

17 cubes

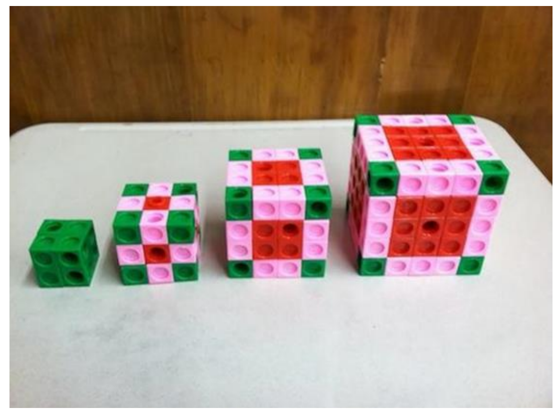
1 side painted: n=19 so you take 2 off of 19 again.
17 is the length of one edge (not including the corners) but the cubes with only 1 side painted are the cubes one row into the square/side. They make up the 17×17 square on the side of the big cube so you have to do 17 squared (17×17).

19 − 2 = 17

17² = 289 (17×17 = 289). Now you have the number of cubes with 1 side painted on one side of the big cube but there's six sides on a cube so you have to multiply 289 by 6. 289×6 = 1734. So your answer is that there would be 1734 cubes with only one side painted.

Expression used: (n-2)² × 6

A Visual Representation of a Solution:



Mathematics related to the coloring of the cubes will emerge:

- Cubes with 3 faces painted: 8
  These are always in the corners and there are always 8 (except on a size 1 cube).
- Cubes with 2 faces painted: $12(n - 2)$
  These are always along the edges but not on the corners, so on each edge, there are 2 less than the size of the cube. There are 12 edges, so the number needs to be multiplied by 12.
- Cubes with 1 face painted: $6(n-2)^2$
  These are in the middle of each face. They are in the shape of a square, two sizes smaller than the face of the original cube. There are 6 faces, so the number needs to be multiplied by 6.
- Cubes with no faces painted: $(n-2)^3$
  These are always in the middle. They form a cube shape that is two sizes smaller than the original cube.

The Painted Cube N1,PR2,SS3,SS4

Notes:

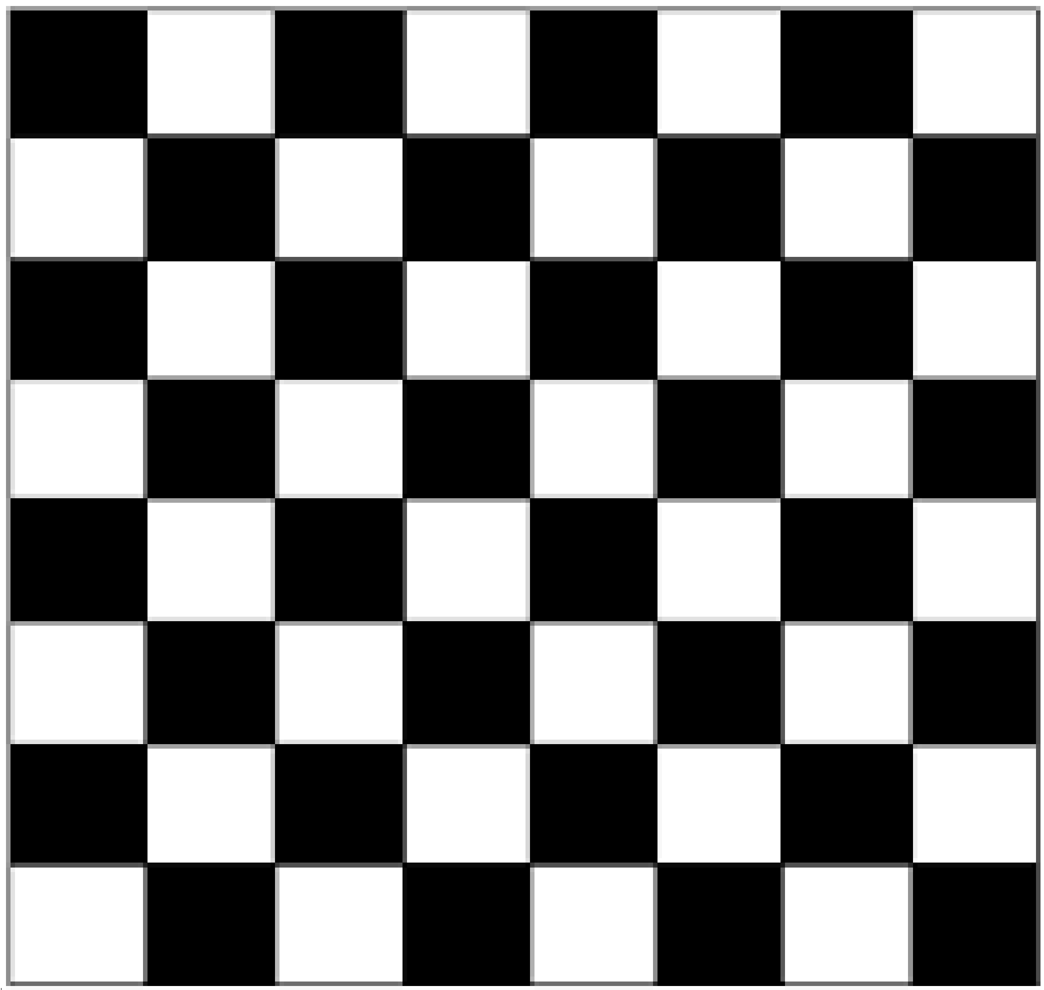
This problem not only illustrates linear relationships, but also introduces and reinforces the idea that algebraic relations come from real situations and that can and should be visualized. Students can also graph the different relations.

Assessment idea: Have students initially solve the problem within a group, but individually write up their own solution which explain the process, tells the story of how the problem was solved, and explains the mathematics!
**Credit to David Pimm*

Checkerboard N1

Problem:

How many squares are there on a standard 8x8 checkerboard? The answer is not 64!



Extension:
If the checkerboard was a different size, could you find a solution?
If the checkerboard was ***NxN*** (any size), could you find an algebraic expression?
What if you had to could the squares that are positioned diagonally (advanced)?

Checkerboard N1

Teacher Notes:

Outcome Objectives:

Number 1-Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers). [C, CN, R, V]

Material Suggestions:

- Picture of an 8 by 8 checkerboard
- Scissors to cut out different sized squares
- Graph Paper and markers
- Flip Tiles

Sample Solutions:

$8^2 + 7^2 + 6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2 = 204$

- Let's say that the checkerboard measures 8cmx8cm.
- There are 64 1x1cm squares, easily recognized as the red and black squares.
- There are also 2x2 cm squares, 3x3 cm squares, 4x4 cm squares etc.
- One way to show this is through the use of graph paper, and students cut out squares measuring 2x2cm, 3x3 cm etc., and moving them along the 8x8 grid.
- There are 7 – 2x2 cm squares vertically and horizontally, totaling 7^2 2x2 squares.
- There are 6 – 3x3 cm squares vertically and horizontally, totaling 6^2 3x3 squares.
- Following the same pattern, we reach the equation presented in the solution above.

Notes:

Students as young as grade three have been successful at solving this problem. Let them struggle and explore.

**Credit to Rina Zazkis*

Open Lockers N1

Problem:

Imagine that you are in a school that has a row of 100 closed lockers. Suppose a student goes along and opens every locker. Then a second student goes along and shuts every second locker. Now a third student changes the state of every third locker (if the locker is open the student closes it, and if the locker is closed, the student opens it). A fourth student changes the state of every fourth locker. This continues until 100 students have followed this pattern with the lockers.

When finished, which lockers are open? How do you know?

Can you determine the list of open lockers all the way to 1000?

Extension:

Make and investigate another toothpick pattern.

Open Lockers N1

Teacher Notes:

Outcome Objectives:

Number 1–Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers). [C, CN, R, V]

Material Suggestions:

- Flip Tiles or two coloured counters
- Lockers and sticky notes
- Markers

Sample Solution:

All square numbered lockers are left open (1,4,9,16,25,…)

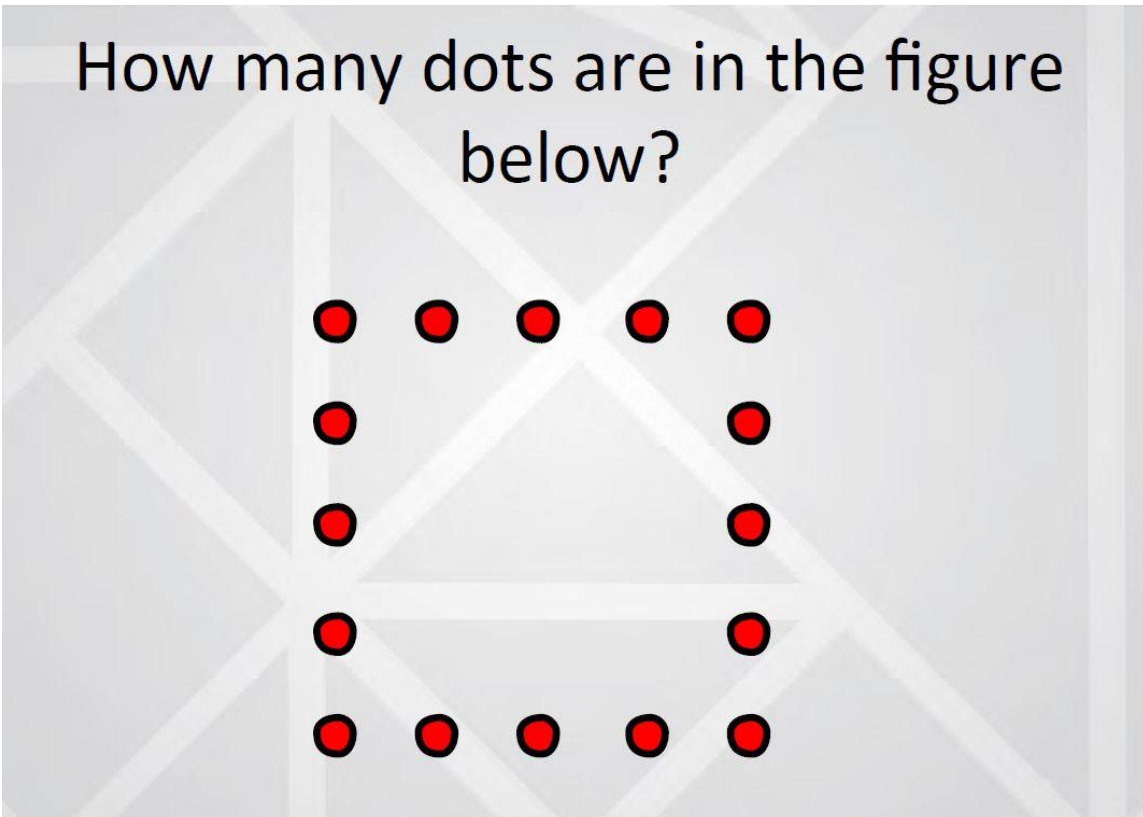
- The first person opens all the lockers, then locker 1 is no longer touched.
- The second person closes every second locker, so all even numbered lockers are closed. Locker 2 is no longer touched.
- The third person changes the state of every third locker, so locker 3 is now closed, and no longer touched.
- The only way a locker can remain open is if it is not touched an even number of times.
- Only numbers with an uneven number of factors are the square numbers.

Notes:

**Credit to Rina Zazkis*

How Many Dots? N1, PR1, PR2

Problem:



How many dots are in the figure? How do you know? What mathematical sentences can you come up with to help you count the dots?

Extension:

What if the side lengths were different? Can you predict the number of dots for any side length? Can you find an algebraic expression that represents how you count the dots? What about other algebraic expressions?

How Many Dots? N1, PR1, PR2

Teacher Notes:

Outcome Objectives:

Number 1-Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers). [C, CN, R, V]

Patterns and Relations (Patterns) 1-Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2–3.3]

Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

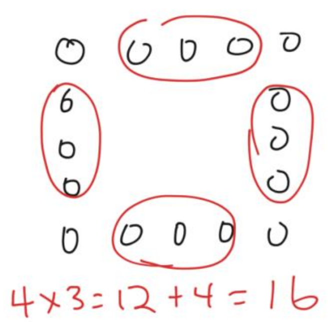
- $ax = b$
- $\frac{x}{a} = b,\ a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c,\ a \neq 0$
- $a(x + b) = c$

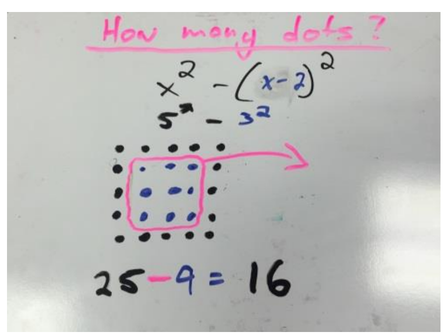
where *a*, *b* and *c* are integers. [C, CN, PS, V]

Material Suggestions:

- counters

Sample Solutions:





Answers will vary.

Notes:

**Credit to NCTM*

Discount Machines N3

Problem:

One invention saves 30% on fuel; a second, 45%; and a third, 25%. If you use all three inventions at once can you save 100%? If not, how much?

Extension:

Discount Machines N3

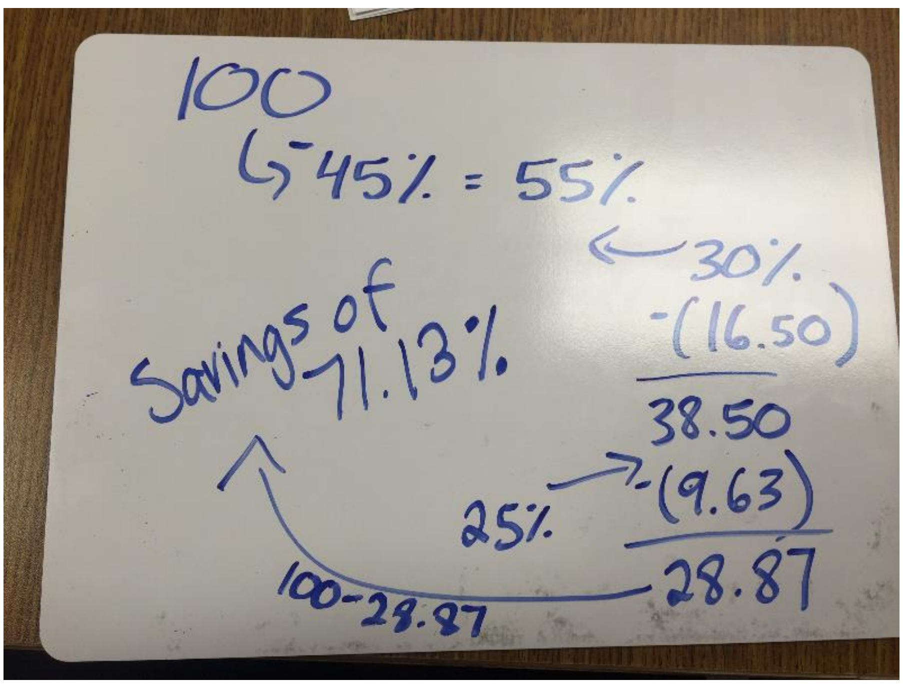
Teacher Notes:

Outcome Objectives:

Number 3-Demonstrate an understanding of percents greater than or equal to 0%, including greater than 100% (CN, PS, R, V).

Material Suggestions:

Sample Solutions:



Notes:

**Credit to Kordemsky: The Moscow Puzzles (Dover)*

Percent Benchmarks N3

Problem:

Percent Benchmarks # 1

1.

100%	50%	25%	10%	5%	$2\frac{1}{2}$%	1%
1000						

a) 75% x 1000 = _____ b) 15% of 1000 is _____

c) 35% x 1000 = _____ d) 60% of 1000 is _____

e) 20% x 1000 = _____ f) 30% of 1000 is _____

g) 51% x 1000 = _____ h) 26% of 1000 is _____

i) 99% x 1000 = _____ j) 105% of 1000 is _____

Make up two of your own.

k) ______ x 1000 = _____ l) ______ of 1000 is _____

Extension:

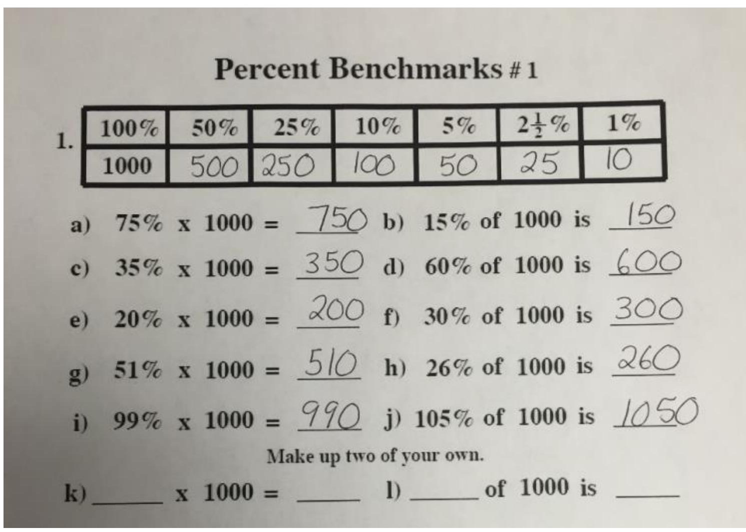
Percent Benchmarks N3

Teacher Notes:

Outcome Objectives:
Number 3-Demonstrate an understanding of percents greater than or equal to 0%, including greater than 100% (CN, PS, R, V).

Material Suggestions:

Sample Solutions:



Percent Benchmarks # 1

1.

100%	50%	25%	10%	5%	$2\frac{1}{2}$%	1%
1000	500	250	100	50	25	10

a) 75% x 1000 = 750 b) 15% of 1000 is 150

c) 35% x 1000 = 350 d) 60% of 1000 is 600

e) 20% x 1000 = 200 f) 30% of 1000 is 300

g) 51% x 1000 = 510 h) 26% of 1000 is 260

i) 99% x 1000 = 990 j) 105% of 1000 is 1050

Make up two of your own.

k) ______ x 1000 = ______ l) ______ of 1000 is ______

Notes:
**Credit to Wheatley and Abshire. Developing Mathematical Fluency.*

Amazing Offer N4,5

Problem:

Your rich friend needs your help with her business for 30 days. She makes you an offer to pay you 1¢ the first day, 2¢ the second day, 4¢ the third day, each day doubling the previous day's pay. Of course, you were insulted and were about to refuse when she said, "Ok, ok, how about $1,000.00 a day with a $1,000.00 raise each day." You gladly accept and your friend bursts out laughing. What is so funny?

Extension:

Is there a point where the first offer is actually better?

Make up another problem that involves exponential growth.

Research a real world situation that deals with exponential growth.

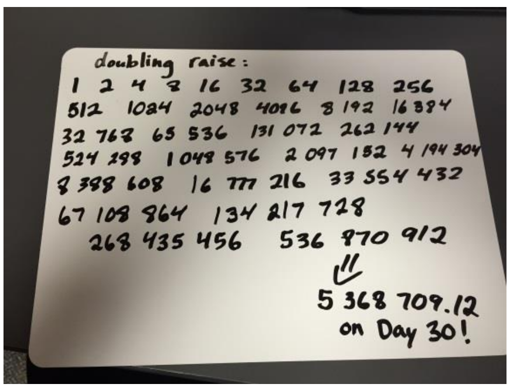
Amazing Offer N4,5

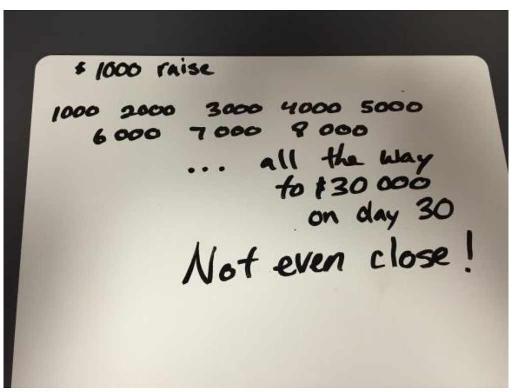
Outcome Objectives:

Number 4 - Demonstrate an understanding of ratio and rate. [C, CN, V]
Number 5 – Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R

Material Suggestions:

Sample Solutions:





Total salary for the first option is $10,737,418.23
Total for the second option is $465,000.00

Notes:

Teacher Notes:

Cell Phone Plans N4, N5

Problem:

Your parents have agreed to buy you a cell phone. However, the deal is that you have to pay for the cell phone plan out of your own pocket. There are three plans to choose from:

PAY AS YOU GO

This truly is "pay as you go". Phone calls are 25¢ a minute and text messages are 15¢ for each message sent.

BASIC PLAN

This plan is $20.00 per month and includes 100 free minutes of "anytime" talk. If you use more than 100 minutes then you will be charged 30¢ per minute for each minute over. Text messages are 15¢ for each message sent.

EASY 4 U PLAN

This plan is $50.00 per month and includes 200 weekday minutes and unlimited weekend minutes of talk time. Each additional minute is 35¢. The first 100 text messages sent are free. Anything above that is 25¢ for each message sent.

Show all of the work that leads to your decision.

Write an explanation that will convince your parents that this is the best plan for you.

Extension:

Cell Phone Plans N4, N5

Teacher Notes:

Outcome Objectives:

Number 4-Demonstrate an understanding of ratio and rate. [C,CN,V]
Number 5-Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]

Material Suggestions:

Sample Solutions:

The solution completely rests upon the assumptions made of air time and text messages, and the best plan will vary depending upon this information.

Notes:

**Credit to Peter Liljedahl*

Please see his website for more Numeracy Tasks.

The Jeweler N4,N5

Problem:

“This man is a mathematician?” asked old Salim. “Then he has come at just the right moment to help me out of a difficult spot. I have just had a dispute with a jeweler. We argued for a long time, and yet we still have a problem that we cannot resolve.”

“This man,” old Salim said, pointing to the jeweler “came from Syria to sell precious stones in Baghdad. He promised he would pay 20 dinars for his lodgings if he sold all of his jewels for 100 dinars, and 35 dinars if he sold them for 200. After several days of wandering about, he ended up selling all of them for 140 dinars. How much does he owe me according to our agreement?”

Extension:

The Jeweler — N4, N5

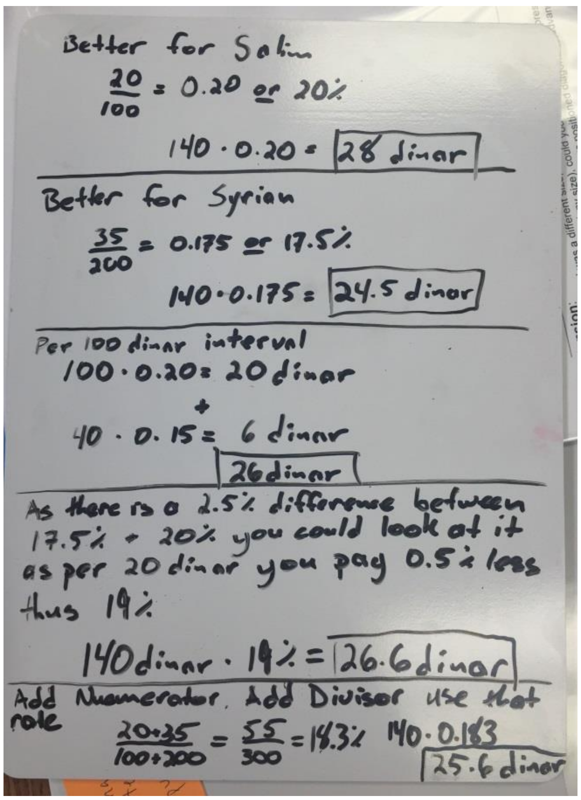
Teacher Notes:

Outcome Objectives:

Number 4 - Demonstrate an understanding of ratio and rate. [C, CN, V]
Number 5 – Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]

Material Suggestions:

Sample Solutions:



Notes:

Great problem to discuss assumptions and reasons for different answers.

**Credit to Malba Tahan,The Man Who Counted, p.23*

12 Cookies N4, N5

Problem:

Four friends buy 42 cookies for $12. Each person contributes the following amount of money:

Tom: $2.00

Jake: $3.00

Ted: $4.00

Sam: $3.00

How many cookies should each person get?

Extension:

What if there were more cookies? What if there were more friends?

12 Cookies **N4, N5**

Teacher Notes:

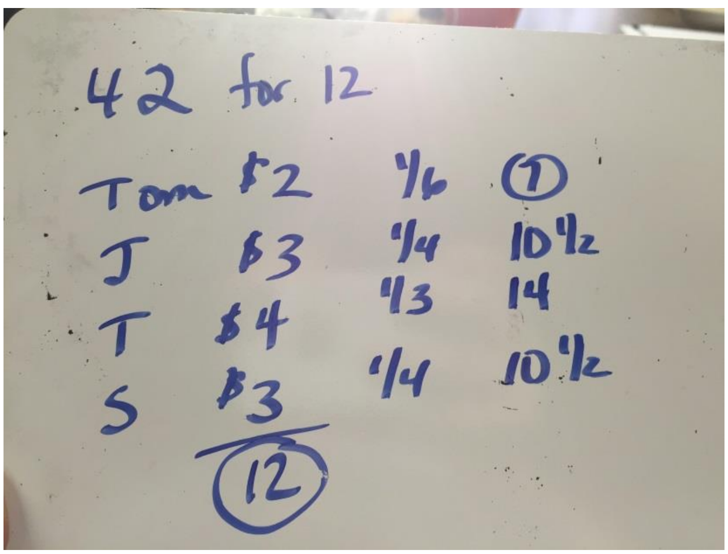
Outcome Objectives:

Number 4 – Demonstrate an understanding of ratio and rate.
[C, CN, V]
Number 5 – Solve problems that involve rates, rations and proportional reasoning. [C, CN, PS, R]

Material Suggestions:

- Manipulatives

Sample Solutions:



Notes:

**Credit to Geri Lorway*

Bathtub N4,N5,N6

Problem:

An old fashioned bathtub has two faucets, a hot water tap and a cold water tap.

The hot water tap can fill the tub in half an hour.

The cold water tap has a blockage and does not run properly, so it takes one hour to fill the tub.

If the two taps run concurrently, how long will it take to fill the tub?

Extension:

Bathtub N4,N5,N6

Teacher Notes:

Outcome Objectives:

Number 4 – Demonstrate an understanding of ratio and rate. [C, CN, V]
Number 5 – Solve problems that involve rates, rations and proportional reasoning. [C, CN, PS, R]
Number 6 – Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

Material Suggestions:

Sample Solution:

If the hot water tap takes 30 minutes to fill the tub, it fills 1/3 of the tub in 10 minutes. The cold water takes 60 minutes to fill the tub, so in ten minutes it can fill 1/6 of the tub.
If both taps run for 10 minutes, the tub would be half full (1/6 + 1/3 = 3/6). We would need to double the time to fill the whole tub. It will take a total of 20 minutes.

Notes:

**Credit to Steven Strogatz*

Retirement N4,N5,N6

Problem:

An assortment of single people and married couples are living in a retirement complex.

$\frac{2}{5}$ of the men are married to $\frac{3}{4}$ of the women

How many men might there be? How many women? How many different answers can you find?

Extension:

Try with different fractions.

Retirement N4,N5,N6

Teacher Notes:

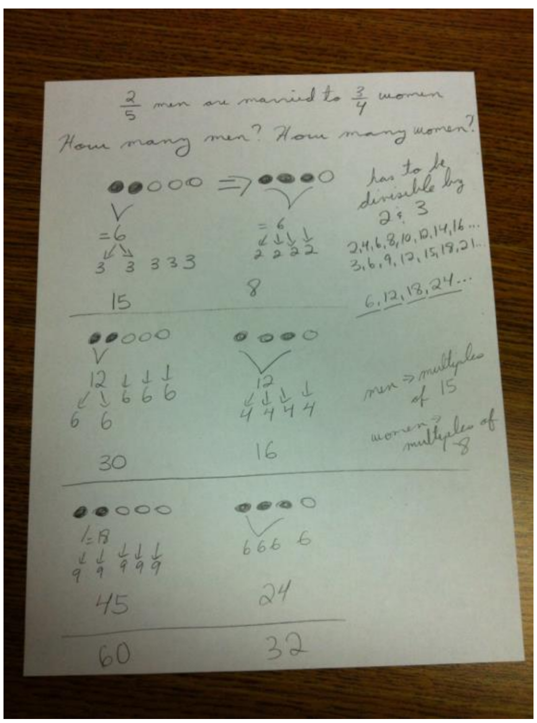
Outcome Objectives:

Number 4 – Demonstrate an understanding of ratio and rate.
[C, CN, V]
Number 5 – Solve problems that involve rates, rations and proportional reasoning. [C, CN, PS, R]
Number 6 – Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

Material Suggestions:

- Manipulatives

Sample Solutions:



Notes:

**Credit to Rina Zazkis*

Bungee Barbie N4, N5, PR1, PR2

Problem:

How many linked elastics do you need for Barbie to bungee jump from a determined distance? Her hair can touch the ground, but not her head. You can only test your prediction twice.

You will need data, a table of values, and a graph as part of your solution before you are allowed to test.

Extension:

Bungee Barbie N4, N5, PR1, PR2

Outcome Objectives:

Number 4-Demonstrate an understanding of ratio and rate. [C,CN,V]

Number 5-Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]

Patterns and Relations (Patterns) 1-Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2–3.3]

Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

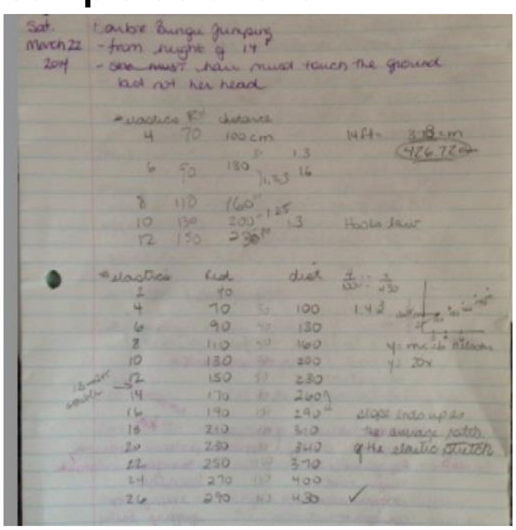
- $ax = b$
- $\frac{x}{a}=b$, $a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c$, $a \neq 0$
- $a(x + b) = c$

where *a*, *b* and *c* are integers. [C, CN, PS, V]

Material Suggestions:

- Barbies
- A box of identical elastics
- A pre-measured high location to bungee jump

Sample Solutions:



The solution varies depending on the elastics used. Elastics stretch uniformly – if one elastic stretches 25 cm, then 2 elastics stretch 50 cm, 3 stretch 75 cm, etc.

Notes:

**Credit to Peter Liljedahl*

Teacher Notes:

Grannie Stitchwork's Teddy Bears N4, N5

Problem:

Grannie Stitchwork made 20 teddy bears each month to sell in the local toyshop. She received a monthly delivery of 6 square metres of fur material, 5 kg of stuffing, 4 metres of coloured ribbon for bow ties and 40 special eye buttons. made 20 identical teddies.

The teddies sold well but customers were asking for teddies twice the size, so Grannie was asked to make 20 large teddies for the following month. Always obliging, Grannie Stitchwork immediately doubled the order of materials for the next month.

How many large teddy bears will she be able to make after doubling her material order?

Extension:

Grannie Stitchwork's Teddy Bears **N4, N5**

Teacher Notes:

Outcome Objectives:

Number 4-Demonstrate an understanding of ratio and rate. [C,CN,V]

Number 5- Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]

Material Suggestions:

Sample Solutions:

She made 5 large teddies.

- The ribbon would take double 4 m or 8 m for the larger bears. That works out.
- The 6 m^2 of fur would require 4x as much, or 24 m^2, because doubling the linear dimensions increases the surface area by a factor of 4. (If the original is 2x3 then doubling those dimensions is 4x6=24 or 4x the amount.)
- The stuffing is a volume measurement, which increases by a factor of 8 when the linear dimensions are doubled which would require 5x8=40kg of stuffing. (If the original is 2x2x2, then doubling becomes 4x4x4 – from 8 to 64, or 8x as large).
- Her new order is 12 m^2 of fur, 10kg of stuffing and 8m of ribbon. She actually needs 24m^2 of fur, 40 kg of stuffing and 16 m of ribbon. The stuffing is the most limiting, allowing her only ¼ of the order to be filled, or 5 teddies

Notes:

**Credit to Mathematical Cavalcade by Brian Bolt, Cambridge University Press, 1992. ISBN 978-0-521-42617-6*

Pennies N5, N6

Problem:

There are some pennies on a table. One fourth of the pennies are heads up. If two pennies are turned over then one third of the pennies are heads up.

How many pennies are on the table? How do you know?

Extension:

Create your own problem. What if your numerators could not be one?

Pennies N5, N6

Teacher Notes:

Outcome Objectives:

Number 5-Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]

Number 6-Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

Material Suggestions:

- pennies
- other manipulatives

Sample Solution:

24 pennies because ¼ of 24 is 6 and 1/3 of 24 is 8.

Notes:

Traffic Lights SP2,N5

Problem:

A complete cycle of a traffic lights takes 60 seconds. During each cycle the light is yellow for 5 seconds and red for 30 seconds. At a randomly chosen time, what is the probability that the light will be green?

Extension:

Traffic Lights SP2,N5

Teacher Notes:

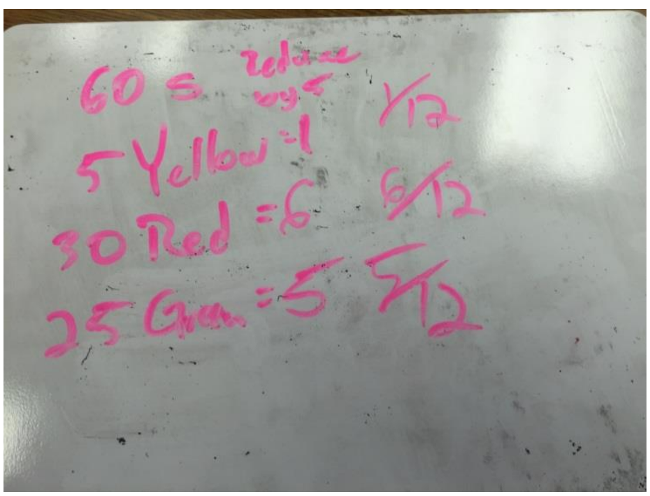
Outcome Objectives:

Statistics and Probability (Chance and Uncertainty) 2-Solve problems involving the probability of independent events.
[C, CN, PS, T] [ICT: P2–3.4]
Number 5-Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]

Material Suggestions:

Sample Solutions:

25/60 or 42% probability that the light will be green.



Notes:

**Credit to NCTM*

Bears and Apples N6

Problem:

"Once upon a time there were three bears named Minnie, Mickey, and Molly. One sunny day the bears went for a walk in the forest. They played games, picked berries and were enjoying themselves so much that they lost all sense of time and were very surprised when it suddenly got dark. It was so dark that they could not find their way home. So they wandered around until they became very tired and very hungry. They sat under a tree to get some rest, and they all fell asleep. At that time a kind fairy was passing by. She saw the three bears and thinking that they looked hungry she left them a basket of apples. In the middle of the night Minnie, the oldest bear woke up. Seeing the basket of tasty red apples she thought, "What a wonderful treat, these apples look so good and I'm so hungry. I want to eat them all." But then she remembered that she was not alone and that her siblings were likely hungry as well. She also remembered what mama bear had taught her about sharing. So Minnie only ate one-third of the apples and immediately fell back to sleep.

Another hour passed by and Mickey woke up. He saw the basket of tasty red apples and thought, "What a wonderful treat, these apples look so good and I'm so hungry. I want to eat them all." But then he remembered that he was not alone, and his siblings were likely hungry as well. He also remembered what mama bear had taught him about sharing. So Mickey only ate one-third of the apples and immediately fell back to sleep. Another hour passed by and Molly woke up. She was so hungry that she ate her apples and went back to sleep.

Slowly, the forest awoke to the sunny morning. The birds were singing and their lovely songs woke the three sleeping bears. They saw a basket under the tree. **What was in the basket?"**

Extension:

If there were 8 apples left in the basket when the bears awoke in the morning, how many apples did the Fairy leave them?

What if there was a different amount of apples left in the basket?

Bears and Apples N6

Teacher Notes:

Outcome Objectives:

Number 6-Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

Material Suggestions:

- Basket
- Cubes or apples

Sample Solutions:

27x apples

- If each bear eats 1/3 of the apples, and there is a whole number of apples in the basket to begin with, then bear #3 eats 1/3 of 1/3 of 1/3 of the apples, or 1/27th of the apples.
- Any multiple of 27 can be divided in 3 and then in 3 and then in 3 again.

if each bear eats 1/3 then the last bear has eaten $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$ of the apples, so there are 27 apples to begin with. - or a multiple of 27.

Start	B1 eats 1/3	# left	B2 eats 1/3	# left	B3 eats 1/3	# left
27	9	18	6	12	4	8
54	18	36	12	24	8	12
81	27	54	18	36	12	24

Alternately, if we assume that bear#3 eats all the apples left in the basket, then there are only 9x apples to begin with, as bear 1 and bear 2 each 1/3, meaning that the number must divide by 3 and then 3 again. If this is the case, then 9 apples to start goes to 6 apples then to 4 apples that the last bear consumes.

Notes:

** Credit to The Story of the Three Bears (adapted from Teaching Mathematics as Storytelling by Rina Zazkis and Peter Liljedahl).*

Birds on a Lake N6

Problem:

A flock of geese are on a lake. 1/5 of the geese fly away. 1/8 of the remaining birds are startled and fly away too. Finally, three times the number that first left now fly away. Twenty-eight geese are left on the lake. How many were there to begin with?

Extension:

Birds on a Lake N6

Teacher Notes:

Outcome Objectives:

Number 6-Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

Material Suggestions:

Sample Solutions:

280 birds were on the lake to begin with.

- 1/5 flew away first.
- 1/8(4/5) flew away next.
- 3(1/5) flew away last.
- 1/5 + 1/8(1/5) + 3(1/5) = total fraction that flew away.
  So 1/5 + 1/40 + 3/5 = 8/40+4/40+24/40 = 36/40
- if 36/40 flew away, then 4/40 stayed.
- 4/40 = 28/n, n being the total number on the lake.
- N=280.

Notes:

Cookie Jar N6

Problem:

There was a jar of cookies on the table. First Ken came along. He was hungry because he missed lunch, so he ate half the cookies in the jar. Mary came in the kitchen next. She didn't have dessert at lunch so she ate one third of the cookies left in the jar. Karen followed her to the cookie jar and she took three fourths of the remaining cookies. Susan ran into the kitchen, grabbed one cookie from the jar and ran out again. That left one cookie in the jar. How many were in the jar to begin with?

Extension:

Could there be another number of cookies?

Cookie Jar N6

Teacher Notes:

Outcome Objectives:

Number 6-Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

Material Suggestions:

- Snap Cubes
- Pattern Blocks
- Graph Paper

Sample Solution:

There were 24 cookies in the jar to begin with.

- Working backwards, there were 2 cookies in the jar before Susan took one.
- 2 cookies is ¼ of the batch that Karen ate (she ate ¾ of the cookies in the jar at that point). So that means there were 8 cookies in the jar, Karen ate 6 of them, and left 2.
- The 8 cookies left represent 2/3 of the cookies because Mary ate 1/3 of the batch, leaving 2/3 there for Karen. 8=2/3 of what? 12. That means Mary ate 4 cookies, leaving 8 in the jar.
- Now, 12 cookies is ½ the batch let after Ken, so that means there were 24 cookies in the jar to start with.

Notes:

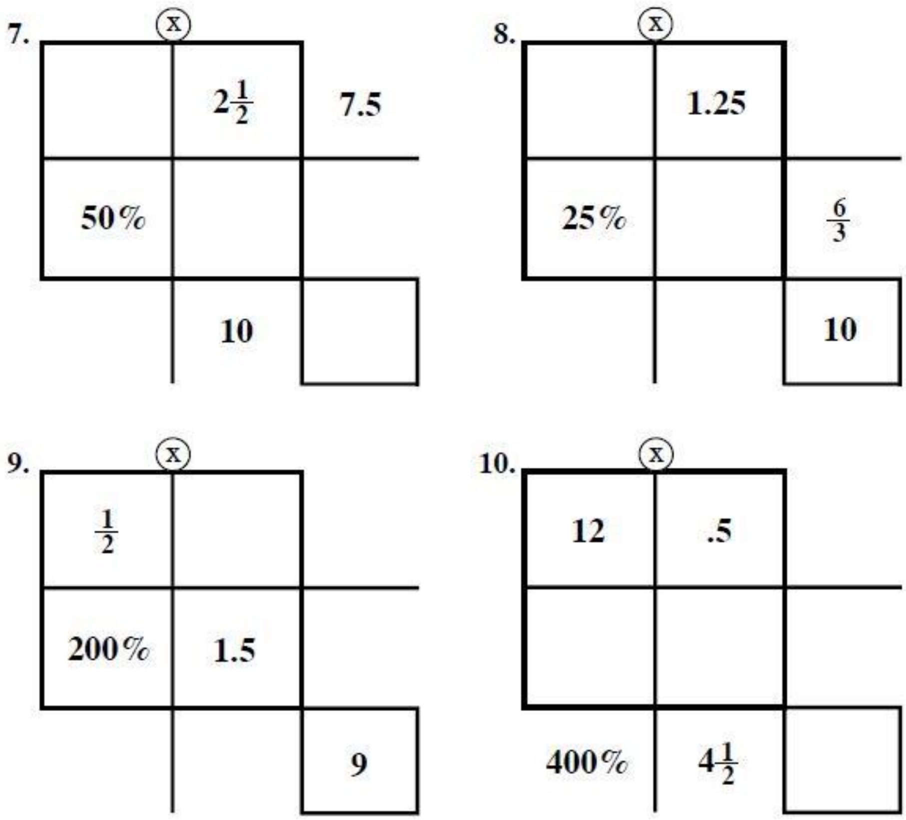
**Credit to Geri Lorway*

Fractions/Decimals/Percents N3,N6

Problem:

Fraction-Decimal-Percent Two Ways # 1

1. Write 0.5 as a percent.
2. Write $\frac{3}{4}$ as a decimal.
3. Write 0.25 as a fraction in simplest terms.
4. Change 10% to a fraction in simplest terms.
5. Write $\frac{1}{5}$ as percent.
6. Change 37% to a decimal.



Extension:

Fractions/Decimals/Percents N3,N6

Teacher Notes:

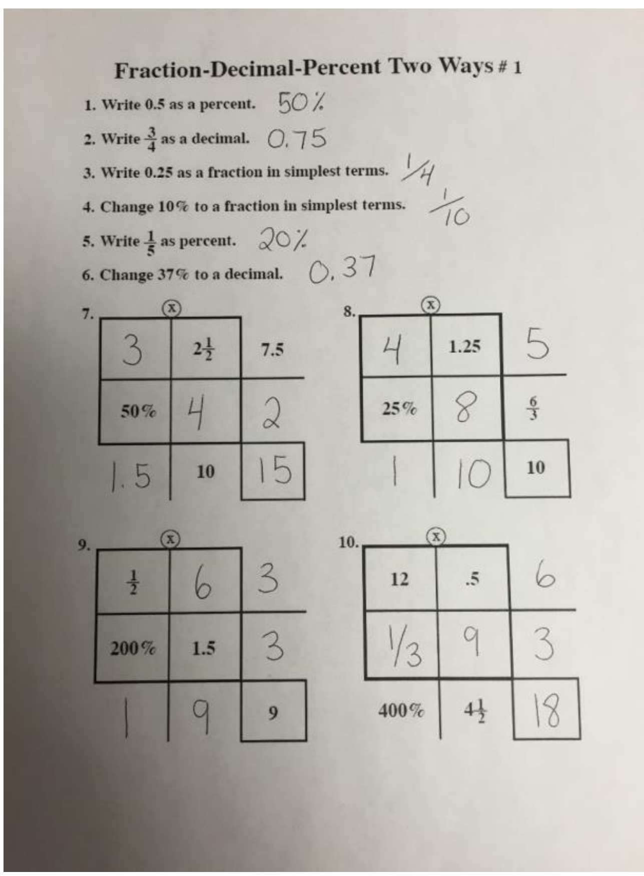
Outcome Objectives:

Number 3-Demonstrate an understanding of percents greater than or equal to 0%, including greater than 100% (CN, PS, R, V).

Number 6-Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

Material Suggestions:

Sample Solutions:



Fraction-Decimal-Percent Two Ways # 1

1. Write 0.5 as a percent. 50%
2. Write $\frac{3}{4}$ as a decimal. 0.75
3. Write 0.25 as a fraction in simplest terms. 1/4
4. Change 10% to a fraction in simplest terms. 1/10
5. Write $\frac{1}{5}$ as percent. 20%
6. Change 37% to a decimal. 0.37

7. (x)

3	$2\frac{1}{2}$	7.5
50%	4	2
1.5	10	15

8. (x)

4	1.25	5
25%	8	$\frac{6}{3}$
1	10	10

9. (x)

$\frac{1}{2}$	6	3
200%	1.5	3
1	9	9

10. (x)

12	.5	6
1/3	9	3
400%	$4\frac{1}{2}$	18

Notes:

**Credit to Wheatley and Abshire. Developing Mathematical Fluency.*

Sharing Bread N6

Problem:

“Three days later, we were approaching the ruins of a small village called Sippar, when we found sprawled on the ground a poor traveler, his clothes in rags and he apparently badly hurt. His condition was pitiful. We went to the aid of the unfortunate man, and he later told us the story of his misfortune. His name was Salem Nasair, and he was one of the riches merchants in Baghdad. On the way back from Basra, a few days before, bound for el-Hilleh, his large caravan had been attacked and looted by a band of Persian desert nomads, and almost everyone had perished at their hands. He, the head, managed to escape miraculously by hiding in the sand. When he had finished his tale of woe, he asked us in a trembling voice, “Do you by some chance have anything to eat? I am dying of hunger. I have three loaves of bread I answered. I have five said the man who counted. Very well answered the sheik. I beg you to share those loaves with me. Let me make an equitable arrangement. I promise to pay for the bread with eight pieces of god, when I get to Baghdad. So we did…”

How did he pay the two strangers fairly with the 8 pieces of gold?

Extension:

Sharing Bread N6

Teacher Notes:

Outcome Objectives:
Number 6 – Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

Material Suggestions:
- Snap Cubes

Sample Solutions:

1 gold coin to the man with 3 loaves and 7 gold coins to the man with 5 loaves.

- 3 loaves and 5 loaves divided onto 3 parts yields 24 parts of loaves of bread.
- Each person eats 8 parts.
- The man with 3 loaves has 1 part to give to the shiek (3 loavesx3parts=9parts, of which he eats 8, leaving 1 for the shiek)
- The man with 5 loaves has 5x3=15, -8=7parts for the shiek.
- The shiek owes 1/8 to the man with 3 loaves and 7/8 gold to the man with 5 loaves.

Notes:
**Credit to Malba Tahan, The Man Who Counted, p.15*

Giving Out Bonuses N6

Problem:

You are the manager of the Text-n-Talk Cell Phone Company that employs a number of independent sales people to sell their phones **seven days a week**. These sales people work as much or as little as they want. As a sales manager you don't care how much they work, but you do care how much they sell. To motivate them to sell more, you give out bonuses based on how productive they have been.

There are two bonus plans:

- the top producing individual receives **$500**.
- the top producing team **shares $500 in a fair manner**.

However, there are also two problems:

- different people have different ways of reporting their productivity.
- the individual sales teams don't have the same number of people on them.

Based on the information provided in the table below, **who should get the bonuses this month, and how much do you think they should get? Justify your answers in writing.**

Sales Person	Team	Sales Reported for the Month of April (30 days)
Tysen	A	300 cell phones sold this month
Peter	B	An average of 56 cell phones sold every 5 days
Lewis	A	An average of 10 1/3 cell phones sold each day
Ainsley	A	598 cell phones sold in the last 60 days
Avery	A	An average of $98\frac{3}{4}$ cell phones sold every 10 days
Jennifer	B	An average of 11 4/15 cell phones sold each day
Steven	B	An average of 55 cell phones each week
Gabrielle	C	4113 cell phone sold in the last year
Diana	C	An average of 10.05 cell phones each day
Matthew	D	An average of 10.87 cell phones each day
Alexa	D	An average of 9 1/6 cell phones each day
Jasmine	C	267 cell phones this month

Extension:

Giving Out Bonuses N6

Teacher Notes:

Outcome Objectives:

Number 6-Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

Material Suggestions:

Sample Solution:

Solutions will vary.

Notes:

**Credit to Peter Liljedahl*

Please see his website for more Numeracy Tasks.

Sharing Sub Sandwiches N6

Problem:

A fifth-grade class traveled on a field trip in four separate cars. The school provided a lunch of submarine sandwiches for each group. When they stopped for lunch, the subs were cut and shared as follows:

-the first group had 4 people and shared 3 subs equally.

-the second group had 5 people and shared 4 subs equally.

-the third group had 8 people and shared 7 subs equally.

-the last group had 5 people and shared 4 subs equally.

When they returned from the field trip, the children began to argue that the distribution of sandwiches had not been fair, that some children got more to eat that the others. Were they right?

Extension:

Sharing Sub Sandwiches N6

Teacher Notes:

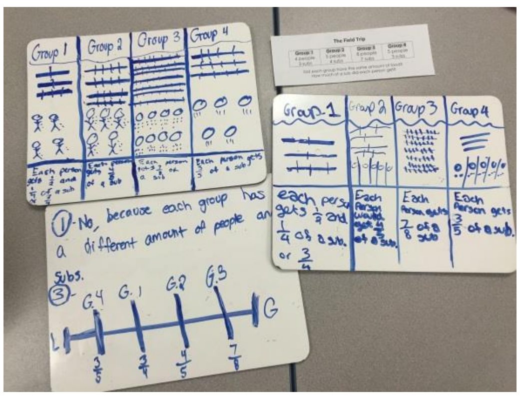
Outcome Objectives:

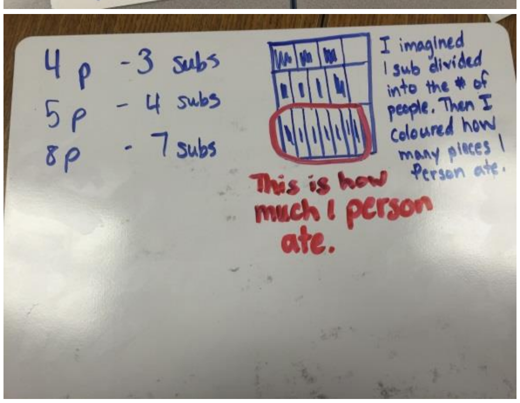
Number 6-Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

Material Suggestions:

- Unifix cubes
- Other manipulatives

Sample Solutions:





Notes:

This can be solved in a variety of ways. Students can use fractions, decimals, percents.

**Credit to Cathy Fosnot*

Integers – Teaching Progression N7

Lesson One: Discuss Integers, what they are and how they are used

Whole-group brainstorm for ideas followed by some research on the ideas. (Elevation, altitude, airplanes, scuba divers, above or below sea-level, temperatures, bank accounts and overdraft, plus and minus scores in hockey, T-Minus 10, 9, 8…blast off!, golf scores, etc.).

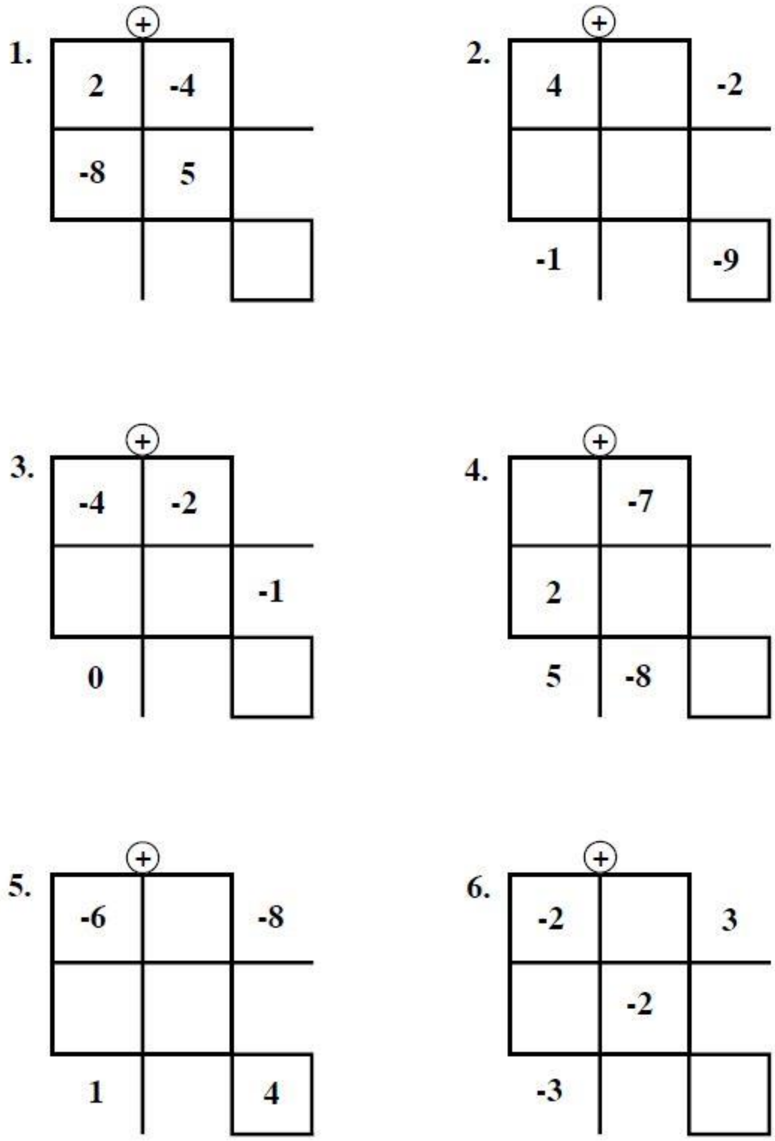
Exit Card: Explain integers and two places you may use them in everyday life.

Integers – Teaching Progression N7

Lesson Two (optional): Review Adding and Subtracting Integers

**Credit to Wheatley and Abshire, Developing Mathematical Fluency*

Integer Two Ways # 1



Exit Card: Explain how to add and subtract integers. Give multiple examples and explain why your strategies work.

Integers – Teaching Progression N7

Lesson Three: Representing Multiplication of Integers with Manipulatives (integer tiles, bingo chips and pictures (number line) with one positive integer and one negative integer.

(+3) x (-4) = (-4) x (+3) = (-3) x (+4) = (+4) x (-3)

3 groups of (-4) = 4 groups of (-3) = (-12)

Exit Card: Explain how to multiply a positive integer and a positive integer. Explain how to multiply a positive integer and a negative integer.

Integers – Teaching Progression N7

Lesson Four: Exploring Mathematical Statements in Groups

The following mathematical statements are true. Can you figure out others that also must be true? Can you figure out some rules about multiplying and dividing integers? Can you test these rules? Use technology.

(-4) x (-5) = (+20) **Therefore (+20) ÷ (-5) = (-4)**

(-2) x (+3) = (-6) **Therefore (-6) ÷ (+3) = (-2)**

(+6) x (-6) = (-36) **Therefore (-36) ÷ (-6) = (+6)**

(+7) x (+2) = (+14) **Therefore (+14) ÷ (+2) = (+7)**

Exit Card: What do you know for sure about multiplying and dividing positive and negative integers?

Integers – Teaching Progression N7

Lesson Five: Following the Patterns and Making Conclusions

4 x 3 =
3 x 3 =
2 x 3 =
1 x 3 =
0 x 3 =
(-1) x 3 = ____
(-2) x 3 =____
(-3) x 3 =____
(-4) x 3 =____

4 x (-3) =
3 x (-3) =
2 x (-3) =
1 x (-3) =
0 x (-3) =
(-1) x (-3) = ____
(-2) x (-3) = ____
(-3) x (-3) = ____
(-4) x (-3) =____

Exit Card: What conclusions can you make about multiplying positive and negative integers? What about dividing positive and negative integers?

Integers – Teaching Progression N7

Lesson Six: A Story to Explain

**Credit to Rinal Zazkis and Peter Liljedahl, Teaching Mathematics as Sorytelling by Sense Publishers, 2009).*

Represent the following situations with a math equation.

"We consider a chemical reaction in which the temperature is rising by 2 degrees every hour. The current temperature is 0. What will the temperature be in 5 hours"?

"Consider a chemical reaction in which the temperature is increasing by 2 degrees every hour. The current temperature is 0. What will the temperature be in 5 hours"?

"Consider a chemical reaction in which the temperature is increasing by 2 degrees every hour. The current temperature is 0. What was the temperature 5 hours ago"?

"Consider a chemical reaction in which the temperature is decreasing by two degrees every hour. The current temperature is 0. What was the temperature 5 hours ago"?

Exit Card: Can we find or invent other situations that could represent the same math equations?

Integers – Teaching Progression N7

Lesson Seven: Multiplying and Dividing Integers

**Credit to Wheatley and Abshire, Developing Mathematical Fluency*

Integer Two Ways # 7

(x)

1.

-4	-3	
	-2	
20		

2.

		2
-5	-5	
-10		

3.

	-4	
	-3	-6
		48

4.

-7	1	
	4	
-21		

5.

-5		-50
	4	
		-200

6.

2	-3	
		-7
	-21	

Exit Card: Explain how to add and subtract integers. Give multiple examples and explain why your strategies work.

Integers – Teaching Progression N7

Lesson Eight: What about division?

Can we make sense of division with integers? Can we find and invent examples of situations where we divide with positive and negative numbers?

Exit Card: Find or invent situations that represent different divisions with integers (positive and negative).

Integers – Teaching Progression N7

Lesson Nine: Showcase your learning using technology to prove the following statement:

I can "demonstrate an understanding of multiplication and division of integers, concretely, pictorially, and symbolically".

.

Extension:

Notes:
Obviously, these aren't really "problems" but a way to teach integers with a problem solving approach! Students need to explore, experiment, try, and verify together in groups to figure out integers. Silly rhymes or rules may help them succeed on a test, but without understanding or long term learning. Let them use bingo chips, number lines, thermometers, etc. to help make sense of operations with integers.
The *Show Me App for the IPad* is a great way to showcase learning and understanding, explaining thinking, etc. Another option is to have the students make a video as they teach/explain how to multiply and divide integers.

Algebra Sort Problems N6, N7

Problem:

Sort each of the following 12 statements into the categories: Always True, Sometimes True, and Never True. Be sure to discuss your thoughts with your group. Be prepared to display your agreed upon reasoning in a poster format. Include examples or reasons for each statement.

Previous Knowledge: Cut out and sort these statements into the categories given.		
Always True	**Sometimes True**	**Never True**
2n + 3 = 3 + 2n	n + 5 is less than 20	2t – 3 = 3 – 2t
5q = 5	2x = 2x	4p is greater than 9 + p
2*3 + s = 6 + s	k + 12 = g + 12	d + 3 = d ÷ 3
2x = 4	q + 2 = q + 16	n + 5 = 11

Algebra Sort Problems N6, N7

Grade 8: Cut out and sort these statements into the categories given.		
Always True	**Sometimes True**	**Never True**
$6x = 2(3x)$	n + 5 is less than 20	$x^2 = 9$
$2n + 3 = 3 + 2n$	$n + 5 = 11$	$k + 12 = g + 12$
$2(3 + s) = 6 + 2s$	4p is greater than 9 + p	$2(d + 3) =2d + 3$
$2t – 3 = 3 – 2t$	$q + 2 = q + 16$	$y * y = 2y$

Algebra Sort Problems N6, N7

Extension: Cut out and sort these statements into the categories given.		
Always True	**Sometimes True**	**Never True**
$9x^2 = (3x)^2$	n + 5 is less than 20	2t – 3 = 3 – 2t
2n + 3 = 3 + 2n	n + 5 = 11	q + 2 = q + 16
2(3 + s) = 6 + 2s	4p is greater than 9 + p	2(d + 3) =2d + 3
k + 12 = g + 12	x^2 is greater than 4	$x^2 = 5x$

Algebra Sort Problems N6, N7

Teacher Notes:

Outcome Objectives:

Number 6-Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

Number 7-Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically. [C, CN, PS, R, V]

Material Suggestions:

- Algebra Sort number cards
- scissors

Sample Solutions:

Previous knowledge

Always	Sometimes	Never
2x = 2x	n + 5 is less than 20	2t – 3 = 3 – 2t
2n + 3 = 3 + 2n	n + 5 = 11	q + 2 = q + 16
2*3 + s = 6 + s	4p is greater than 9 + p	d + 3 = d ÷ 3
	2x = 4	
	5q = 5	
	k + 12 = g + 12	

Grade 8

Always	Sometimes	Never
6x = 2(3x)	n + 5 is less than 20	2t – 3 = 3 – 2t
2n + 3 = 3 + 2n	n + 5 = 11	q + 2 = q + 16
2(3 + s) = 6 + 2s	4p is greater than 9 + p	2(d + 3) =2d + 3
	x^2 = 9	
	y * y = 2y	
	k + 12 = g + 12	

Extension:

Always	Sometimes	Never
$9x^2 = (3x)^2$	n + 5 is less than 20	2t – 3 = 3 – 2t
2n + 3 = 3 + 2n	n + 5 = 11	q + 2 = q + 16
2(3 + s) = 6 + 2s	4p is greater than 9 + p	2(d + 3) =2d + 3
	x^2 is greater than 4	
	$x^2 = 5x$	

Notes:

Have the students cut out the statements, mix them up and sort them.

**Credit to Geri Lorway, Marj Farris, NRLC*

Moving Squares PR1,PR2

Problem:

You have 16 people in a group filling a 4 x 4 grid (4 rows of 4). If you remove the person in the top right corner, how can you move the person in the bottom left corner to that empty space? You can only move vertically or horizontally, not diagonally. How many moves will this take? Is this the minimum number of moves?

Extension:

What about in different sizes of grids?
How do we calculate the minimum number of moves for any size grid?

Moving Squares PR1,PR2

Teacher Notes:

Outcome Objectives:

Patterns and Relations 1 - Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V]

Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

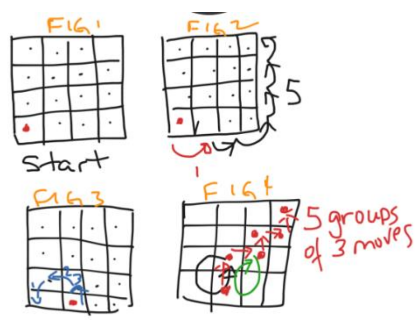
- $ax = b$
- $\frac{x}{a}$=b, $a \neq 0$
- $ax + b = c$
- $\frac{x}{a}$ +b=c, $a \neq 0$
- $a(x + b) = c$

where *a*, *b* and *c* are integers. [C, CN, PS, V]

Material Suggestions:

- People in a grid (made of tape on the floor)
- Cubes or counters
- Graph paper

Sample Solutions:



It takes 6+ (3x5) moves =21 moves

- First must move the empty space adjacent to the person moving, in this picture, the Cheerio. On a 4x4 grid, this takes 5 moves, plus 1 more move of the Cheerio
- Then the dance of 3 begins – it takes 5 dances of 3 steps to move it to the correct corner.

	Move the blank and move the person once	#of sets of 3 dances	total
2x2	2	1	5
3x3	4	3	13
4x4	6	5	21
5x5	8	7	29

Notes:

Take your time to act it out as a class with this one. It is worth it!

**Credit to David Pimm*

Growing Pattern PR1, PR2

Problem:

What changes as this pattern grows?

Use a table, graph, equation, and story to describe the change.

Extension:

Growing Pattern PR1, PR2

Teacher Notes:

Outcome Objectives:

Patterns and Relations (Patterns) 1-Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2–3.3]

Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

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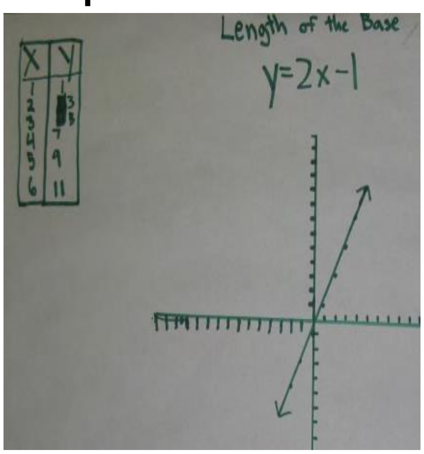
- $\frac{x}{a}=b,\ a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c,\ a \neq 0$
- $a(x + b) = c$

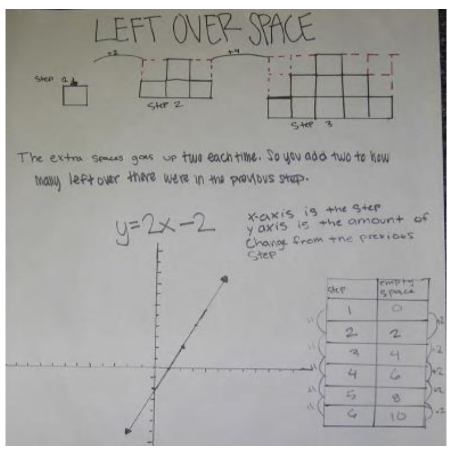
where *a*, *b* and *c* are integers. [C, CN, PS, V]

Material Suggestions:

- toothpicks
- graph paper

Sample Solutions:





Notes:

Have students come up with ideas to investigate (ie. perimeter, height, width, size of enclosing rectangle, # of toothpicks, # of interior toothpicks, # of intersections, # of corners, # of squares, # or rectangles, # of parallel lines, # of diagonals, left over space, etc.

**Credit to NCTM*

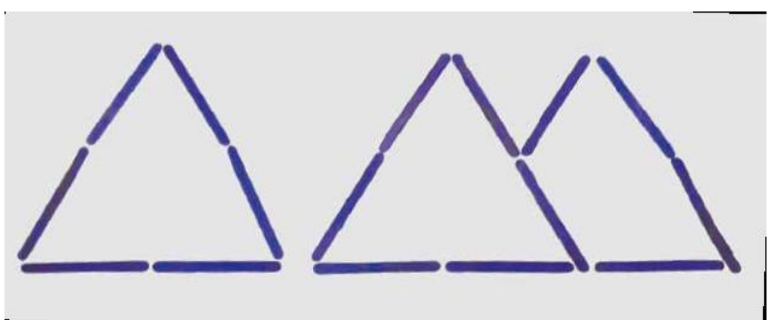
Mountain Range Challenge PR1, PR2

Problem:

Using popsicle sticks, create a mountain range of equilateral triangles. First predict and then determine how many sticks will be needed to build a mountain range with 5 peaks, 8 peaks and 10 peaks. How many sticks would be needed to build a mountain range with 100 peaks?

Use isometric dot paper to create a diagram that represents your mountain range models. Be sure to record your response in a systematic manner and demonstrate on your model how you solved the problem.

Can you determine how many sticks would be needed to build a model of a mountain range with any number of peaks?



Extension:

Mountain Range Challenge PR1, PR2

Teacher Notes:

Outcome Objectives:

Patterns and Relations (Patterns) 1-Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2–3.3]

Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

- $ax = b$
- $\frac{x}{a}=b$, $a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c$, $a \neq 0$
- $a(x + b) = c$

where *a*, *b* and *c* are integers. [C, CN, PS, V]

Material Suggestions:

- toothpicks or popsicle sticks
- triangular dot paper

Sample Solutions:

Figure	# popsicle sticks
1	6
2	10
3	14
4	18
5	22

- 4 more popsicle sticks are added each time
- If the first figure is the constant (6 sticks), then the chart looks like this:

Figure (n)	constant	Additional sticks
1	6	0
2	6	4
3	6	8
4	6	12
n	6	4(n-1)

Notes:

There are a few different expressions, so answers may vary.

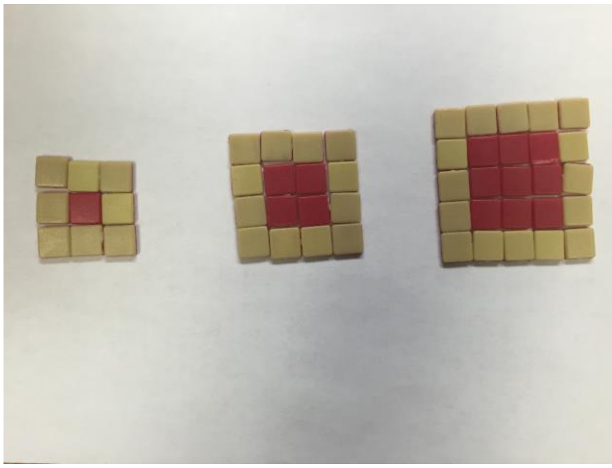
**Credit to NCTM*

Picture Frames PR1, PR2

Problem:

Build a sequence of squares with a different colour border.

What different patterns are there? How can you represent these different patterns using a table, a graph and algebra?



Extension:

Picture Frames PR1, PR2

Teacher Notes:

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Outcome Objectives:

Patterns and Relations (Patterns) 1-Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2–3.3]

Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

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- $\frac{x}{a}=b$, $a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c$, $a \neq 0$
- $a(x + b) = c$

where *a*, *b* and *c* are integers. [C, CN, PS, V]

Material Suggestions:

- square tiles
- graph paper
- markers

Sample Solutions:

This exploration is wide open. Students may look at the inside tiles, the border tiles or the total tiles.

For example, the number of inside tiles are square numbers. This can be shown in a chart or a sequence of numbers.

The total tiles is $(n+2)^2$ when n represents the figure number.

The number of outside tiles is $(n+2)^2 – n^2$. – the total tiles less the inside tiles.

Notes:

**Credit to Mathematics Assessment Sampler, Grades 6-8,NCTM*

Pool Border Problem PR1, PR2

Problem:

Find the number of one-by-one tiles required to surround a 4 by 3 pool. Find a rule to predict the number of tiles required to surround a retangular pool of any size. See if you can express the rule as a visual representation and as an expression.

Extension:

What if the pool was not rectangular?

Pool Border Problem PR1, PR2

Teacher Notes:

Outcome Objectives:

Patterns and Relations (Patterns) 1-Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2–3.3]

Patterns and Relations (Variables and Equations) 2-Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

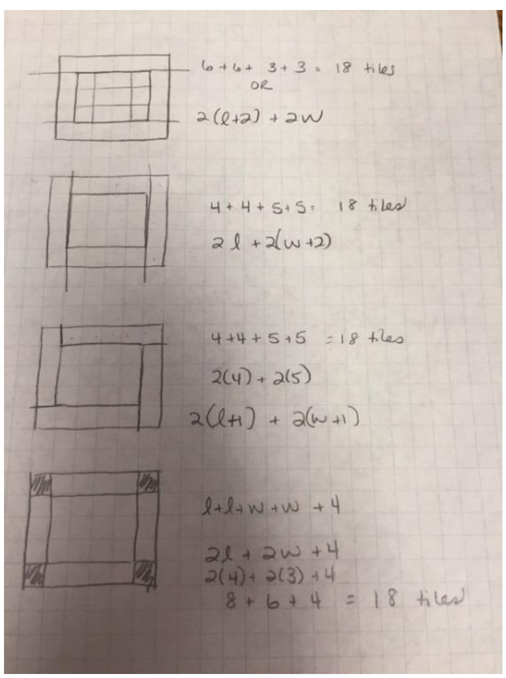
- $ax = b$
- $\frac{x}{a}=b$, $a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c$, $a \neq 0$
- $a(x + b) = c$

where *a*, *b* and *c* are integers. [C, CN, PS, V]

Material Suggestions:

- tiles
- graph paper

Sample Solutions:



Notes:

See sample solution first for the connection between the different expressions and their visual representations.

**Credit to David Pimm*

Staircases PR1, PR2

Problem:

A group of students are building staircases out of wooden cubes. The 1-step staircase consists of one cube, and the 2-step staircase consists of three cubes stacked (see fig. 1).

How many cubes will be needed to build a 3-step staircase? A 6-step staircase? A 50-step staircase? An n-step staircase?

Extension:

What about these?

Staircases PR1, PR2

Teacher Notes:

Outcome Objectives:

Patterns and Relations (Patterns) 1-Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2–3.3]

Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

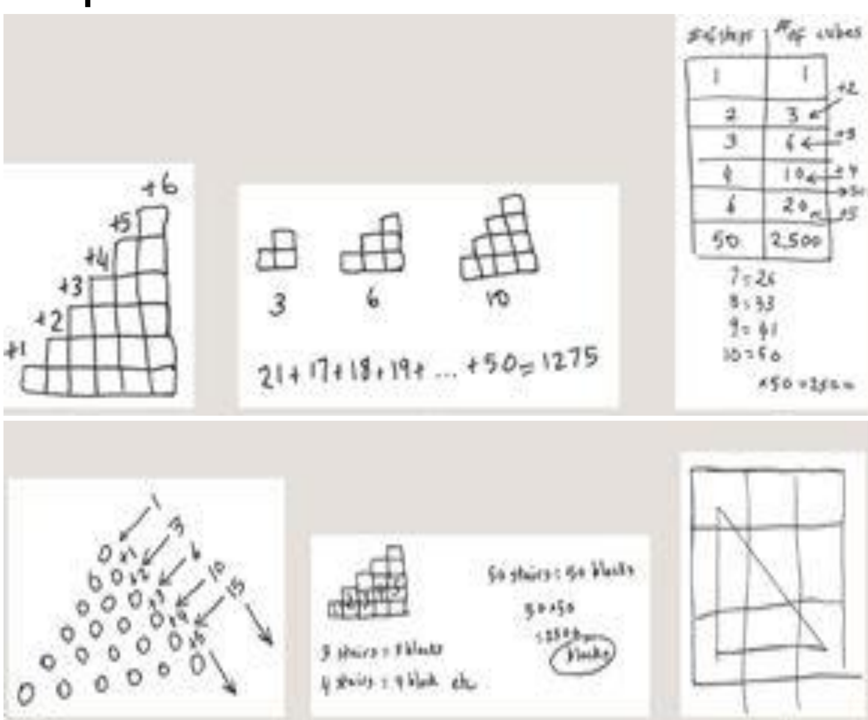
- $ax = b$
- $\frac{x}{a}=b,\ a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c,\ a \neq 0$
- $a(x + b) = c$

where a, b and c are integers. [C, CN, PS, V]

Material Suggestions:

- unifix cubes or blocks
- graph paper

Sample Solutions:



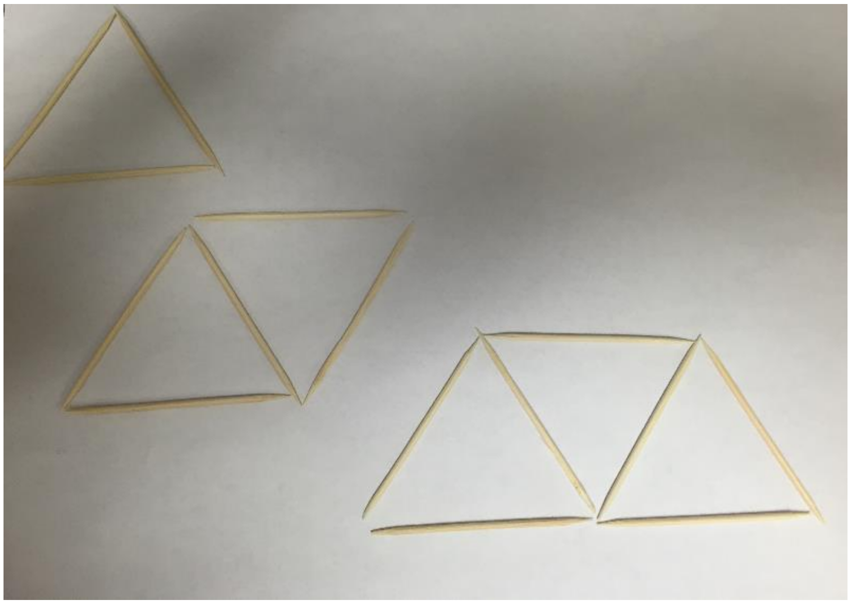
Notes:

**Credit to Vol. 108, No. 9 • May 2015* | *MATHEMATICS TEACHER 663*

Toothpick Pattern PR1, PR2

Problem:

While waiting for my food at a restaurant I began playing with toothpicks. This is what I came up with:



Then I started wondering about my pattern…

How many toothpicks are in each figure?

How is my pattern growing?

How would I know how many toothpicks I would need for the next structure?

If I followed the pattern, which one would require 27 toothpicks to build?

Can I find an expression that represents the rule for the number of toothpicks in Figure ***n***?

Extension:

Make and investigate another toothpick pattern.

Toothpick Pattern PR 1, PR2

Outcome Objectives:

Patterns and Relations (Patterns) 1-Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2–3.3]

Patterns and Relations (Variables and Equations) 2-Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

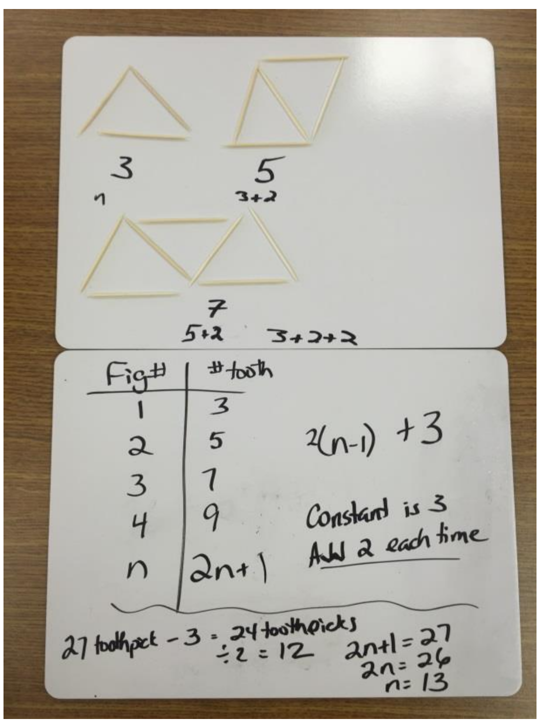
- $ax = b$
- $\frac{x}{a}=b$, $a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c$, $a \neq 0$
- $a(x + b) = c$

where *a*, *b* and *c* are integers. [C, CN, PS, V]

Material Suggestions:

- toothpicks
- triangular dot paper

Sample Solutions:



Notes:

**Credit to Mathematics Assessment Sampler, Grades 6-8, NCTM*

Teacher Notes:

Skyscraper Windows PR1, PR2, SS2

Problem:

A building is 12 stories high and is covered entirely by windows on all four sides. Each floor has 38 windows on it. Once a year, all the windows are washed. The cost for washing each window depends upon the window's floor:

Floor	Cost per window
1	$2.00
2	$2.50
3	$3.00
...	...

For example, each window on Floor 2 costs $2.50 to wash. This pricing scale continues for the windows on each floor.

1. How much will it cost to wash all the windows of this building?
2. If the building is 30 stories tall, how much will it cost to wash all the windows?
3. If the building is n stories tall, how much will it cost to wash all the windows?

Extension:

Skyscraper Windows PR1, PR2, SS2

Teacher Notes:

Outcome Objectives:

Shape and Space (Measurement) 2-Draw and construct nets for 3-D objects. [C, CN, PS, V]

Patterns and Relations (Patterns) 1-Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2–3.3]

Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

- $ax = b$
- $\frac{x}{a} = b$, $a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c$, $a \neq 0$
- $a(x + b) = c$

where *a*, *b* and *c* are integers. [C, CN, PS, V]

Material Suggestions:

Sample Solutions:

1. It would cost $2394.00 to wash the windows of this building.

Floor	Cost per Floor	Cost of 38 windows
1	$2.50	$95.00
2	$3.00	$114.00
3	$3.50	$133.00
4	$4.00	$152.00
5	$4.50	$171.00
6	$5.00	$190.00
7	$5.50	$209.00
8	$6.00	$228.00
9	$6.50	$247.00
10	$7.00	$266.00
11	$7.50	$285.00
12	$8.00	$304.00
Total		$2,394.00

A pairing strategy can also be used:
6 groups of $10.50 x 38 windows (cost of 12th floor plus 1st floor is $10.50, 2nd floor plus 11th floor is $10.50…)

Skyscraper Windows PR1, PR2, SS2

Teacher Notes:

Sample Solutions Continued:

2. For 30 windows there would be 15 pairs of $19.50 x 38 = $11 115.00

3. n windows would cost …$2.00 + $0.50n where n is the floor.

There is a basic constant of $2.00, plus an increase of 50¢ per floor, expressed as $0.50n.

Floor	Cost per Floor	Cost of 38 Windows	Constant	.50n	Cost per Floor
1	$2.50	$95.00	2	$0.50	$2.50
2	$3.00	$114.00	2	$1.00	$3.00
3	$3.50	$133.00	2	$1.50	$3.50
4	$4.00	$152.00	2	$2.00	$4.00
5	$4.50	$171.00	2	$2.50	$4.50
6	$5.00	$190.00	2	$3.00	$5.00
7	$5.50	$209.00	2	$3.50	$5.50
8	$6.00	$228.00	2	$4.00	$6.00
9	$6.50	$247.00	2	$4.50	$6.50
10	$7.00	$266.00	2	$5.00	$7.00
11	$7.50	$285.00	2	$5.50	$7.50
12	$8.00	$304.00	2	$6.00	$8.00
Total		$2,394.00			

Notes:

**Credit to Driscoll*

51-Foot Ladder SS1

Problem:

Is a 51-foot ladder tall enough to get over a 50-foot wall?
Prove your solution mathematically.

What is a safe ladder angle? What are the safety recommendations?
How far away do you place the base of the ladder from the wall?

Ladder Trucks can rescue people from different floors of a building. Some ladder trucks have 100-foot ladders. What floor would you want to stay on to ensure that you could be rescued if there was a fire?

How high are ceilings? How tall is the ladder truck?

Extension:

How tall would your ladder need to be to get over a 50-foot wall?

51-Foot Ladder SS1

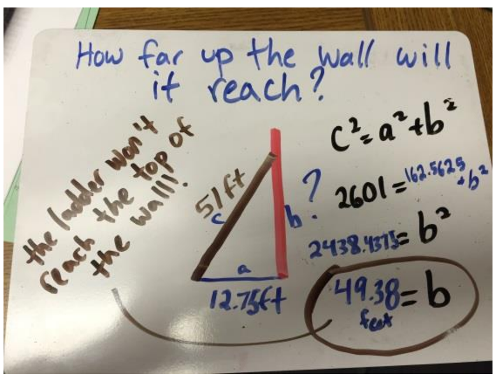
Teacher Notes:

Outcome Objectives:

Shape and Space (Measurement) 1-Develop and apply the Pythagorean theorem to solve problems. [CN, PS, R, T, V] [ICT: P2–3.4]

Material Suggestions:

Sample Solutions:



Solutions will vary, especially for the ladder truck part of the problem which depends largely on assumptions (height of ladder truck, ceiling heights of building, etc.). Lots of discussion needed during and after.

Notes:

**Credit to Mathalicious*

Pringles SS1,SS3

Problem:

Create a paper cone that fits exactly inside a snack-size Pringles can. You will be able to exactly fit the circumference of your cone in the bottom of the can and the point will touch the lid exactly.
.

Extension:
What if it was a regular sized Pringles can?

Pringles

SS1,SS3

Teacher Notes:

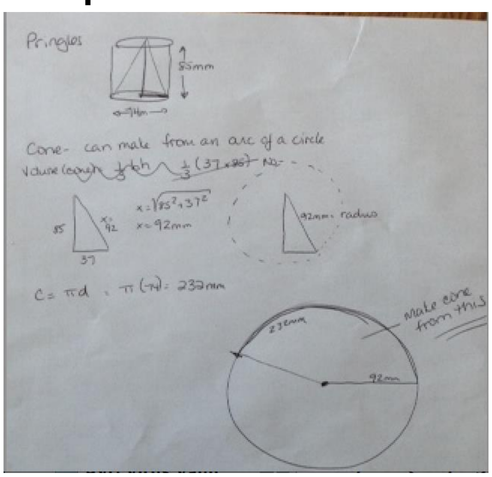
Outcome Objectives:

Shape and Space 1- Develop and apply the Pythagorean theorem to solve problems. [CN, PS, R, T, V]
Shape and Space 3- Determine the surface area of:
• right rectangular prisms
• right triangular prisms
• right cylinders to solve problems.
[C, CN, PS, R, V]

Material Suggestions:

- Small Pringles Can Snack Size
- Printer paper
- Compass
- Ruler, scissors, tape

Sample Solutions:



- The key is to recognize first that a circle of some sort is needed to form the cone.
- Next is to realize that the hypotenuse of the triangle formed by the height of the pringles' container and the half the diameter is actually the radius of the circle needed.
- Finally, the whole circle is not required – only the length of arc equal to the circumference of the pringles container – this can be measured with a string.
- Cut out that piece of the circle, and voila! – tape the sides of the cone and insert into container.

Notes:

**Credit to Peter Liljedahl*

How Much Popcorn? SS2,SS3,SS4

Problem:

Which holds more popcorn? A cylinder made of a rectangular piece of paper rolled widthwise or a cylinder made of the same size of paper rolled lengthwise? Explain why.

Extension:
What if the containers weren't cylinders?

How Much Popcorn? SS2,SS3,SS4

Teacher Notes:

Outcome Objectives:

Shape and Space 2 – Draw and construct nets for 3-D objects. [PS, R, T, V]

Shape and Space 3 – Determine the surface area of: right rectangular prisms, right triangular prisms, right cylinders to solve problems. [C, CN, PS, V]

Shape and Space 4 – Develop and apply formulas for determining the volume of right rectangular prisms, right triangular prisms, and right cylinders. [C, CN, PS, R, V]

Material Suggestions:

- Popcorn
- Rectangular paper

Sample Solutions:

The students will discover that even though the surface area of the two cylinders is the same, their volumes are different. The shorter cylinder will hold more popcorn because the surface area of the base is larger.

Notes:

**Credit to Figure This!*

Pentominoes SS5,SS6

Problem:

A **pentomino** is a plane geometric figure formed by joining five equal squares edge to edge. **Using snap cubes, how many different 3-D "pentominoes" can you make?** (The pentomino obtained by reflecting or rotating a pentomino does not count as a different pentomino).

Extension:

Can you tile a rectangular box with each pentomino? Can you tile a rectangular box with a mix of pentominoes? Can you use all of the pentominoes once to tile a rectangular box?

Pentominoes SS5, SS6

Teacher Notes:

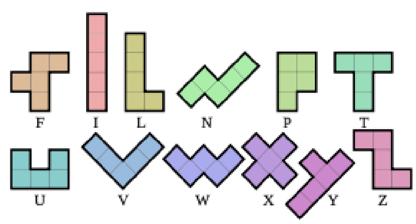
Outcome Objectives:

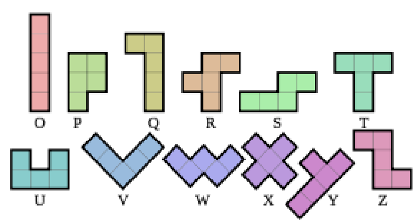
Shape and Space 5 – Draw and interpret top, front and side views of 3-D objects composed of right rectangular prisms.
[C, CN, R, T, V]
Shape and Space 6 – Demonstrate an understanding of the congruence of polygons. [CN, R, V]

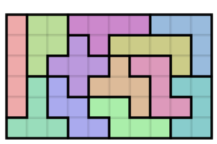
Material Suggestions:

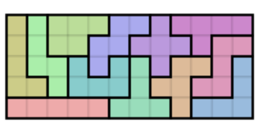
- Snap cubes
- Masking Tape to make a rectangle

Sample Solutions:






6 × 10


5 × 12


4 × 15


3 × 20

Notes:

**Credit to Peter Liljedahl and Wikipedia*

Eyewitness SP2

Problem:

In a certain town, 90% of the cars are purple, 10% are blue. A crime is committed. An eye-witness says the thief drove off in a blue car. Testing shows that the witness can correctly identify colour 80% of the time. What is the probability that the car really was blue?

Extension:

Eyewitness SP2

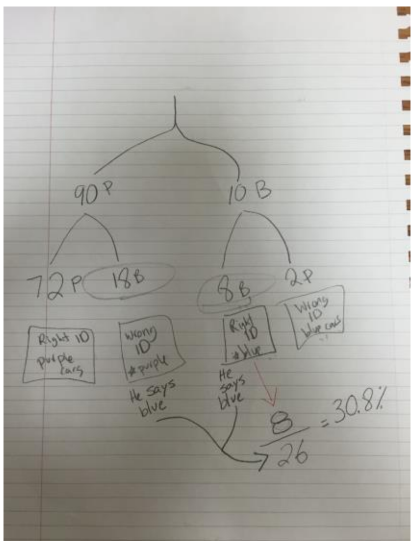
Teacher Notes:

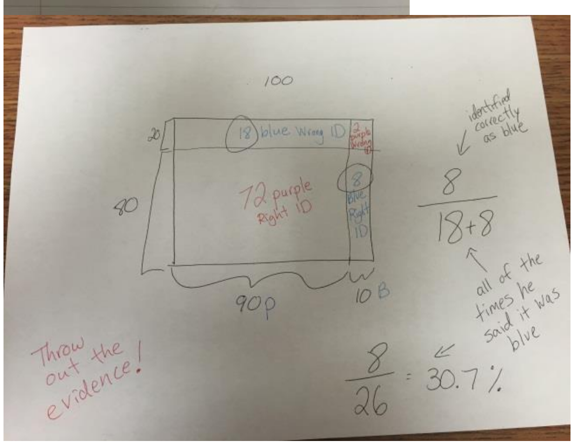
Outcome Objectives:

Statistics and Probability 2 – Solve problems involving the probability of independent events. [C, CN, PS, R]

Material Suggestions:

Sample Solutions:





Notes:

**Credit to James Tanton*

Hat and Rabbit SP2

Problem:

A magician has a hat with a rabbit in it. The rabbit is either black or white. She then places a white rabbit into the hat to join the other rabbit. What is the probability that a white rabbit is left in the hat after one has been pulled out?

Extension: What if there was a different number of rabbits?

Hat and Rabbit SP2

Teacher Notes:

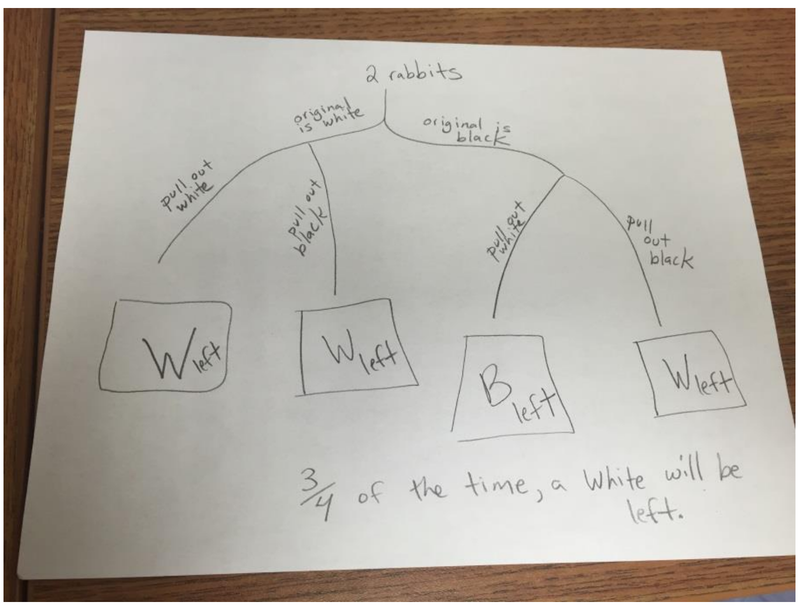
Outcome Objectives:

Statistics and Probability 2 – Solve problems involving the probability of independent events. [C, CN, PS, R]

Material Suggestions:

- A top hat
- Rabbits

Sample Solution:



Note:

**Credit to Ian Stewart*

The Monte Hall Problem SP2

Problem:

A TV host shows you three numbered doors (all three equally likely), one hiding a car and the other two hiding goats. You get to pick a door, winning whatever is behind it. Regardless of the door you choose, the host, who knows where the car is, then opens one of the other two doors to reveal a goat, and invites you to switch your choice if you so wish. Does switching increases your chances of winning the car?

Should you stick or switch? Why?

Link to online simulation. http://www.grand-illusions.com/simulator/montysim.htm

Extension:

The Monte Hall Problem SP2

Teacher Notes:

Outcome Objectives:

Statistics and Probability 2 – Solve problems involving the probability of independent events. [C, CN, PS, R]

Material Suggestions:

- manipulatives

Sample Solutions:

The probability of winning the car is 1/3 if you stick and 2/3 if you switch, so you should always switch.

Assume that you always start by picking Door #1, and the host then always shows you some other door which does *not* contain the car, and you then always switch to the remaining door.
If the car is behind Door #1, then after you pick Door #1, the host will open another door (either #2 or #3), and you will then switch to the remaining door (either #3 or #2), thus LOSING.
If the car is behind Door #2, then after you pick Door #1, the host will be forced to open Door #3, and you will then switch to Door #2, thus WINNING.
If the car is behind Door #3, then after you pick Door #1, the host will be forced to open Door #2, and you will then switch to Door #3, thus WINNING.
Hence, in 2 of the 3 (equally-likely) possibilities, you will win. Ergo, the probability of winning by switching is 2/3.
(*Sample solution from Jeffrey S. Rosenthal, 2006)

Notes: **Credit to* **American Mathematical Monthly, January 1992**