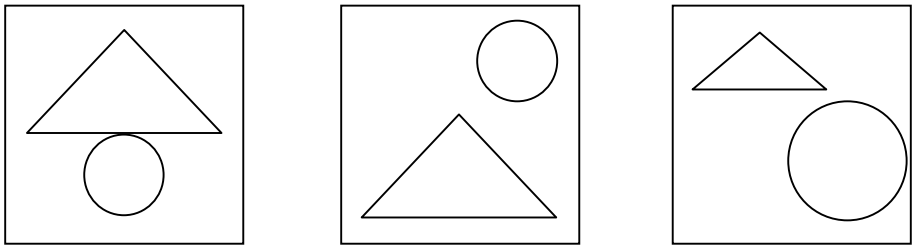
Algebra III – Polynomial and Polynomial Functions

A Picture Is Worth a Thousand Words

Have you ever played a game where a person describes a picture and you must draw it using only the instructions given? For example, the instructions could be: draw a square; then draw a triangle in the square; then draw a circle inside the square but outside the triangle. What kind of picture will you get? Here are a few examples that have been drawn from the description:



Could better directions have been given to create pictures that more closely resemble each other? How specific must your instructions be so that every picture is exactly the same? Is there a better way to use numbers and/or symbols to describe a picture exactly? This is a worthwhile question because if a picture can be described exactly, it can be placed into a computer and the computer can make an exact copy. This process is used for computer-animated movies, many types of mathematical analysis and reproduction of experiments that would otherwise be too dangerous and/or costly to conduct. Not to mention all the images that you've encountered playing video games.

It is up to you to find a better way. You are the artist and the mathematician. Your job will be to use polynomial functions to create a picture. If you describe your polynomial function exactly, anyone can create an exact replica of your picture.

Chapter 6 Polynomials and Polynomial Functions shows how to graph and describe polynomial functions. As you have seen with quadratic functions, you can describe polynomials by their end behavior; x-intercepts; number of turns of the graph; and their relative maxima and minima. Points on each graph can also be determined by evaluating the function for different values of *x*.

To begin your project, you might choose to experiment with different polynomial functions to determine one that might serve as the bases for your art project. Then you can then change the coefficients and exponents as well as any constants to achieve a pleasing picture. Lastly, you will describe your functions using as many of the mathematical concepts as you can. You may not reach 1000 words but you will quickly see how describing a function using an equation is more efficient than describing that same function using words.

The Polynomial Art Project will be assessed on three levels

GRAPH: MATH IN THE PICTURE

- Title
- Art Picture (Draw using markers, colored pencils, and or computer program. Use a standard size graph paper. Fill the page. One should be able to clearly see the *x*-intercepts; relative maxima and minima; the points you chose to identify, and etc.)
- Label Functions (Label each function with its correct equation. This can be done by using a key with different colors.)
- Suggestion: turn in *two* drawings –
    - The first showing the functions in "raw" form. Should be on graph/gridded paper.
    - The second showing how the polynomials have been combined to create the final product. Doesn't have to be on graph paper

PAPER PART I: MATH IN EQUATIONS AND WORK

- Describe each function: Intervals of increase and decrease, end behavior, real zeroes, relative minima and maxima. To exceed, describe any complex zeroes
- Select several points on the graph: Evaluate the functions for the selected values of *x*. Listing a table of values is not sufficient. You must show your work (show your algebra!)

PAPER PART II: MATH IN WORDS

- Comparison of functions: Compare each function to a least one other function. Describe any similarities or differences and describe how the similarities or differences can be determined from the function itself.
- Summary of process and thinking: What mathematical concepts were illustrated in the Polynomial Art Project? What challenges did you encounter as you completed the project? How did you solve these challenges? What did you learn from this project? I.E. Think about your thinking (meta-cognition)

Here are some topics from Chapter to 6 to review before creating your Polynomial Art Project.

- Graphs of Polynomial Functions (pp. 306 – 311)
- End Behavior (p. 312)
- Writing Polynomials from roots/zeroes (pp. 313-315)
- Zeroes & Multiplicity of zeroes (pp. 313-319)
- Relative minima/maxima (pp. 313-319)
- Remainder Theorem (pp. 323 – 325)
- Fundamental Theorem of Algebra (pp. 335, 337, 338, 341 – 344)

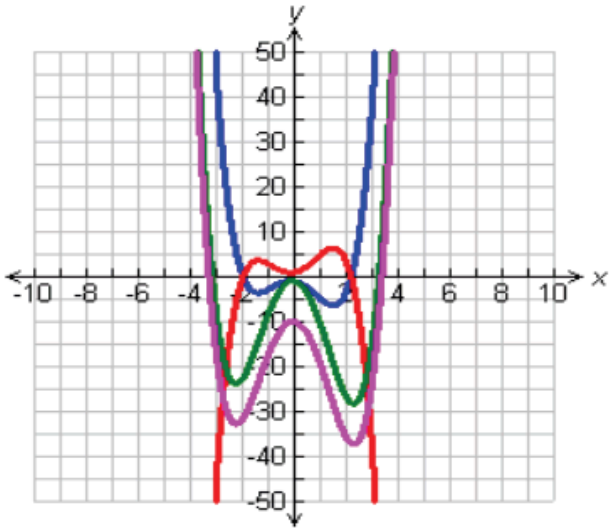
Here is a list of problems that might help you make sense of the material and check your understanding of the vocabulary and concepts. You do not need to do all theses problems, but they allow you to practice the pieces required for the project.

p. 306 (tech activity)
p. 310 (#33 – 54)
p. 312
p. 317 (#7-37)
p. 324 (#23-28)
p. 331 (#42-60)
p. 339 (#25-28)
p. 343 (#21-27)

To help you get started, here is an example of Polynomial Art and how two of the functions compare. (*Your* project will describe each function in more detail)

Graph 1) $y = x^4 - 4x^2 - x - 1$
Graph 2) $y = -x^4 + 4x^2 + x + 1$
Graph 3) $y = x^4 - 10x^2 - x - 1$
Graph 4) $y = x^4 - 10x^2 - x - 10$

Graph 2 is a reflection of **Graph 1** over the x-axis by changing the sign of each term.
Graph 4 is a shift down of **Graph 3** by changing the constant.



Project Details:

Rubric: Use the attached rubric to help you organize and complete the project. The rubric can be used as an outline of what the project is assessing. You will hand this rubric in with your project.

Objective: To make sense of the mathematics of polynomials and their properties in a visual way that is meaningful to your. Think about your thinking and processing of mathematics of polynomials. Communicate with your peers about your understanding and in turn listen to and push the thinking of your peers as you continue your study of the topics listed from chapter 6. Each student will do his/her own work, but is encouraged to work with other students to understand concepts, share ideas, and etc.)

Plan: How will you organize the graph to show your understanding? How will you organize the paper to show your work and summarize your learning? (You will have two class periods to work on the project and concepts needed for the project. Come prepared and use your time wisely.)

Materials: Graphing calculator, graph paper, colored writing utensils … a creative mind

Algebra III – Polynomial Art Project

NAME:

	Exceeding	Meeting	Approaching	Beginning	No Evidence
F-IF 7 Graph polynomial functions expressed symbolically and show key features of the graph.	Fills the page, is neatly and correctly graphed, includes title and correct labeling of axes, functions and scales. Includes correct domain and range of each function. Includes at least four functions of third degree or higher.	Fills the page, is neatly graphed, includes title, labeling of axes, functions, and scales with minimal errors. Includes correct domain and range for at least one function. Includes at least three functions of third degree or higher	Fills most of the page. Effort is made to complete graph with labels, functions and scales but is less than 60% complete or correct. Includes at least two functions of third degree or higher.	Effort is made to complete graph with labels, but is less than 50% complete or correct.	Little or no effort to follow the requirements of the project.
F-IF 4 Interpret key features of graphs and tables in terms of quantities and sketch graphs showing key features (intervals of increase and decrease; relative maximums and minimums; and end behavior)	Labels and identifies intervals of increase and decrease; relative maximums and minimums; and end behavior for each function. Clearly based on the polynomial functions and verified with selected points from the graphs.	Labels and identifies intervals of increase and decrease; relative maximums and minimums; and end behavior for most functions. Clearly based on the polynomial functions and verified with selected points from the graphs. Minor errors present.	Labels and identifies the 2 of 4 key features for each function. Incorrect, or is based on the graphed function only.	Labels and identifies 1 of the 4 key features for the each function.	Little or no effort to follow the requirements of the project.
A-APR 3 Identify zeroes of polynomials, factor polynomials, use the zeroes to graph polynomial functions. (These zeroes can also be called *x*-intercepts.)	Labels and identifies the real zeroes of each function. Complex zeroes are given for the polynomials in which they are present.	Labels and identifies the real zeroes of each function. Minor errors present.	Labels and identifies real zeroes for at least half of the functions. Less than 60% of the zeroes are correctly identified.	Labels and identifies few zeroes for the functions. Less than 50% of the zeroes are correctly identified	Little or no effort to follow the requirements of the project.
A-APR 2 Remainder Theorem	Clearly explained in own words. Use correctly to find the coordinates of one point of interest for each of your functions.	Explained in own words. Use correctly to find the coordinates of one point of interest on at least 2 of your functions.	No explanation or "book" explanation. Attempt to find the coordinates of one point of interest on at least one of your functions.	Little or no explanation. Attempt to find the coordinates of one point of interest on 1 of your functions.	Little or no effort to follow the requirements of the project.
Paper Part II • Comparison of Functions • Reflection on Learning	Compare each function to at least two other functions. Describe any similarities and differences Answers all four reflection questions thoroughly.	Compare each function to a least one other function. Describe any similarities or differences. Answers three reflection questions thoroughly.	Compares most of the functions to at least one other function. Some errors in description of similarities and/or differences Reflects on project	Compares one function to another function. Some errors in description of similarities and/or differences. Comments on project.	Little or no effort to follow the requirements of the project.