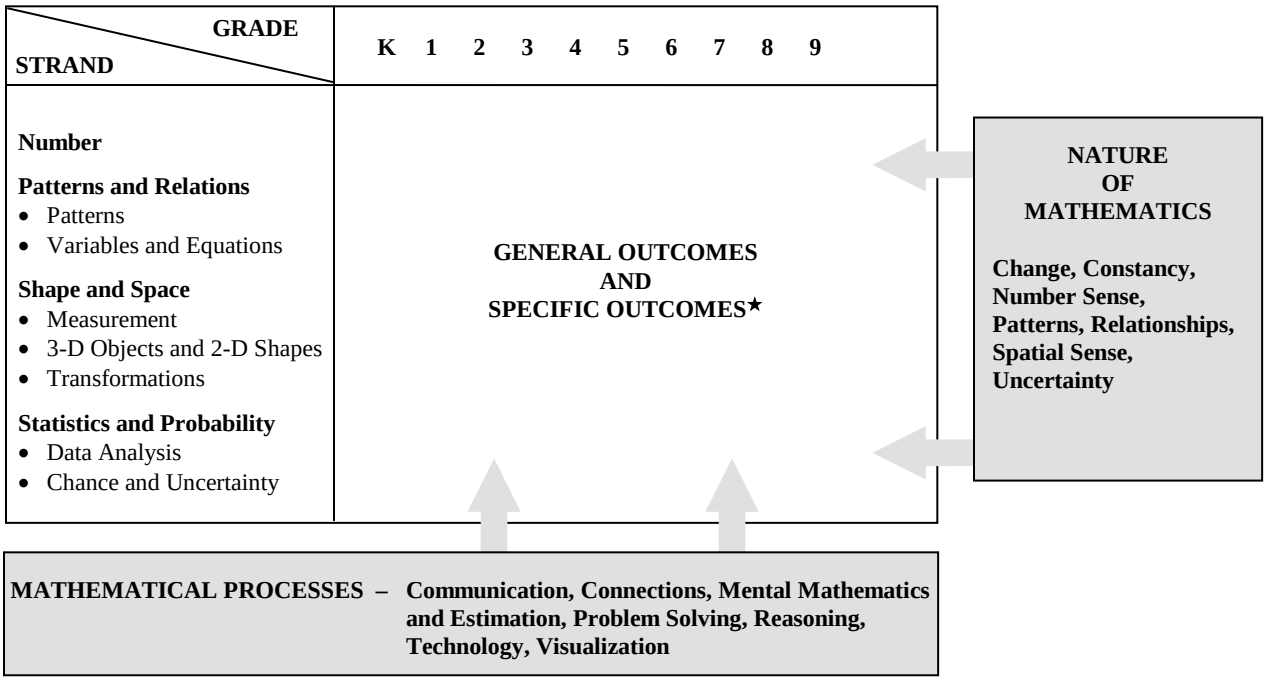
CONCEPTUAL FRAMEWORK FOR K–9 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



★ Achievement indicators for the prescribed program of studies outcomes are provided in the companion document *Alberta K–9 Mathematics Achievement Indicators*, 2014.

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and embrace lifelong learning in mathematics.

Students are expected to:

Communication [C]	• communicate in order to learn and express their understanding
Connections [CN]	• connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines
Mental Mathematics and Estimation [ME]	• demonstrate fluency with mental mathematics and estimation
Problem Solving [PS]	• develop and apply new mathematical knowledge through problem solving
Reasoning [R]	• develop mathematical reasoning
Technology [T]	• select and use technologies as tools for learning and for solving problems
Visualization [V]	• develop visualization skills to assist in processing information, making connections and solving problems.

The program of studies incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

COMMUNICATION [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing and modifying ideas, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication helps students make connections among concrete, pictorial, symbolic, oral, written and mental representations of mathematical ideas.

CONNECTIONS [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. This can be particularly true for First Nations, Métis and Inuit learners. When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding.… Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching" (Caine and Caine, 1991, p. 5).

MENTAL MATHEMATICS AND ESTIMATION [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

"Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math" (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001, p. 442).

Mental mathematics "provides the cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers" (Hope, 1988, p. v).

Estimation is used for determining approximate values or quantities or for determining the reasonableness of calculated values. It often uses benchmarks or referents. Students need to know when to estimate, how to estimate and what strategy to use.

Estimation assists individuals in making mathematical judgements and in developing useful, efficient strategies for dealing with situations in daily life.

PROBLEM SOLVING [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type *How would you …?* or *How could you …?*, the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies.

A problem-solving activity must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk takers.

REASONING [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for students to develop their ability to reason. Students can explore and record results, analyze observations, make and test generalizations from patterns, and reach new conclusions by building upon what is already known or assumed to be true.

Reasoning skills allow students to use a logical process to analyze a problem, reach a conclusion and justify or defend that conclusion.

TECHNOLOGY [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts
- develop personal procedures for mathematical operations
- create geometric patterns
- simulate situations
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels.

VISUALIZATION [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and which estimation strategies to use (Shaw and Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this program of studies. The components are change, constancy, number sense, patterns, relationships, spatial sense and uncertainty.

CHANGE

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- the number of a specific colour of beads in each row of a beaded design
- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

CONSTANCY

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The ratio of the circumference of a teepee to its diameter is the same regardless of the length of the teepee poles.
- The sum of the interior angles of any triangle is 180°.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

NUMBER SENSE

Number sense is an intuition about numbers. Number sense develops when students connect numbers to their own real-life experiences and when students use benchmarks and referents. This results in students who are computationally fluent and flexible with numbers.

A true sense of number includes and goes beyond the skills of counting, memorizing facts and the situational rote use of algorithms. Mastery of number facts occurs when students understand and recall facts and is expected to be attained by students as they develop their number sense. This mastery allows for application of number facts and facility with more complex computations.

Number sense can be developed by providing rich mathematical tasks that allow students to make connections to their own experiences and their previous learning.

PATTERNS

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands of this program of studies.

Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with, and understanding of, their environment.

Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another.

Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and nonroutine problems.

Learning to work with patterns in the early grades helps students develop algebraic thinking, which is foundational for working with more abstract mathematics in higher grades.

RELATIONSHIPS

Mathematics is one way to describe interconnectedness in a holistic worldview. Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves collecting and analyzing data and describing relationships visually, symbolically, orally or in written form.

SPATIAL SENSE

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics.

Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes and to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of shapes and objects. Spatial sense allows students to make predictions about the results of changing these dimensions; e.g., doubling the length of the side of a square increases the area by a factor of four. Ultimately, spatial sense enables students to communicate about shapes and objects and to create their own representations.

UNCERTAINTY

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty.

The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Strands

The learning outcomes in the program of studies are organized into four strands across the grades K–9. Some strands are subdivided into substrands. There is one general outcome per substrand across the grades K–9.

The strands and substrands, including the general outcome for each, follow.

NUMBER
- Develop number sense.

PATTERNS AND RELATIONS
Patterns
- Use patterns to describe the world and to solve problems.

Variables and Equations
- Represent algebraic expressions in multiple ways.

SHAPE AND SPACE
Measurement
- Use direct and indirect measurement to solve problems.

3-D Objects and 2-D Shapes
- Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Transformations
- Describe and analyze position and motion of objects and shapes.

STATISTICS AND PROBABILITY
Data Analysis
- Collect, display and analyze data to solve problems.

Chance and Uncertainty
- Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

An across-the-grades listing of outcomes by strand is provided in Appendix 1.

Outcomes

The program of studies is stated in terms of general outcomes and specific outcomes.

General outcomes are overarching statements about what students are expected to learn in each strand/substrand. The general outcome for each strand/substrand is the same throughout the grades.

Specific outcomes are statements that identify the specific skills, understanding and knowledge that students are required to attain by the end of a given grade.

In the specific outcomes, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for illustrative purposes or clarification, and are not requirements that must be addressed to fully meet the learning outcome. Students investigate a variety of strategies and become proficient in at least one appropriate and efficient strategy that they understand. Strategies may include traditional algorithms such as long division and vertical addition; however, specific strategies are not prescribed in the outcomes. The teaching professional has the flexibility and responsibility to meet the learning needs of each of his or her students. Over time, students refine their strategies to increase their accuracy and efficiency.

Links to Information and Communication Technology (ICT) Outcomes

Some curriculum outcomes from Alberta Education's Information and Communication Technology (ICT) Program of Studies can be linked to outcomes in the mathematics program so that students will develop a broad perspective on the nature of technology, learn how to use and apply a variety of technologies, and consider the impact of ICT on individuals and society. The connection to ICT outcomes supports and reinforces the understandings and abilities that students are expected to develop through the

general and specific outcomes of the mathematics program. Effective, efficient and ethical application of ICT outcomes contributes to the mathematics program vision.

Links to the ICT outcomes have been identified for some specific outcomes. These links appear in square brackets below the process codes for an outcome, where appropriate. The complete wording of the relevant outcomes for ICT is provided in Appendix 2.

Summary

The conceptual framework for K–9 mathematics describes the nature of mathematics, mathematical processes and the mathematical concepts to be addressed in Kindergarten to Grade 9 mathematics. The components are not meant to stand alone. Activities that take place in the mathematics classroom should stem from a problem-solving approach, be based on mathematical processes and lead students to an understanding of the nature of mathematics through specific knowledge, skills and attitudes among and between strands.

INSTRUCTIONAL FOCUS

The program of studies is arranged into four strands. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connection between concepts both within and across strands.

Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- Learning mathematics includes a balance between understanding, recalling and applying mathematical concepts.
- Problem solving, reasoning and connections are vital to increasing mathematical fluency and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using manipulatives and be developed concretely, pictorially and symbolically.
- Students bring a diversity of learning styles and cultural backgrounds to the classroom. They will be at varying developmental stages.

GRADE 9

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

NUMBER

General Outcome
Develop number sense.

Specific Outcomes

1. Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:
   - representing repeated multiplication, using powers
   - using patterns to show that a power with an exponent of zero is equal to one
   - solving problems involving powers.

   [C, CN, PS, R]

2. Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents:
   - $(a^m)(a^n) = a^{m+n}$
   - $a^m \div a^n = a^{m-n}, m > n$
   - $(a^m)^n = a^{mn}$
   - $(ab)^m = a^m b^m$
   - $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0.$

   [C, CN, PS, R, T]
   [ICT: P2–3.4]

3. Demonstrate an understanding of rational numbers by:
   - comparing and ordering rational numbers
   - solving problems that involve arithmetic operations on rational numbers.

   [C, CN, PS, R, T, V]
   [ICT: P2–3.4]

4. Explain and apply the order of operations, including exponents, with and without technology.
   [PS, T]
   [ICT: P2–3.4]

5. Determine the square root of positive rational numbers that are perfect squares.
   [C, CN, PS, R, T]
   [ICT: P2–3.4]

6. Determine an approximate square root of positive rational numbers that are non-perfect squares.
   [C, CN, PS, R, T]
   [ICT: P2–3.4]

PATTERNS AND RELATIONS (Patterns)

General Outcome
Use patterns to describe the world and to solve problems.

Specific Outcomes
1. Generalize a pattern arising from a problem-solving context, using a linear equation, and verify by substitution.
   [C, CN, PS, R, V]
2. Graph a linear relation, analyze the graph, and interpolate or extrapolate to solve problems.
   [C, CN, PS, R, T, V]
   [ICT: C7–3.1, P2–3.3]

PATTERNS AND RELATIONS (Variables and Equations)

General Outcome
Represent algebraic expressions in multiple ways.

Specific Outcomes
3. Model and solve problems, using linear equations of the form:
   - $ax = b$
   - $\frac{x}{a} = b$, $a \neq 0$
   - $ax + b = c$
   - $\frac{x}{a} + b = c$, $a \neq 0$
   - $ax = b + cx$
   - $a(x + b) = c$
   - $ax + b = cx + d$
   - $a(bx + c) = d(ex + f)$
   - $\frac{a}{x} = b$, $x \neq 0$

   where a, b, c, d, e and f are rational numbers.
   [C, CN, PS, V]
4. Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.
   [C, CN, PS, R, V]
5. Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2).
   [C, CN, R, V]
6. Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2).
   [C, CN, PS, R, V]
7. Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically.
   [C, CN, R, V]

SHAPE AND SPACE (Measurement)

General Outcome
Use direct and indirect measurement to solve problems.

Specific Outcomes
1. Solve problems and justify the solution strategy, using the following circle properties:
   - the perpendicular from the centre of a circle to a chord bisects the chord
   - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
   - the inscribed angles subtended by the same arc are congruent
   - a tangent to a circle is perpendicular to the radius at the point of tangency.

   [C, CN, PS, R, T, V]
   [ICT: C6–3.1, C6–3.4]

SHAPE AND SPACE (3-D Objects and 2-D Shapes)

General Outcome
Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Specific Outcomes
2. Determine the surface area of composite 3-D objects to solve problems.
   [C, CN, PS, R, V]

3. Demonstrate an understanding of similarity of polygons.
   [C, CN, PS, R, V]

SHAPE AND SPACE (Transformations)

General Outcome
Describe and analyze position and motion of objects and shapes.

Specific Outcomes
4. Draw and interpret scale diagrams of 2-D shapes.
   [CN, R, T, V]
   [ICT: C6–3.4]

5. Demonstrate an understanding of line and rotation symmetry.
   [C, CN, PS, V]

STATISTICS AND PROBABILITY (Data Analysis)

General Outcome
Collect, display and analyze data to solve problems.

Specific Outcomes
1. Describe the effect of:
   - bias
   - use of language
   - ethics
   - cost
   - time and timing
   - privacy
   - cultural sensitivity

   on the collection of data.
   [C, CN, R, T]
   [ICT: F4–3.2, F4–3.3]

STATISTICS AND PROBABILITY (Data Analysis) (continued)

2. Select and defend the choice of using either a population or a sample of a population to answer a question.
   [C, CN, PS, R]

3. Develop and implement a project plan for the collection, display and analysis of data by:
   - formulating a question for investigation
   - choosing a data collection method that includes social considerations
   - selecting a population or a sample
   - collecting the data
   - displaying the collected data in an appropriate manner
   - drawing conclusions to answer the question.

   [C, PS, R, T, V]
   [ICT: C1–3.5, C4–3.1, C6–3.1, C6–3.2, C7–3.1, C7–3.2, P1–3.4, P2–3.1]

STATISTICS AND PROBABILITY (Chance and Uncertainty)

General Outcome
Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Outcomes
4. Demonstrate an understanding of the role of probability in society.
   [C, CN, R, T]
   [ICT: F4–3.3]

Number

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Grade 5	Grade 6	Grade 7	Grade 8	Grade 9
General Outcome Develop number sense.	**General Outcome** Develop number sense.	**General Outcome** Develop number sense.	**General Outcome** Develop number sense.	**General Outcome** Develop number sense.
Specific Outcomes 1. Represent and describe whole numbers to 1 000 000. [C, CN, V, T] [ICT: C6–2.2] 2. Use estimation strategies, such as: • front-end rounding • compensation • compatible numbers in problem-solving contexts. [C, CN, ME, PS, R, V]	**Specific Outcomes** 1. Demonstrate an understanding of place value, including numbers that are: • greater than one million • less than one thousandth. [C, CN, R, T] 2. Solve problems involving whole numbers and decimal numbers. [ME, PS, T] [ICT: C6–2.4] 3. Demonstrate an understanding of factors and multiples by: • determining multiples and factors of numbers less than 100 • identifying prime and composite numbers • solving problems using multiples and factors. [CN, PS, R, V] 4. Relate improper fractions to mixed numbers and mixed numbers to improper fractions. [CN, ME, R, V]	**Specific Outcomes** 1. Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and why a number cannot be divided by 0. [C, R] 2. Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals to solve problems (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected). [ME, PS, T] [ICT: P2–3.4] 3. Solve problems involving percents from 1% to 100%. [C, CN, PS, R, T] [ICT: P2–3.4] 4. Demonstrate an understanding of the relationship between positive terminating decimals and positive fractions and between positive repeating decimals and positive fractions. [C, CN, R, T] [ICT: P2–3.4]	**Specific Outcomes** 1. Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers). [C, CN, R, V] 2. Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers). [C, CN, ME, R, T] [ICT: P2–3.4] 3. Demonstrate an understanding of percents greater than or equal to 0%, including greater than 100%. [CN, PS, R, V] 4. Demonstrate an understanding of ratio and rate. [C, CN, V]	**Specific Outcomes** 1. Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by: • representing repeated multiplication, using powers • using patterns to show that a power with an exponent of zero is equal to one • solving problems involving powers. [C, CN, PS, R] 2. Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents: • $(a^m)(a^n) = a^{m+n}$ • $a^m \div a^n = a^{m-n}, m > n$ • $(a^m)^n = a^{mn}$ • $(ab)^m = a^m b^m$ • $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0.$ [C, CN, PS, R, T] [ICT: P2–3.4]

Number (continued)

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Grade 5	Grade 6	Grade 7	Grade 8	Grade 9
General Outcome Develop number sense.	**General Outcome** Develop number sense.	**General Outcome** Develop number sense.	**General Outcome** Develop number sense.	**General Outcome** Develop number sense.
Specific Outcomes 3. Apply mental mathematics strategies and number properties, such as: • skip counting from a known fact • using doubling or halving • using patterns in the 9s facts • using repeated doubling or halving in order to understand and recall basic multiplication facts (multiplication tables) to 81 and related division facts. [C, CN, ME, R, V] Understand, recall and apply multiplication and related division facts to 9×9. 4. Apply mental mathematics strategies for multiplication, such as: • annexing then adding zero • halving and doubling • using the distributive property. [C, CN, ME, R, V]	**Specific Outcomes** 5. Demonstrate an understanding of ratio, concretely, pictorially and symbolically. [C, CN, PS, R, V] 6. Demonstrate an understanding of percent (limited to whole numbers), concretely, pictorially and symbolically. [C, CN, PS, R, V] 7. Demonstrate an understanding of integers, concretely, pictorially and symbolically. [C, CN, R, V] 8. Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors). [C, CN, ME, PS, R, V]	**Specific Outcomes** 5. Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences). [C, CN, ME, PS, R, V] 6. Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically. [C, CN, PS, R, V] 7. Compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using: • benchmarks • place value • equivalent fractions and/or decimals. [CN, R, V]	**Specific Outcomes** 5. Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R] 6. Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS] 7. Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically. [C, CN, PS, R, V]	**Specific Outcomes** 3. Demonstrate an understanding of rational numbers by: • comparing and ordering rational numbers • solving problems that involve arithmetic operations on rational numbers. [C, CN, PS, R, T, V] [ICT: P2–3.4] 4. Explain and apply the order of operations, including exponents, with and without technology. [PS, T] [ICT: P2–3.4] 5. Determine the square root of positive rational numbers that are perfect squares. [C, CN, PS, R, T] [ICT: P2–3.4]

Number (continued)

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Grade 5	Grade 6	Grade 7	Grade 8	Grade 9
General Outcome Develop number sense.	**General Outcome** Develop number sense.			**General Outcome** Develop number sense.
Specific Outcomes 5. Demonstrate, with and without concrete materials, an understanding of multiplication (2-digit by 2-digit) to solve problems. [C, CN, PS, V] 6. Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit), and interpret remainders to solve problems. [C, CN, ME, PS, R, V] 7. Demonstrate an understanding of fractions by using concrete, pictorial and symbolic representations to: • create sets of equivalent fractions • compare fractions with like and unlike denominators. [C, CN, PS, R, V]	**Specific Outcomes** 9. Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers). [C, CN, ME, PS, T] [ICT: C6–2.4, C6–2.7]			**Specific Outcomes** 6. Determine an approximate square root of positive rational numbers that are non-perfect squares. [C, CN, PS, R, T] [ICT: P2–3.4]

Patterns and Relations (Patterns)

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Grade 5	Grade 6	Grade 7	Grade 8	Grade 9
General Outcome Use patterns to describe the world and to solve problems.	**General Outcome** Use patterns to describe the world and to solve problems.	**General Outcome** Use patterns to describe the world and to solve problems.	**General Outcome** Use patterns to describe the world and to solve problems.	**General Outcome** Use patterns to describe the world and to solve problems.
Specific Outcomes 1. Determine the pattern rule to make predictions about subsequent elements. [C, CN, PS, R, V]	**Specific Outcomes** 1. Represent and describe patterns and relationships, using graphs and tables. [C, CN, ME, PS, R, V] [ICT: C6–2.3] 2. Demonstrate an understanding of the relationships within tables of values to solve problems. [C, CN, PS, R] [ICT: C6–2.3]	**Specific Outcomes** 1. Demonstrate an understanding of oral and written patterns and their equivalent linear relations. [C, CN, R] 2. Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems. [C, CN, PS, R, V] [ICT: C7–3.1]	**Specific Outcomes** 1. Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2–3.3]	**Specific Outcomes** 1. Generalize a pattern arising from a problem-solving context, using a linear equation, and verify by substitution. [C, CN, PS, R, V] 2. Graph a linear relation, analyze the graph, and interpolate or extrapolate to solve problems. [C, CN, PS, R, T, V] [ICT: C7–3.1, P2–3.3]

Patterns and Relations (Variables and Equations)

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Grade 5	Grade 6	Grade 7	Grade 8	Grade 9
General Outcome Represent algebraic expressions in multiple ways.	**General Outcome** Represent algebraic expressions in multiple ways.	**General Outcome** Represent algebraic expressions in multiple ways.	**General Outcome** Represent algebraic expressions in multiple ways.	**General Outcome** Represent algebraic expressions in multiple ways.
Specific Outcomes 2. Express a given problem as an equation in which a letter variable is used to represent an unknown number (limited to whole numbers). [C, CN, PS, R] 3. Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions. [C, CN, PS, R]	**Specific Outcomes** 3. Represent generalizations arising from number relationships, using equations with letter variables. [C, CN, PS, R, V] 4. Express a given problem as an equation in which a letter variable is used to represent an unknown number. [C, CN, PS, R] 5. Demonstrate and explain the meaning of preservation of equality, concretely and pictorially. [C, CN, PS, R, V]	**Specific Outcomes** 3. Demonstrate an understanding of preservation of equality by: • modelling preservation of equality, concretely, pictorially and symbolically • applying preservation of equality to solve equations. [C, CN, PS, R, V] 4. Explain the difference between an expression and an equation. [C, CN] 5. Evaluate an expression, given the value of the variable(s). [CN, R] 6. Model and solve, concretely, pictorially and symbolically, problems that can be represented by one-step linear equations of the form $x + a = b$, where a and b are integers. [CN, PS, R, V]	**Specific Outcomes** 2. Model and solve problems concretely, pictorially and symbolically, using linear equations of the form: • $ax = b$ • $\frac{x}{a} = b$, $a \neq 0$ • $ax + b = c$ • $\frac{x}{a} + b = c$, $a \neq 0$ • $a(x + b) = c$ where a, b and c are integers. [C, CN, PS, V]	**Specific Outcomes** 3. Model and solve problems, using linear equations of the form: • $ax = b$ • $\frac{x}{a} = b$, $a \neq 0$ • $ax + b = c$ • $\frac{x}{a} + b = c$, $a \neq 0$ • $ax = b + cx$ • $a(x + b) = c$ • $ax + b = cx + d$ • $a(bx + c) = d(ex + f)$ • $\frac{a}{x} = b$, $x \neq 0$ where a, b, c, d, e and f are rational numbers. [C, CN, PS, V] 4. Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context. [C, CN, PS, R, V]

Patterns and Relations (Variables and Equations) (continued)

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Grade 5	Grade 6	Grade 7	Grade 8	Grade 9
		General Outcome Represent algebraic expressions in multiple ways.		**General Outcome** Represent algebraic expressions in multiple ways.
		Specific Outcomes 7. Model and solve, concretely, pictorially and symbolically, problems that can be represented by linear equations of the form: • $ax + b = c$ • $ax = b$ • $\frac{x}{a} = b$, $a \neq 0$ where a, b and c are whole numbers. [CN, PS, R, V]		**Specific Outcomes** 5. Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2). [C, CN, R, V] 6. Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2). [C, CN, PS, R, V] 7. Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically. [C, CN, R, V]

Shape and Space (Measurement)

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Grade 5	Grade 6	Grade 7	Grade 8	Grade 9
General Outcome Use direct and indirect measurement to solve problems.	**General Outcome** Use direct and indirect measurement to solve problems.	**General Outcome** Use direct and indirect measurement to solve problems.	**General Outcome** Use direct and indirect measurement to solve problems.	**General Outcome** Use direct and indirect measurement to solve problems.
Specific Outcomes 1. Identify 90º angles. [ME, V] 2. Design and construct different rectangles, given either perimeter or area, or both (whole numbers), and make generalizations. [C, CN, PS, R, V] 3. Demonstrate an understanding of measuring length (mm) by: • selecting and justifying referents for the unit mm • modelling and describing the relationship between mm and cm units, and between mm and m units. [C, CN, ME, PS, R, V]	**Specific Outcomes** 1. Demonstrate an understanding of angles by: • identifying examples of angles in the environment • classifying angles according to their measure • estimating the measure of angles, using 45°, 90° and 180° as reference angles • determining angle measures in degrees • drawing and labelling angles when the measure is specified. [C, CN, ME, V] 2. Demonstrate that the sum of interior angles is: • 180° in a triangle • 360° in a quadrilateral. [C, R]	**Specific Outcomes** 1. Demonstrate an understanding of circles by: • describing the relationships among radius, diameter and circumference • relating circumference to pi • determining the sum of the central angles • constructing circles with a given radius or diameter • solving problems involving the radii, diameters and circumferences of circles. [C, CN, PS, R, V] 2. Develop and apply a formula for determining the area of: • triangles • parallelograms • circles. [CN, PS, R, V]	**Specific Outcomes** 1. Develop and apply the Pythagorean theorem to solve problems. [CN, PS, R, T, V] [ICT: P2–3.4] 2. Draw and construct nets for 3-D objects. [C, CN, PS, V] 3. Determine the surface area of: • right rectangular prisms • right triangular prisms • right cylinders to solve problems. [C, CN, PS, R, V] 4. Develop and apply formulas for determining the volume of right rectangular prisms, right triangular prisms and right cylinders. [C, CN, PS, R, V]	**Specific Outcomes** 1. Solve problems and justify the solution strategy, using the following circle properties: • the perpendicular from the centre of a circle to a chord bisects the chord • the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc • the inscribed angles subtended by the same arc are congruent • a tangent to a circle is perpendicular to the radius at the point of tangency. [C, CN, PS, R, T, V] [ICT: C6–3.1, C6–3.4]

Shape and Space (3-D Objects and 2-D Shapes)

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Grade 5	Grade 6	Grade 7	Grade 8	Grade 9
General Outcome Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.	**General Outcome** Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.	**General Outcome** Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.	**General Outcome** Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.	**General Outcome** Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.
Specific Outcomes 6. Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are: • parallel • intersecting • perpendicular • vertical • horizontal. [C, CN, R, T, V] [ICT: C6–2.2, P5–2.3] 7. Identify and sort quadrilaterals, including: • rectangles • squares • trapezoids • parallelograms • rhombuses according to their attributes. [C, R, V]	**Specific Outcomes** 4. Construct and compare triangles, including: • scalene • isosceles • equilateral • right • obtuse • acute in different orientations. [C, PS, R, V] 5. Describe and compare the sides and angles of regular and irregular polygons. [C, PS, R, V]	**Specific Outcomes** 3. Perform geometric constructions, including: • perpendicular line segments • parallel line segments • perpendicular bisectors • angle bisectors. [CN, R, V]	**Specific Outcomes** 5. Draw and interpret top, front and side views of 3-D objects composed of right rectangular prisms. [C, CN, R, T, V] [ICT: C6–3.4]	**Specific Outcomes** 2. Determine the surface area of composite 3-D objects to solve problems. [C, CN, PS, R, V] 3. Demonstrate an understanding of similarity of polygons. [C, CN, PS, R, V]

Shape and Space (Transformations)

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Grade 5	Grade 6	Grade 7	Grade 8	Grade 9
General Outcome Describe and analyze position and motion of objects and shapes.	**General Outcome** Describe and analyze position and motion of objects and shapes.	**General Outcome** Describe and analyze position and motion of objects and shapes.	**General Outcome** Describe and analyze position and motion of objects and shapes.	**General Outcome** Describe and analyze position and motion of objects and shapes.
Specific Outcomes 8. Identify and describe a single transformation, including a translation, rotation and reflection of 2-D shapes. [C, T, V] [ICT: C6–2.1] 9. Perform, concretely, a single transformation (translation, rotation or reflection) of a 2-D shape, and draw the image. [C, CN, T, V] [ICT: C6–2.1]	**Specific Outcomes** 6. Perform a combination of translations, rotations and/or reflections on a single 2-D shape, with and without technology, and draw and describe the image. [C, CN, PS, T, V] 7. Perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations. [C, CN, T, V] 8. Identify and plot points in the first quadrant of a Cartesian plane, using whole number ordered pairs. [C, CN, V] 9. Perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices). [C, CN, PS, T, V] [ICT: C6–2.1]	**Specific Outcomes** 4. Identify and plot points in the four quadrants of a Cartesian plane, using integral ordered pairs. [C, CN, V] 5. Perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices). [C, CN, PS, T, V] [ICT: C6–3.4]	**Specific Outcomes** 6. Demonstrate an understanding of the congruence of polygons. [CN, R, V]	**Specific Outcomes** 4. Draw and interpret scale diagrams of 2-D shapes. [CN, R, T, V] [ICT: C6–3.4] 5. Demonstrate an understanding of line and rotation symmetry. [C, CN, PS, V]

Statistics and Probability (Data Analysis)

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Grade 5	Grade 6	Grade 7	Grade 8	Grade 9
General Outcome Collect, display and analyze data to solve problems.	**General Outcome** Collect, display and analyze data to solve problems.	**General Outcome** Collect, display and analyze data to solve problems.	**General Outcome** Collect, display and analyze data to solve problems.	**General Outcome** Collect, display and analyze data to solve problems.
Specific Outcomes 1. Differentiate between first-hand and second-hand data. [C, R, T, V] [ICT: C1–2.2, P5–2.3] 2. Construct and interpret double bar graphs to draw conclusions. [C, PS, R, T, V] [ICT: C6–2.2, P5–2.3]	**Specific Outcomes** 1. Create, label and interpret line graphs to draw conclusions. [C, CN, PS, R, V] 2. Select, justify and use appropriate methods of collecting data, including: • questionnaires • experiments • databases • electronic media. [C, CN, PS, R, T] [ICT: C4–2.2, C6–2.2, C7–2.1, P2–2.1, P2–2.2] 3. Graph collected data, and analyze the graph to solve problems. [C, CN, PS, R, T] [ICT: C6–2.5, C7–2.1, P2–2.1, P2–2.2]	**Specific Outcomes** 1. Demonstrate an understanding of central tendency and range by: • determining the measures of central tendency (mean, median, mode) and range • determining the most appropriate measures of central tendency to report findings. [C, PS, R, T] [ICT: P2–3.4] 2. Determine the effect on the mean, median and mode when an outlier is included in a data set. [C, CN, PS, R] 3. Construct, label and interpret circle graphs to solve problems. [C, CN, PS, R, T, V] [ICT: P2–3.3]	**Specific Outcomes** 1. Critique ways in which data is presented in circle graphs, line graphs, bar graphs and pictographs. [C, R, T, V] [ICT: C7–3.1, C7–3.2, F4–3.3]	**Specific Outcomes** 1. Describe the effect of: • bias • use of language • ethics • cost • time and timing • privacy • cultural sensitivity on the collection of data. [C, CN, R, T] [ICT: F4–3.2, F4–3.3] 2. Select and defend the choice of using either a population or a sample of a population to answer a question. [C, CN, PS, R]

Statistics and Probability (Data Analysis) (continued)

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Grade 5	Grade 6	Grade 7	Grade 8	Grade 9
				General Outcome Collect, display and analyze data to solve problems.
				Specific Outcomes 3. Develop and implement a project plan for the collection, display and analysis of data by: • formulating a question for investigation • choosing a data collection method that includes social considerations • selecting a population or a sample • collecting the data • displaying the collected data in an appropriate manner • drawing conclusions to answer the question. [C, PS, R, T, V] [ICT: C1–3.5, C4–3.1, C6–3.1, C6–3.2, C7–3.1, C7–3.2, P1–3.4, P2–3.1]

Statistics and Probability (Chance and Uncertainty)

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Grade 5	Grade 6	Grade 7	Grade 8	Grade 9
General Outcome Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.	**General Outcome** Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.	**General Outcome** Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.	**General Outcome** Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.	**General Outcome** Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.
Specific Outcomes 3. Describe the likelihood of a single outcome occurring, using words such as: • impossible • possible • certain. [C, CN, PS, R] 4. Compare the likelihood of two possible outcomes occurring, using words such as: • less likely • equally likely • more likely. [C, CN, PS, R]	**Specific Outcomes** 4. Demonstrate an understanding of probability by: • identifying all possible outcomes of a probability experiment • differentiating between experimental and theoretical probability • determining the theoretical probability of outcomes in a probability experiment • determining the experimental probability of outcomes in a probability experiment • comparing experimental results with the theoretical probability for an experiment. [C, ME, PS, T] [ICT: C6–2.1, C6–2.4]	**Specific Outcomes** 4. Express probabilities as ratios, fractions and percents. [C, CN, R, T, V] [ICT: P2–3.4] 5. Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events. [C, ME, PS] 6. Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or other graphic organizer) and experimental probability of two independent events. [C, PS, R, T] [ICT: C7–3.2, P2–3.4]	**Specific Outcomes** 2. Solve problems involving the probability of independent events. [C, CN, PS, T] [ICT: P2–3.4]	**Specific Outcomes** 4. Demonstrate an understanding of the role of probability in society. [C, CN, R, T] [ICT: F4–3.3]

APPENDIX 2: INFORMATION AND COMMUNICATION TECHNOLOGY (ICT) OUTCOMES

The following excerpts from the Information and Communication Technology (ICT) Program of Studies provide the complete wording for outcomes that are linked to the mathematics program of studies. For the complete ICT Program of Studies, go to the Alberta Education Web site at http://education.alberta.ca/teachers/program/ict/programs.aspx.

ICT Outcomes, Division 1

General Outcomes	Specific Outcomes
C4 – Students will use organizational processes and tools to manage inquiry.	1.3 organize information from more than one source
C7 – Students will use electronic research techniques to construct personal knowledge and meaning.	1.1 develop questions that reflect a personal information need 1.3 draw conclusions from organized information 1.4 make predictions based on organized information
P2 – Students will organize and manipulate data.	1.1 read information from a prepared database

ICT Outcomes, Division 2

C1 – Students will access, use and communicate information from a variety of technologies.	2.2 organize information gathered from the Internet, or an electronic source, by selecting and recording the data in logical files or categories; and by communicating effectively, through appropriate forms, such as speeches, reports and multimedia presentations, applying information technologies that serve particular audiences and purposes
C4 – Students will use organizational processes and tools to manage inquiry.	2.2 organize information, using such tools as a database, spreadsheet or electronic webbing
C6 – Students will use technology to investigate and/or solve problems.	2.1 select and use technology to assist in problem solving 2.2 use data gathered from a variety of electronic sources to address identified problems 2.3 use graphic organizers, such as mind mapping/webbing, flow charting and outlining, to present connections between ideas and information in a problem-solving environment 2.4 solve problems, using numerical operations and such tools as calculators and spreadsheets 2.5 solve problems requiring the sorting, organizing, classifying and extending of data, using such tools as calculators, spreadsheets, databases or hypertext technology 2.7 generate alternative solutions to problems by using technology to facilitate the process

General Outcomes	Specific Outcomes
C7 – Students will use electronic research techniques to construct personal knowledge and meaning.	2.1 use a variety of technologies to organize and synthesize researched information
P2 – Students will organize and manipulate data.	2.1 enter and manipulate data by using such tools as spreadsheets or databases for a specific purpose 2.2 display data electronically through graphs and charts
P5 – Students will navigate and create hyperlinked resources.	2.3 navigate the Internet with appropriate software

ICT Outcomes, Division 3

C1 – Students will access, use and communicate information from a variety of technologies.	3.5 analyze and synthesize information to create a product
C4 – Students will use organizational processes and tools to manage inquiry.	3.1 create a plan for an inquiry that includes consideration of time management
C6 – Students will use technology to investigate and/or solve problems.	3.1 articulate clearly a plan of action to use technology to solve a problem 3.2 identify the appropriate materials and tools to use in order to accomplish a plan of action 3.4 pose and test solutions to problems by using computer applications, such as computer-assisted design or simulation/modelling software
C7 – Students will use electronic research techniques to construct personal knowledge and meaning.	3.1 identify patterns in organized information 3.2 make connections among related, organized data, and assemble various pieces into a unified message
F4 – Students will become discerning consumers of mass media and electronic information.	3.2 understand the nature of various media and how they are consciously used to influence an audience 3.3 identify specific techniques used by the media to elicit particular responses from an audience
P1 – Students will compose, revise and edit text.	3.4 use appropriate communication technology to elicit feedback from others
P2 – Students will organize and manipulate data.	3.1 design, create and modify a database for a specific purpose 3.3 use a variety of technological graphing tools to draw graphs for data involving one or two variables 3.4 use a scientific calculator or a computer to solve problems involving rational numbers