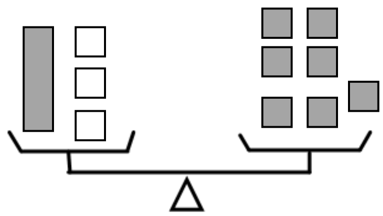
Grade 9 Math **Unit 6: Solving equations and Inequalities**

Introduction

In previous grades, 7 and 8, you learned how to solve one-step and some two-step equations, using models and algebra.

Review examples:

1. $x - 3 = 7$



You may have seen a balance scale.

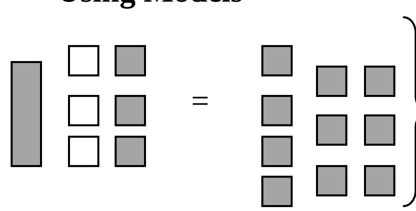
We must keep both sides of the scale balanced or equal. Whatever you do to one side of the equation, you must do to the other side.

We need to isolate *x* - which means get *x* by itself. Therefore, we must get rid of the –3. We do this by making –3 zero.

What can we add to –3 to get zero? + 3. Just remember to add 3 to both sides of the equation.

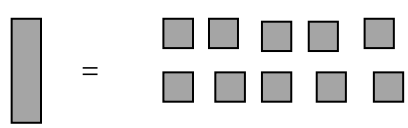
Using Algebra **Using Models**

$x - 3 + 3 = 7 + 3$



cancel zero pairs

$x = 10$



Note:

To "undo" the subtract 3, we did the opposite operation and we added 3. When we do an opposite operation, it is known as the **inverse operation**.

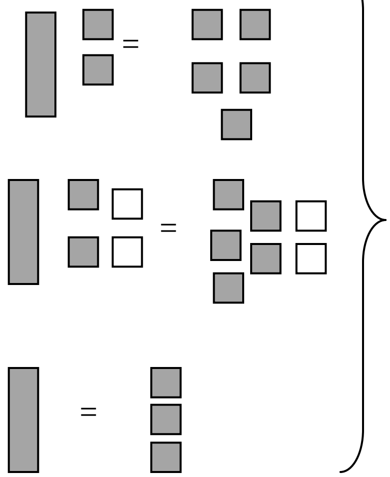
2. **Algebra**

$x + 2 = 5$

$x + 2 - 2 = 5 - 2$

$x = 3$

Algebra Tiles



We need to get rid of the +2, use the opposite/inverse operation.

Subtract 2 from both sides.

Cancel zero pairs.

3. 2x = 10 "means 2 multiplied by something is 10"

Since the operation is multiply, the inverse operation is divide.

We only want 1x, so split into two groups...or divide into two groups.

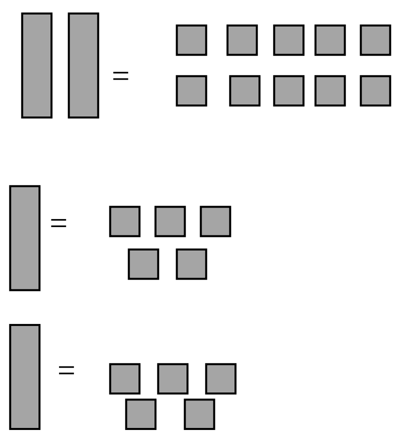
Algebra

2x = 10

$\frac{2x}{2} = \frac{10}{2}$

x = 5

Algebra Tiles



How many tiles are with 1x?

There are 5!

Therefore, x = 5

4. Try without models.

$\frac{x}{5} = 3$ "means something divided by 5 is 3"

Therefore, since the operation is divide by 5, we do the inverse operation and multiply by 5 to solve the equation.

$\frac{x}{5} \times 5 \;=\; 3 \times 5$ Multiply both sides of the equation (both numerators) by the denominator, 5.

$\frac{5x}{5} = 15$ $\frac{\cancel{5}x}{\cancel{5}} = 15$ x = 15

The reason this works is because a whole number multiplied by its reciprocal is one.

$\frac{1}{5} \times 5 = 1$ $\frac{1}{8} \times 8 = 1$

Examples: Solve for the variable, using algebra (remember the inverse operation).

1). $x + 4 = -7$

x + 4 – 4 = 1 – 7 – 4
x = – 11

2). $x - 6 = 15$

x – 6 + 6 = 15 + 6
x = 21

3). $4m = 12$

$\frac{4m}{4} = \frac{12}{4}$

m = 3

4). $-2x = 16$

$\frac{-2x}{-2} = \frac{16}{-2}$

x = – 8

5). $\frac{p}{3} = -2$

$\frac{p}{3} \times 3 \;=\; -2 \times 3$

p = – 6

6). $\frac{1}{6}x = 4$

$\frac{x}{6} \times 6 = 4 \times 6$

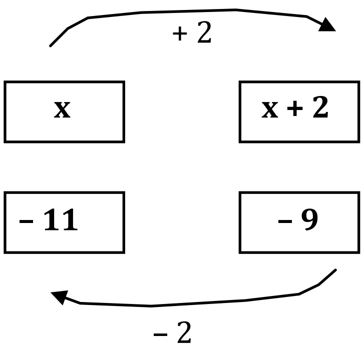
x = 24

Sec 6.1: Solving Equations Using Inverse Operations

Solve these examples using inverse operations (your textbook uses the following diagram). Show all steps.

1. $x + 2 = -9$

The operation is: **add 2**
The inverse operation is: **subtract 2**



Using Algebra

$\mathbf{x + 2 = -9}$

$\mathbf{x + 2 - 2 = -9 - 2}$

$\mathbf{x = -11}$

You should always **verify** your answer. This means put your answer of – 11, back into your original equation. **The right side of the equation should equal the left side.**

Verify: $x + 2 = -9$
$\mathbf{-11} + 2 = -9$
$-9 = -9$

2. $y + 2.4 = 6.5$

The operation is: **add 2.4**
The inverse operation is: **subtract 2.4**

$y + 2.4 - 2.4 = 6.5 - 2.4$
$y = 4.1$

Verify: $y + 2.4 = 6.5$
$\mathbf{4.1} + 2.4 = 6.5$
$6.5 = 6.5$

3. Write the equation and solve: " Three times a number is – 3.6"

$3x = -3.6$

The operation is: **multiply by 3**
The inverse operation is: **divide by 3**

$$\frac{3x}{3} = \frac{-3.6}{3}$$

$x = -1.2$

Verify: $3x = -3.6$
$3(\mathbf{-1.2}) = -3.6$
$-3.6 = -3.6$

4. Write the equation and solve: "A number divided by 4 is 1.5"

$$\frac{m}{4} = 1.5$$

The operation is: **divide by 4**
The inverse operation is: **multiply by 4**

$$\frac{m}{4} \times 4 = 1.5 \times 4$$

$$m = 6$$

Verify: $\frac{m}{4} = 1.5$ $\quad \frac{6}{4} = 1.5$

$1.5 = 1.5$ ☺

5. $3p - 4 = 5$

The operations are: **subtract 4 and multiply by 3**
The inverse operations are: **add 4 and divide by 3**

$$3p - 4 + 4 = 5 + 4$$
$$3p = 9$$

$$\frac{3p}{3} = \frac{9}{3}$$

$$p = 3$$

Verify: $3p - 4 = 5$
$3(3) - 4 = 5$
$9 - 4 = 5$
$5 = 5$ ☺

6. $2a + 7 = 12$

The operations are: **add 7 and multiply by 2**
The inverse operations are: **subtract 7 and divide by 2**

$$2a + 7 - 7 = 12 - 7$$
$$2a = 5$$

$$\frac{2a}{2} = \frac{5}{2}$$

$a = \frac{5}{2}$ or 2.5

Verify: $2a + 7 = 12$
$2\,(2.5) + 7 = 12$
$5 + 7 = 12$
$12 = 12$ ☺

7. $1.9 + \frac{n}{3} = 6.8$

The operations are: **add 1.9 and divide by 3**
The inverse operations are: **subtract 1.9 and multiply by 2**

$$1.9 - 1.9 + \frac{n}{3} = 6.8 - 1.9$$

$$\frac{n}{3} = 4.9$$

$$\frac{n}{3} \times 3 = 4.9 \times 3$$

$$n = 14.7$$

Verify: $1.9 + \frac{n}{3} = 6.8$ $\quad 1.9 + \frac{14.7}{3} = 6.8$

$1.9 + 4.9 = 6.8$
$6.8 = 6.8$ ☺

More Examples of Solving Equations

Equations with rational numbers in fraction or decimal form **cannot** be modelled easily, but we can still solve these equations using inverse operations – even if there is a **variable in the denominator. Remember: the variable cannot be zero in the denominator.**

Examples: Solve and verify.

1. $\frac{4.2}{x} = 3$

The operation is: **divide by x**
The inverse operation is: **multiply by x**

$\frac{4.2}{x} \times x = 3 \times x$

now the equation is $4.2 = 3x$
we still need to solve for x

The operation is: **multiply by 3**
The inverse operation is: **divide by 3**

$\frac{4.2}{3} = \frac{3x}{3}$

$1.4 = x$

Verify: $\frac{4.2}{x} = 3 \qquad \frac{4.2}{1.4} = 3$

$3 = 3$ ☺

TRY this one!

2. $\frac{2}{x} = 0.5$

The operation is: **divide by x**
The inverse operation is: **multiply by x**

$\frac{2}{x} \times x = 0.5 \times x$

now the equation is $2 = 0.5x$
we still need to solve for x

The operation is: **multiply by 0.5**
The inverse operation is: **divide by 0.5**

$\frac{2}{0.5} = \frac{0.5x}{0.5}$

$4 = x$

Verify: $\frac{2}{x} = 0.5 \qquad \frac{2}{4} = 0.5$

$0.5 = 0.5$ ☺

Equations can also contain brackets. If you remember from the unit on polynomials, this requires we use the **distributive property**. Every term in the bracket is multiplied by the number in front of the bracket. (This number could even be a fraction or decimal).

Examples: Solve and verify.

1. $2(3.7 + x) = 13.2$

$$2(3.7 + x) = 13.2$$

$$7.4 + 2x = 13.2$$

The operation is: add 7.4 and multiply by 2
The inverse operation: subtract 7.4 and divide by 2

$$7.4 - 7.4 + 2x = 13.2 - 7.4$$
$$2x = 5.8$$

$$\frac{2x}{2} = \frac{5.8}{2}$$

$$x = 2.9$$

Verify:
$$2(3.7 + x) = 13.2$$
$$2(3.7 + 2.9) = 13.2$$
$$2(5.8) = 13.2$$
$$13.2 = 13.2$$ ☺

2. $6 = 1.5(x - 6)$

$$6 = 1.5(x - 6)$$
$$6 = 1.5x - 9$$

The operation is: subtract 9 and multiply by 1.5
The inverse operation: add 9 and divide by 1.5

$$6 + 9 = 1.5x - 9 + 9$$
$$15 = 1.5x$$

$$\frac{15}{1.5} = \frac{1.5x}{1.5}$$

$$10 = x$$

Verify:
$$6 = 1.5(x - 6)$$
$$6 = 1.5(10 - 6)$$
$$6 = 1.5(4)$$
$$6 = 6$$ ☺

3. On a test, a student solved the following equation. Were they correct?

$$3(x - 5) = 2$$
$$3(x) - 3(5) = 3(2)$$
$$3x - 15 = 6$$
$$3x - 15 + 15 = 6 + 15$$
$$3x = 21$$

$$\frac{3x}{3} = \frac{21}{3}$$

$$x = 7$$

NO! You only multiply the number in front of the bracket by the terms inside the bracket.

$$3(x - 5) = 2$$
$$3(x) - 3(5) = 2$$
$$3x - 15 = 2$$
$$3x - 15 + 15 = 2 + 15$$
$$3x = 17$$
$$x = \frac{17}{3}$$

Can you verify this answer?

$3(x - 5) = 2$ answer: $x = \frac{17}{3}$

$3\left(\frac{17}{3} - 5\right) = 2$

$3\left(\frac{17}{3} - \frac{15}{3}\right) = 2$

$3\left(\frac{2}{3}\right) = 2$

$\frac{3}{1}\left(\frac{2}{3}\right) = 2$

$\frac{6}{3} = 2$

$2 = 2$

Solving Equations with Fractions

The easiest way to solve equations which contain fractions is to **eliminate the denominator.** If we can get rid of all the fractions, the equation will be easier to solve.

To solve equations containing fractions, multiply each term by the whole number you choose. This whole number **MUST BE A COMMON DENOMINATOR** for all the fractions in the equations.

Examples: Solve each equation – by eliminating the denominator first.

1. $\frac{x}{12} = \frac{1}{3} \longrightarrow$ choose 12

$$\longrightarrow \frac{x}{12} \times 12 = \frac{1}{3} \times 12$$

$$\longrightarrow \frac{12\,x}{12} = \frac{12}{3} \longrightarrow 1x = 4$$

Verify: $\frac{x}{12} = \frac{1}{3}$ $\qquad \frac{4}{12} = \frac{1}{3}$ ☺

2. $\frac{-1}{9} = \frac{3x}{27} \longrightarrow$ choose 27

$$\longrightarrow \frac{-1}{9} \times 27 = \frac{3x}{27} \times 27$$

$$\longrightarrow \frac{-27}{9} = \frac{81x}{27} \longrightarrow -3 = 3x$$

$$\longrightarrow \frac{-3}{3} = \frac{-3x}{-3} \longrightarrow -1 = x$$

Verify: $\frac{-1}{9} = \frac{3x}{27}$ $\qquad \frac{-1}{9} = \frac{3(-1)}{27}$

$$\frac{-1}{9} = \frac{-3}{27}$$ ☺

3. $\frac{x}{4} + \frac{1}{5} = \frac{1}{2}$ $\longrightarrow$ choose 20

$$\longrightarrow \quad \frac{x}{4} \times 20 + \frac{1}{5} \times 20 = \frac{1}{2} \times 20$$

$$\longrightarrow \quad \frac{20x}{4} + \frac{20}{5} = \frac{20}{2}$$

$$\longrightarrow \quad 5x + 4 = 10$$

$$\longrightarrow \quad 5x + 4 - 4 = 10 - 4$$

$$\longrightarrow \quad 5x = 6$$

$$\longrightarrow \quad \frac{5x}{5} = \frac{6}{5} \quad x = \frac{6}{5}$$

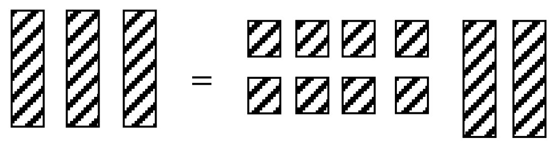
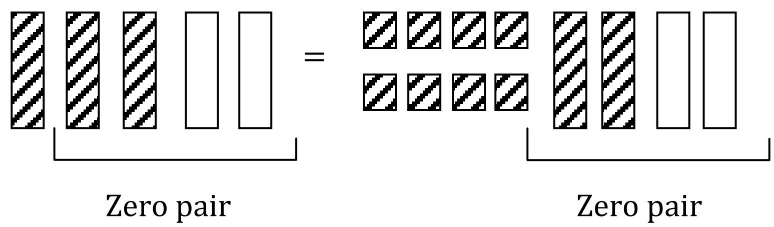
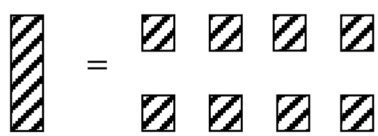
Solving Equations with Variables on Both Sides of the Equation

* Remember, when solving equations our goal is the get the variable by itself. Therefore, the variable cannot be on both sides of the equation. We must get the variable on one side of the equal sign and the constant term(s) on the other.

Example 1: Solve for x, using algebra and algebra tiles.

$3x = 8 + 2x$

Hint: you will need to use the zero property rule on the 2x.

$3x = 8 + 2x$	
$3x - 2x = 8 + \underbrace{2x - 2x}_{\text{zero}}$	
$1x = 8$	

You can also verify this type of equation. Left side = right side.

$3x = 8 + 2x$

$3\,(8) = 8 + 2\,(8)$

$24 = 8 + 16$

$24 = 24$ ☺

Example 2: Solve and verify. Use algebra only.

A. $16 - 3x = 5x$

$16 - 3x + 3x = 5x + 3x$

$16 = 8x$

$\frac{16}{8} = \frac{8x}{8}$ $\quad 2 = x$

Verify: $16 - 3x = 5x$

$16 - 3(2) = 5(2)$

$16 - 6 = 10$

$10 = 10$

B. $w = 9 - 2w$

$w + 2w = 9 - 2w + 2w$

$3w = 9$

$\frac{3w}{3} = \frac{9}{3}$ $\quad w = 3$

Verify: $w = 9 - 2w$

$3 = 9 - 2(3)$

$3 = 9 - 6$

$3 = 3$

C. $2x = -30 + 5x$

$2x - 5x = -30 + 5x - 5x$

$-3x = -30$

$\frac{-3x}{-3} = \frac{-30}{-3}$

$x = 10$

Verify: $2x = -30 + 5x$

$2(10) = -30 + 5(10)$

$20 = -30 + 50$

$20 = 20$

D. $2x + 3x = 8x - 3$

$5x = 8x - 3$

$5x - 8x = 8x - 8x - 3$

$-3x = -3$

$\frac{-3x}{-3} = \frac{-3}{-3}$

$x = 1$

Verify: $2x + 3x = 8x - 3$

$2(1) + 3(1) = 8(1) - 3$

$2 + 3 = 8 - 3$

$5 = 5$

More . . . . Solving Equations with Variables on Both Sides

Examples:

1. Solve and verify

$$4x + 7 = 21 - 3x$$

- Whenever you have a variable on both sides of the equation and a constant term on both sides of the equation, you will need to use the zero property idea, as inverse operations, TWICE.
- One time will be to get rid of the constant term from one side.
- Second time will be to get rid of the variable from the other side.
- In an example like this, our goal is to get all numbers on one side of the equal sign and all variables on the other side of the equal sign.

Solve for x:

$$\left.\begin{array}{l} 4x + 7 - 7 = 21 - 7 - 3x \\ 4x = 14 - 3x \end{array}\right\}$$

so far I have used the inverse operation on +7 this gets rid of the number on the left side.

$$\left.\begin{array}{l} 4x + 3x = 14 - 3x + 3x \\ 7x = 14 \end{array}\right\}$$

now I used the inverse operation on $-3x$ this gets rid of the variable on the right side.

$$\frac{7x}{7} = \frac{14}{7}$$

$$x = 2$$

Verify:

$$4x + 7 = 21 - 3x$$
$$4(2) + 7 = 21 - 3(2)$$
$$8 + 7 = 21 - 6$$
$$15 = 15$$

Right side = left side so the answer of x = 2 is correct.

2. $2x + 10 = 20 - 3x$

$2x + 10 - 10 = 20 - 10 - 3x$

$2x = 10 - 3x$

$2x + 3x = 10 - 3x + 3x$

$\frac{5x}{5} = \frac{10}{5}$ $\quad x = 2$

Verify: $2x + 10 = 20 - 3x$

$2(2) + 10 = 20 - 3(2)$

$4 + 10 = 20 - 6$

$14 = 14$

3. $7y + 53 = 14 - 6y$

$7y + 53 - 53 = 14 - 53 - 6y$

$7y = -39 - 6y$

$7y + 6y = -39 - 6y + 6y$

$\frac{13y}{13} = \frac{-39}{13}$

$y = -3$

Verify: $7y + 53 = 14 - 6y$

$7(-3) + 53 = 14 - 6(-3)$

$-21 + 53 = 14 + 18$

$32 = 32$

4. $4x + 2 = -2x + 8$

$4x + 2 - 2 = -2x + 8 - 2$

$4x = -2x + 6$

$4x + 2x = -2x + 2x + 6$

$\frac{6x}{6} = \frac{6}{6}$

$x = 1$

Verify: $4x + 2 = -2x + 8$

$4(1) + 2 = -2(1) + 8$

$4 + 2 = -2 + 8$

$6 = 6$

5. $3(x + 1) = 5(x - 1)$

$3x + 3 = 5x - 5$

$3x + 3 - 3 = 5x - 5 - 3$

$3x = 5x - 8$

$3x - 5x = 5x - 5x - 8$

$-2x = -8$

$\frac{-2x}{-2} = \frac{-8}{-2}$

$x = 4$

Verify: $3x + 3 = 5x - 5$

$3(4) + 3 = 5(4) - 5$

$12 + 3 = 20 - 5$

$15 = 15$

6. Two different taxi companies charge the following:

Company A: \$3.00 plus \$0.20 per km

Company B: \$2.50 plus \$0.25 per km

At what distance will the cost be the same?

A). Model the problem with an equation. $0.20k + 3 = 0.25k + 2.50$

B). Solve the problem.

$$0.20k - 0.25k = 2.50 - 3$$

$$\frac{-0.05k}{-0.05} = \frac{-0.50}{-0.05}$$

$$k = 10 \text{ kilometers}$$

C). Verify the solution.

$$0.20\,(10) + 3 = 0.25\,(10) + 2.50$$
$$2 + 3 = 2.5 + 2.5$$
$$5 = 5$$

7. An internet company offers two plans.

Plan A: no monthly fee, \$0.75 per minute

Plan B: \$15 monthly fee, plus \$0.25 per minute

When will the companies result in the same cost?

A). Model the problem with an equation. $0.75m = 15 + 0.25m$

B). Solve the problem.

$$0.75m - 0.25m = 15$$
$$0.50m = 15$$

$$\frac{0.50m}{0.50} = \frac{15}{0.50}$$

$$m = 30 \text{ minutes}$$

C). Verify the solution

$$0.75\,(30) = 15 + 0.25(30)$$
$$22.50 = 15 + 7.50$$
$$22.50 = 22.50$$

Solving Multi-Step Equations **Grade 9 Math**

We have already solved equations with distributive property, fractions and variables on both sides.

What if we had all of these steps in one equation?

Examples: Solve and Verify.

1. $3(x + 1) = 2(4 - x)$

$$\begin{aligned} 3x + 3 &= 8 - 2x \\ 3x + 3 - 3 &= 8 - 3 - 2x \\ 3x &= 5 - 2x \\ 3x + 2x &= 5 - 2x + 2x \\ \frac{5x}{5} &= \frac{5}{5} \\ x &= 1 \end{aligned}$$

Left = Right

$$\begin{aligned} 3(x + 1) &= 2(4 - x) \\ 3(1 + 1) &= 2(4 - 1) \\ 3(2) &= 2(3) \\ 6 &= 6 \quad \checkmark \end{aligned}$$

Therefore, x = 1 is the correct answer.

2. $\frac{1}{2}(x - 1) = \frac{2}{3}(1 - x)$ * multiply by a common denominator to eliminate the denominator first!

$$6 \times \frac{1}{2}(x - 1) = 6 \times \frac{2}{3}(1 - x)$$

$$\frac{6}{2}(x - 1) = \frac{12}{3}(1 - x)$$

$3(x - 1) = 4(1 - x)$ * now do distributive property

$3x - 3 = 4 - 4x$ * now zero pairs!

$$\begin{aligned} 3x - 3 + 3 &= 4 - 4x + 3 \\ 3x &= 7 - 4x \\ 3x + 4x &= 7 - 4x + 4x \\ \frac{7x}{7} &= \frac{7}{7} \end{aligned}$$

x = 1

Left		**Right**
$\frac{1}{2}(x - 1)$	=	$\frac{2}{3}(1 - x)$
$\frac{1}{2}(1 - 1)$	=	$\frac{2}{3}(1 - 1)$
$\frac{1}{2}(0)$	=	$\frac{2}{3}(0)$
0	=	0

Therefore, x = 1 is correct.

3. $\frac{(2x-3)}{2} = \frac{(-x-1)}{4}$

$4 \times \frac{(2x-3)}{2} = 4 \times \frac{(-x-1)}{4}$

2 (2x – 3) = 1 (- x – 1)

4x – 6 = –1x – 1

4x – 6 + 6 = – 1x – 1 + 6

4x = – 1x + 5

4x + 1x = 1x – 1x + 5

5x = 5

x = 1

Left	=	Right
$\frac{(2x-3)}{2}$	$=$	$\frac{(-x-1)}{4}$
$\frac{(2(1)-3)}{2}$	$=$	$\frac{(-1-1)}{4}$
$\frac{(-1)}{2}$	$=$	$\frac{(-2)}{4}$
$\frac{(-1)}{2}$	$=$	$\frac{(-1)}{2}$ ✓

Therefore, x = 1

is correct.

Sec 6.3: Introduction to Linear Inequalities

What are Inequalities?

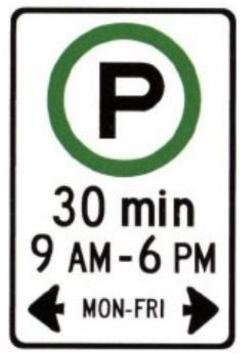
We use inequalities to model a situation that can be described by a range of numbers instead of a single number.

We use specific symbols:

When one quantity is **greater or equal to** the other quantity: $\geq$
When one quantity is **greater than** the other quantity: $>$
When one quantity is **less or equal to** the other quantity: $\leq$
When one quantity is **less than** the other quantity: $<$

Example 1:

Which inequality describes the time, t, for which a car could be legally parked?



$t > 30$

$t \geq 30$

$t < 30$

$t \leq 30$

Example 2:

Define a variable and write an inequality for each situation:

Answers:

a). $s \leq 55$
b). $h \geq 102$
c). $t < 4$
d). $R \geq 14$

Here are some examples of inequality statements:
One expression is less than another. Ex: a is less than 3, $\mathbf{a < 3}$
One expression is greater than another Ex: b is greater than -4, $\mathbf{b > -4}$
One expression is less than or equal to another.
Ex: c is less than or equal to 3.1, $\mathbf{c \leq 3.1}$
One expression is greater than or equal to another.
Ex: d is greater than or equal to 5.4, $\mathbf{d \geq 5.4}$

Example 3:
Define a variable and write an inequality to describe each situation:

a) Contest entrants must be at least 18 years old **$E \geq 18$**

b) The temperature has been below -5°C for the last week **$T < -5$**

c) You must have 10 items or less to use the express checkout line at a grocery store
$g \leq 10$

d) Scientist have identified over 400 species of dinosaurs **$d > 400$**

- A **linear equation** is true for only **ONE** value of the variable.
- A **linear inequality** may be true for **MANY** values of the variable.
- The solution of an inequality is any value of the variable that makes the inequality true.
- There are usually too many numbers to list, so we may show them on a number line.

Example 4:
Is each number a solution of the inequality $b \geq -4$? Justify the answers.
a) -8 b) -3.5 c) -4 d) 4.5 e) 0

Method 1: Use a number line

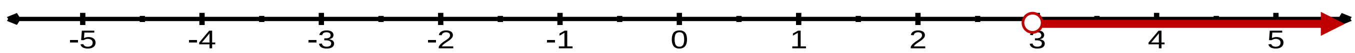
-8 is not included -3.5 is included -4 is included 4.5 is included 0 is included

Method 2: Substitute each number for b

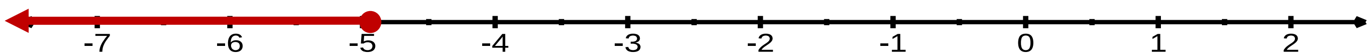
$-8 \geq -4$	$-3.5 \geq -4$	$-4 \geq -4$	$4.5 \geq -4$	$0 \geq -4$
NO	**YES**	**YES**	**YES**	**YES**

GRAPHING INEQUALITIES

For example, what would a > 3 look like on a number line?



What about b ≤ -5



NOTE:
Since 3 is **NOT** part of the solution, we draw an **Hollow** circle at 3 to indicate this.

Since -5 **IS** part of the solution, we draw a **Solid** circle at -5 to indicate this.

Example 5:
Graph each inequality on a number line and list 4 numbers that are solutions of the inequality.

a) t > – 5 Answer: – 4, –3, –2, –1, etc

b) -2 ≥ x

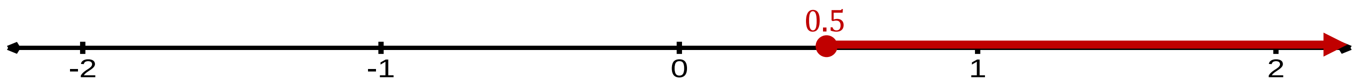
This inequality reads "–2 is greater than and equal to a number." So which numbers are –2 greater than........ Answer: –2, –3, –4, –5, etc

When an inequality is written in this direction you can switch it around to be x ≤ – 2. The inequality still opens up toward the –2, so we didn't change its meaning. This says that x is less than or equal to –2.

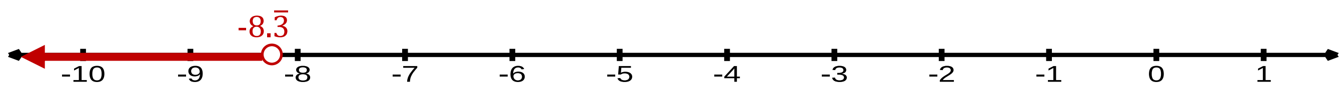
c) 0.5 ≤ a

This reads “0.5 is less than or equal to a number” or switch it to be a > 0.5 which reads “ a number is greater than or equal to 0.5”   Answer: 0.5, 1, 1.5, 2, etc

Indicate where 0.5 is on the number line.

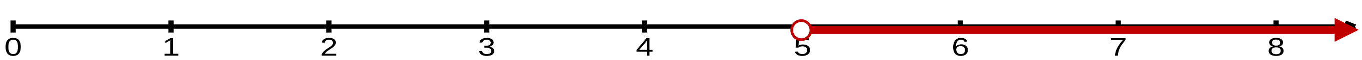


d) $p < \frac{-25}{3}$ $\frac{-25}{3} = -8.\overline{3}$ Answer: -9, -10, -11, -12, etc.



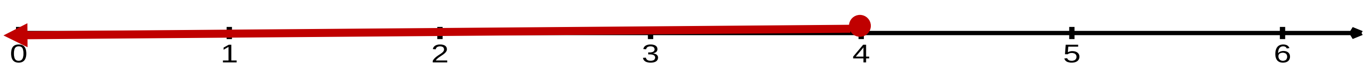
Example 6:
Write an inequality to describe each situation, then graph the solution on a number line.

a). a number is greater than 5.   Answer: $x > 5$



Note: Any number greater than 5 has to be included in this answer, but NOT 5 itself. We represent it on a number line by putting a hollow dot on 5 and shading the entire line as well as the arrow … to show it continues.

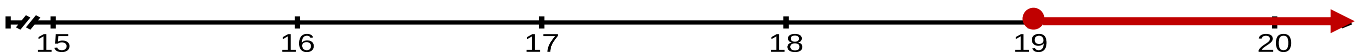
b). a number is less than or equal to 4.   Answer: $x \leq 4$



Note: Any number less than or equal to 4 has to be included in this answer. This time 4 IS included. We represent it on a number line by putting a solid dot on 4 and shading the entire line as well as the arrow to show it is continuous and continues forever.

c). the temperature is below – 1^0C today.   Answer: $x < -1$

d). to enter a bar you must be at least 19 years of age. Answer: $x \geq 19$



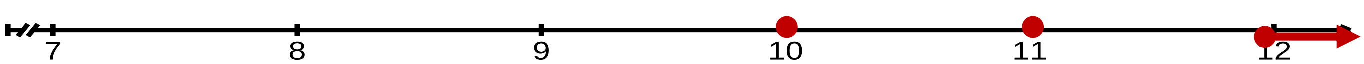
e). You must have 10 items of less to go to the express lane at the grocery store.

Answer: $x \leq 10$

Note: Any **whole** number greater than or equal to 10, but not greater than 10, has to be included in this answer. This problem includes discrete data. This time decimals or fractions are NOT included, due to the situation. (Ex: You can't buy half an item at the grocery store). We represent it on a number line by putting a solid dot on each included possible number.

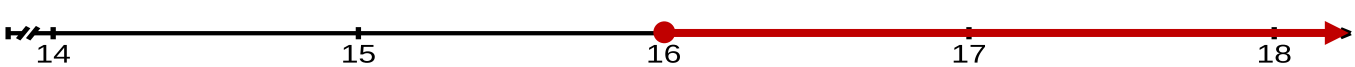
f). Chantal's mom said she should invite at least 10 people to her birthday party.

Answer: $x \geq 10$



Note: Again we use solid dots (discrete data) to represent each possible number. This time the numbers can keep going so we use a shaded arrow to represent that it continues.

g). In most provinces, you have to be at least 16 years old to drive. Answer: $x \geq 16$



Summary:

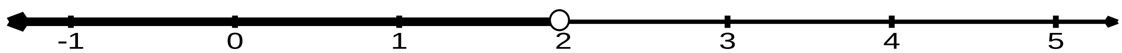
When graphing inequalities: $>$ or $<$ use hollow dots on the number
$\geq$ or $\leq$ use solid dots on the number

The line could be: Continuous , so shade the entire line.
Discrete, so use only dots.

Example 7:

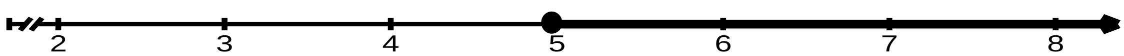
Write the inequality for each number line.

a)



Answer: $x < 2$

b)



Answer: $x \geq 5$

c)

Answer: $x \leq -1$

d)

Answer: $x > -6$

Section 6.4 - Solving Linear Inequalities using Addition and Subtraction

Consider the inequality:

▶ $-2 < 4$ Is it true? Yes! -2 is less than 4

Can we add the same number to both sides and it still be true?

Choose a positive number	Choose a negative number
$-2 < 4$ $-2 \mathbf{+ 5} < 4 \mathbf{+ 5}$ $3 < 9$ Still true!	$-2 < 4$ $-2 \mathbf{+ -3} < 4 \mathbf{+ -3}$ $-5 < 1$ Still true!

▶ $-2 < 4$

Can we subtract the same number from both sides and it still be true?

Choose a positive number	Choose a negative number
$-2 < 4$ $-2 \mathbf{- 6} < 4 \mathbf{- 6}$ $-8 < -2$ Still true!	$-2 < 4$ $-2 \mathbf{- (-3)} < 4 \mathbf{- (-3)}$ $1 < 7$ Still True!

When the same number is added or subtracted from each side of an inequality, the resulting inequality is still true. <u>Therefore, we can still use the zero pair idea to solve inequalities.</u>

Compare an Equation with an Inequality:

Equation
$h + 3 = 5$
$h + 3 - 3 = 5 - 3$
$h = 2$

There is ONE solution.
$h = 2$

Inequality
$h + 3 < 5$
$h + 3 - 3 < 5 - 3$
$h < 2$

There are an infinite number of solutions.
Any number less than 2 is a solution.
0, -3, -4.6, etc.

<u>Example 1:</u>

Solve the inequality, verify the solution and graph on a number line:

$$6.2 \leq x - 4.5$$

<u>Solve</u>: $6.2 + 4.5 \leq x - 4.5 + 4.5$

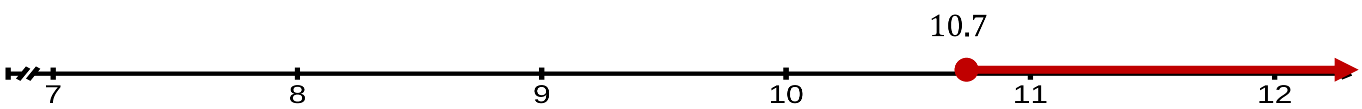
$10.7 \leq x$

<u>Verify</u>: $6.2 \leq x - 4.5$

$6.2 \leq 10.7 - 4.5$

$6.2 \leq 6.2$ True

<u>Graph</u>:



<u>Example 2</u> Solve and graph: $4.8 + c > -3.2$

$4.8 - 4.8 + c > -3.2 - 4.8$

$c > -8$

<u>Example 3:</u>

Jake plans to board his dog while he is away on vacation.

- Boarding House A charges $90 plus $5 per day = 90 + 5d
- Boarding House B charges $100 plus $4 per day = 100 + 4d

For how many days must Jake board his dog in House A to be less expensive than House B?

a) choose a variable and write an inequality
b) Solve the problem
c) Graph

$90 + 5d < 100 + 4d$

$90 - 90 + 5d < 100 - 90 + 4d$

$5d < 10 + 4d$

$5d - 4d < 10 + 4d - 4d$

$d < 10$

For less than 10 days, Boarding House will be cheaper than Boarding House B.

Section 6.5 - Solving Linear Inequalities using Multiplication and Division

► Consider the inequality: -3 < 6 Is it true?

Yes! -3 is less than 6

Multiply each side by 3: Is it still true?	Divide each side by 3: Is it still true?
$-3 < 6$ $-3 \times 3 < 6 \times 3$ $-9 < 18$ Still true!	$\frac{-3}{3} < \frac{6}{3}$ $-1 < 2$ Still true!

NOTES:
When each side of an inequality is multiplied or divided by the same positive number, the resulting inequality is still true. **This means we can still ``eliminate fractions`` by multiplying by a positive common denominator and we can still ``split into groups`` . For example: go from 3x to x by dividing by 3.**

► Consider the inequality: -3 < 6

Multiply each side by -3: Is it still true?	Divide each side by -3: Is it still true?
$-3 < 6$ $-3 \times -3 < 6 \times -3$ $9 < -18$ NOT true! **Reverse inequality: 9 > -18** **Now it's true!**	$\frac{-3}{-3} < \frac{6}{-3}$ $1 < -2$ NOT true! **Reverse inequality: 1 > -2** **Now it's true!**

What can be done to make these inequalities true?

VERY IMPORTANT NOTE:

► We must reverse the inequality sign when multiplying or dividing both sides by a negative number to keep the inequality true!

To solve an inequality, we use the same strategy as for solving an equation.

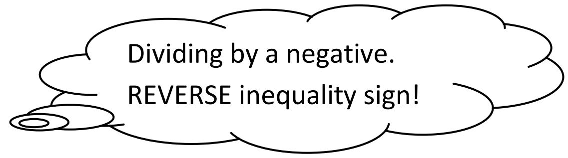
However, when we multiply or divide by a negative number, we MUST **reverse** the inequality sign.

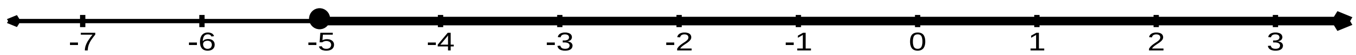
Example 1: Solve the inequality and graph the solution

a) $-5s \leq 25$

$$\frac{-5s}{-5} \leq \frac{25}{-5}$$

$$s \geq -5$$

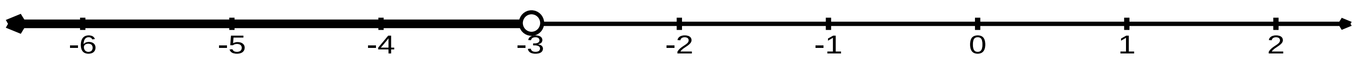




b) $7a < -21$

$$\frac{7a}{7} < \frac{-21}{7}$$

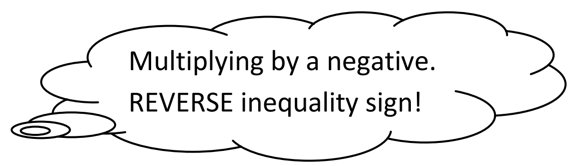
$$a < -3$$



c) $\frac{x}{-4} > -3$

$$\frac{x}{-4} \times -4 > -3 \times -4$$

$$x < 12$$

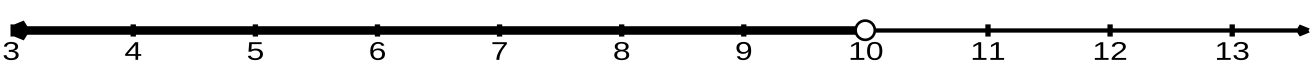


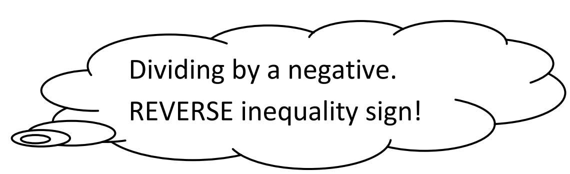
d) $3x - 12 < 18$

$$3x - 12 + 12 < 18 + 12$$

$$\frac{3x}{3} < \frac{30}{3}$$

$$x < 10$$



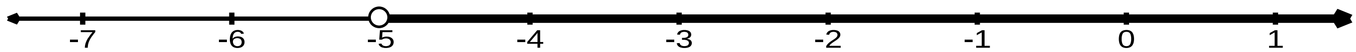


e) $13 - 4x < 33$

$13 - 13 - 4x < 33 - 13$

$\dfrac{-4x}{-4} < \dfrac{20}{-4}$

$x > -5$



<u>Example 2:</u> Solve and verify: $-2.6a + 14.6 > -5.2 + 1.8a$

Solve: $-2.6a + 14.6 - 14.6 > -5.2 + 1.8a - 14.6$

$-2.6a > -19.8 + 1.8a$

$-2.6a - 1.8a > -19.8 + 1.8a - 1.8a$

$\dfrac{-4.4a}{-4.4} > \dfrac{-19.8}{-4.4}$

$a < 4.5$

Dividing by a negative.
REVERSE inequality sign!

Verify:
Choose a number less than 4.5
Try: $a = 0$
$-2.6a + 14.6 > -5.2 + 1.8a$
$14.6 > -5.2$ it's true!

<u>Example 3:</u>
A super-slide charges $1.25 to rent a mat and $0.75 per ride. Jason has $10.25. How many rides can Jason go on?

a) choose a variable and write an inequality

b) Solve the problem

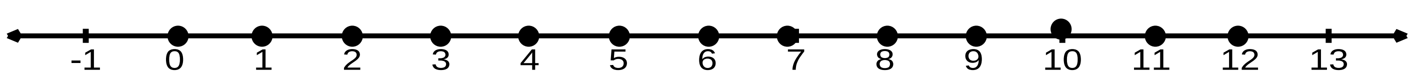
c) Graph

Answer: $1.25 + 0.75r \leq 10.25$

$1.25 - 1.25 + 0.75r \leq 10.25 - 1.25$

$\dfrac{0.75r}{0.75} \leq \dfrac{9.00}{0.75}$

$r \leq 12$ Therefore, Jason can get on 12 or less rides for $10.75



This is discrete! You can't go
on half a ride, so use dots!

Summary for solving Inequalities

The process for solving an inequality is the same as for solving equations.

Our goal: x by itself

→ adding or subtracting positive or negative numbers from each side of an inequality keeps the inequality true.

**** This means we can still use zero pairs! ****

→ when dividing both sides of the inequality by a positive number, the inequality is still true.

**** This means we can still "split into groups." ****

→ when multiplying both sides by a positive number, the inequality is still true.

**** This means we can still eliminate denominators with fractions. ****

HOWEVER

→ When dividing or multiplying each side by a negative you MUST REVERSE the inequality sign to keep the inequality true.

Ex: $-2x < 10$

$\frac{-2x}{-2} < \frac{10}{-2}$ $\quad x > -5$ the inequality sign must switch direction.

→ An equation has one answer, while an inequality has a range of answers.

Solve an Equation	**Solve an Inequality**
$7x = 2x + 15$ $7x - 2x = 2x - 2x + 15$ $5x = 15$ $\frac{5x}{5} = \frac{15}{5}$ $x = 3$ only one answer for x	$7x < 2x + 15$ $7x - 2x < 2x - 2x + 15$ $5x < 15$ $\frac{5x}{5} < \frac{15}{5}$ $x < 3$ more than one answer for x (a range of answers)
Verify Equation	**Verify Inequality**

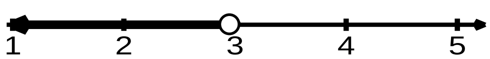
$7x = 2x + 15$ $7(3) = 2(3) + 15$ $21 = 6 + 15$ $21 = 21$ ✓	$7x < 2x + 15$ Since the solution says $x < 3$, choose any value and substitute in for x. Try $x = 2$ $7(2) < 2(2) + 15$ $14 < 4 + 15$ $14 < 19$ ✓

Extra Examples

Solve each inequality and graph the solution on a number line.

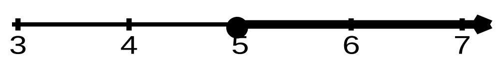
a). $x + 4 < 7$

$x + 4 - 4 < 7 - 4$
$x < 3$



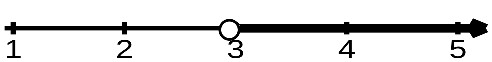
b). $2x \geq 10$

$\frac{2x}{2} \geq \frac{10}{2}$
$x \geq 5$



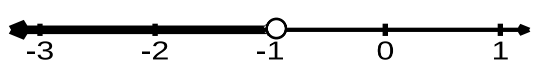
c). $3x + 1 > 10$

$3x + 1 - 1 > 10$
$\frac{3x}{3} > \frac{9}{3}$
$x > 3$



d). $-3(x - 2) > 9$

$-3x + 6 > 9$
$-3x + 6 - 6 > 9 - 6$
$\frac{-3x}{-3} > \frac{3}{-3}$
$x < -1$



e). $a - 10 \geq 2 + 4a$

$a - 10 + 10 \geq 2 + 10 + 4a$

$a \geq 12 + 4a$

$a - 4a \geq 12 + 4a - 4a$

$\frac{-3a}{-3} \geq \frac{12}{-3}$

$a \leq -4$

f). $\frac{y}{3} + \frac{1}{3} < \frac{-4}{6}$

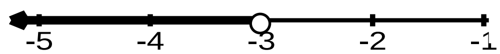
$\frac{y}{3} \times 6 + \frac{1}{3} \times 6 < \frac{-4}{6} \times 6$

$2y + 2 < -4$

$2y + 2 - 2 < -4 - 2$

$\frac{2y}{2} < \frac{-6}{2}$

$y < -3$



2. For the inequality $2(5 - 3x) \geq -7x + 2$, Karen says the solution is $x \geq -8$. Choose values to verify whether or not this is correct.

Try x = 0, its greater than -8 and should be true. $2(5-3(0)) \geq -7(0) + 2$ $2(5) \geq + 2$ $10 \geq 2$ TRUE!	Try x = -8, this should result in both sides being equal which is part of the solution. $2(5-3(-8)) \geq -7(-8) + 2$ $2(5 + 24) \geq +56 + 2$ $2(29) \geq 58$ $58 \geq 58$ EQUAL so TRUE!	Try x = – 10, this is less than -8 and should not be a solution. $2(5-3(-10)) \geq -7(-10) + 2$ $2(5 + 30) \geq +70 + 2$ $2(35) \geq 72$ $70 \geq 72$ FALSE!

Word Problems: Linear Equations and Inequalities

1. Write an equation or inequality for each statement and solve. Sketch number lines to show the solution for all inequalities.

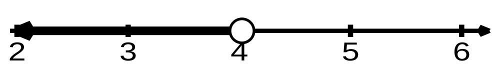
a). Triple a number decreased by one is less than 11.

$3x - 1 < 11$

$3x - 1 + 1 < 11 + 1$

$\frac{3x}{3} < \frac{12}{3}$

$x < 4$



b). A number multiplied by 4, increased by 5 is 1.

$4n + 5 = 1$ $\quad$ $4n + 5 - 5 = 1 - 5$ $\quad$ $\frac{4n}{4} = \frac{-4}{4}$ $\quad$ $n = -1$

c). You can invite at most 5 friends over to your house Saturday evening.

$x \leq 5$

d). Five subtract 3 times a number is equal to 3.5 times the same number subtract eight.

$5 - 3x = 3.5x - 8$

$5 - 5 - 3x = 3.5x - 8 - 5$

$-3x = 3.5x - 13$

$-3x - 3.5x = 3.5x - 3.5x - 13$

$\frac{-6.5x}{-6.5} = \frac{-13}{-6.5}$ $\quad$ $x = 2$

e). Henry has a choice of two companies to rent a car.
Company A charges \$199 per week plus \$0.20 kilometers driven.
Company B charges \$149 per week plus \$0.25 kilometers driven.
At what distance will both companies cost the same?

$199 + 0.20k = 149 + 0.25k$

$199 - 199 + 0.20k = 149 - 199 + 0.25k$

$0.20k - 0.25k = -50 + 0.25k - 0.25k$

$\frac{-0.05k}{-0.05} = \frac{-50}{-0.05}$ $\quad$ $k = 1000$ kilometers

2. Write an expression and solve.

a). Three times a number is -3.6

$3x = -3.6$

$X = -1.2$

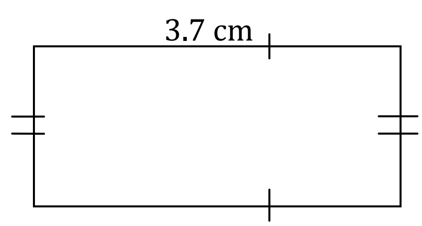
$\frac{3x}{3} = \frac{-3.6}{3}$

b). A number divided by 4 is 1.5

$\frac{n}{4} = 1.5$ $\quad$ $\frac{n}{4} \times 4 = 1.5 \times 4$

$n = 6$

3. A rectangle has length 3.7cm and perimeter 13.2cm.



a) Write an equation that can be used to determine the width of the rectangle.

$w + w + 3.7 + 3.7 = 13.2$

b) Solve the equation

$2w + 7.4 = 13.2$

$2w + 7.4 - 7.4 = 13.2 - 7.4$

$\frac{2w}{2} = \frac{5.8}{2}$

$w = 2.9$ cm

c) Verify the solution

$2.9 + 2.9 + 3.7 + 3.7 = 13.2$

$13.2 = 13.2$ ✓

4. Seven percent of a number is 56.7

7% of x = 56.7   7% = 0.07

a) Write, then solve an equation to determine the number

$0.07\,x = 56.7$ $\frac{0.07\,x}{0.07} = \frac{56.7}{0.07}$ $x = 810$

b) Check the solution

$7\% \times 810 = 56.7$

$0.07 \times 810 = 56.7$

$56.7 = 56.7$ ✓

5. Two different taxi companies charge the following:

Company A: $2.50 plus $0.25 per km   **Company B**: $3.00 plus $0.20 per km

At what distance will the cost be the same?

a) Write an equation for each company. $2.50 + 0.25k = 3 + 0.20k$

b) Solve the problem.

$2.50 + 0.25k = 3 + 0.20k$

$2.50 - 2.50 + 0.25k = 3 - 2.50 + 0.20k$

$0.25k = 0.5 + 0.20k$

$0.25k - 0.20\,k = 0.5 + 0.20k - 0.20k$

$\frac{0.05k}{0.05} = \frac{0.5}{0.05}$

$K = 10$ km

Both companies will have the same cost at 10 km.

c) Verify the solution.

$2.50 + 0.25k = 3 + 0.20k$

$2.50 + 0.25(10) = 3 + 0.20(10)$

$2.50 + 2.5 = 3 + 2$

$5 = 5$ ✓

6. A cell phone company offers two plans.

Plan A: 20 free minutes, \$0.75 per additional minute
Plan B: 30 free minutes, \$0.25 per additional minutes

Which time for calls will result in the same cost for both plans?
a) Write an equation for each company. $20 + 0.75m = 30 + 0.25m$

b) Solve the problem

$20 + 0.75m = 30 + 0.25m$
$20 - 20 + 0.75m = 30 - 20 + 0.25m$
$0.75m = 10 + 0.25m$
$0.75m - 0.25m = 10 + 0.25m - 0.25m$
$\frac{0.5m}{0.5} = \frac{10}{0.5}$ $m = 20$

Both Plans will have the same cost at 20 minutes.

c) Verify the solution.

$20 + 0.75(20) = 30 + 0.25(20)$ $20 + 15 = 30 + 5$
$35 = 35$ ✓

7. Chris is 7 years younger than his sister, Rachel. How old must each be if the sum of their ages is greater than 25?

Chris : $x - 7$
Rachel: x

$x + x - 7 > 25$
$2x - 7 > 25$
$2x - 7 + 7 > 25 + 7$
$\frac{2x}{2} > \frac{32}{2}$
Rachel: $x > 16$
Colin: $16 - 7 > 9$

Rachel must be greater than 16 and Colin must be greater than 9.

8. Debbie rents a car for \$350 plus \$12.50 per day on her vacation. If she has budgeted \$900 for her car rental, how many days can she rent the car? Graph the solution.

$350 + 12.50d \leq 900$
$350 - 350 + 12.50d \leq 900 - 350$
$\frac{12.50d}{12.50} \leq \frac{550}{12.50}$
$d \leq 44$

Debbie can rent the car for 44 or less days.