

GRADE 9 MATH - CIRCLE GEOMETRY

KEY TERMS:

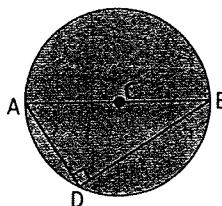
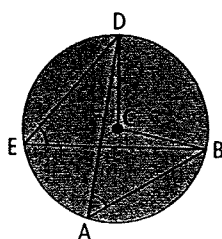
- chord
- central angle
- inscribed angle
- arc
- perpendicular bisector
- tangent

Students will be able to solve problems and justify the solution strategy using the following circle properties:

- the perpendicular from the centre of a circle to a chord bisects the chord
- the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
- the inscribed angles subtended by the same arc are congruent
- a tangent to a circle is perpendicular to the radius at the point of tangency

Key Ideas

- Inscribed angles subtended by the same arc of a circle are equal. $\angle DEB = \angle DAB$
- A central angle is twice the measure of an inscribed angle subtended by the same arc. $\angle DCB = 2\angle DAB$
- An inscribed angle is one half the measure of a central angle subtended by the same arc. $\angle DAB = \frac{1}{2}\angle DCB$
- When the inscribed angle is subtended by a diameter of the circle, the inscribed angle is equal to 90° .

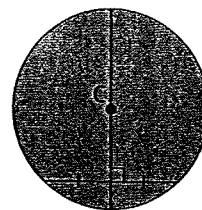


Literacy Link

The identical markings at the inscribed angles $\angle DEB$ and $\angle DAB$ indicate that the measures of these angles are equal.

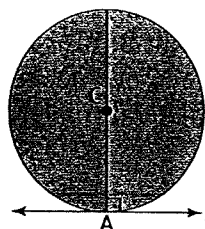
Key Ideas

- The perpendicular bisector of a chord passes through the centre of the circle.
- The perpendicular bisectors of two distinct chords intersect at the centre of the circle.
- If a bisector of a chord in a circle passes through the centre, then the bisector is perpendicular to the chord.
- If a line passes through the centre of a circle and intersects a chord at right angles, then the line bisects the chord.
- The shortest path between the centre of a circle and a chord is a line that is perpendicular to the chord.



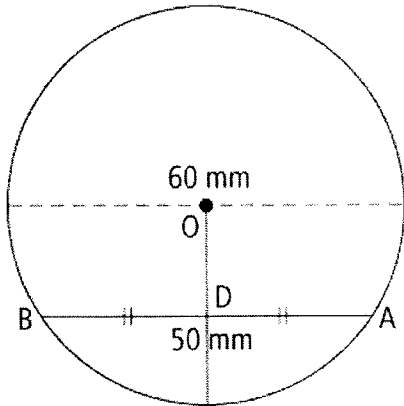
Key Ideas

- A line that touches a circle at exactly one point is tangent to the circle.
- Point A is known as the point of tangency.
- A line l that is tangent to a circle at point A is perpendicular to the radius AC.
- A chord drawn perpendicular to a tangent line at the point of tangency contains the centre of the circle, and is a diameter.

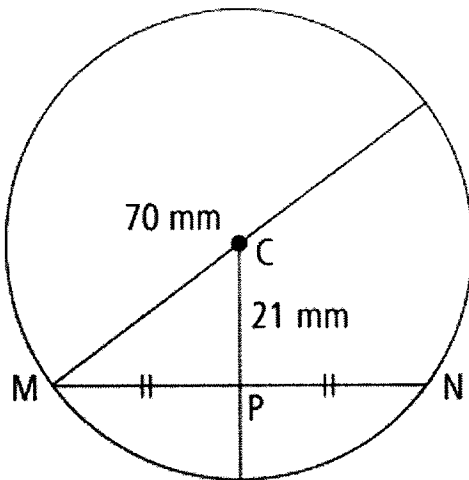


Circle Geometry Review

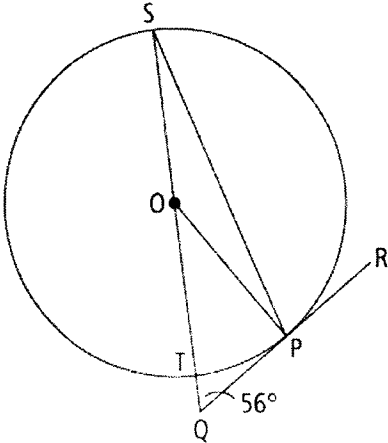
- Construct a circle with centre C and a chord AB . On chord AB , construct a central angle. Construct an inscribed angle with vertex labeled D that is subtended by arc AB . What is the relationship between $\angle ACB$ and $\angle ADB$?
- Construct circle C with a diameter of 12.5 cm. Draw line AB through the centre of the circle so that points A and B lie on the circumference of the circle. Draw a 10-cm chord extending from point A . Label the other endpoint D . Join points D and B .
 - What is the measure of $\angle ADB$? Explain your answer.
 - What is the length of line DB ? Explain your answer.
- A circle with centre O has a diameter of 60 mm. Chord AB is 50 mm long. The chord is bisected at point D . What is the distance between point O and point D ? Revise the diagram to show your thinking. Express your answer to the nearest millimetre.



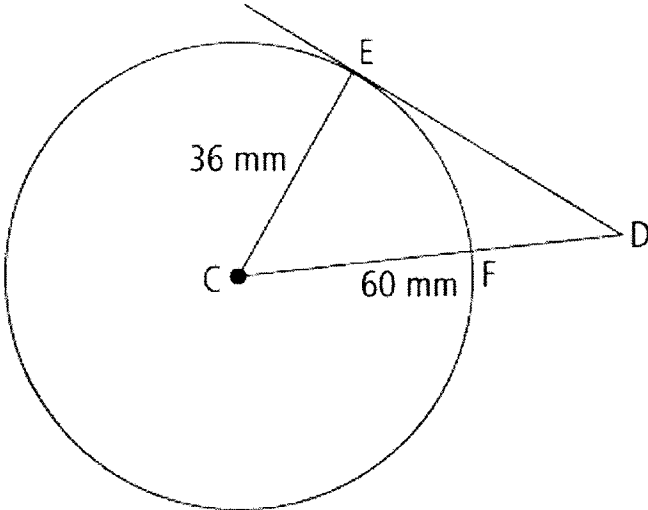
- A circle with centre C has a diameter 70 mm. Chord MN is drawn in the circle. The centre of the chord, P , is 21 mm from the centre of the circle. What is the length of the chord? Express your answer to the nearest millimetre.



5. Determine the measure of $\angle RPS$.

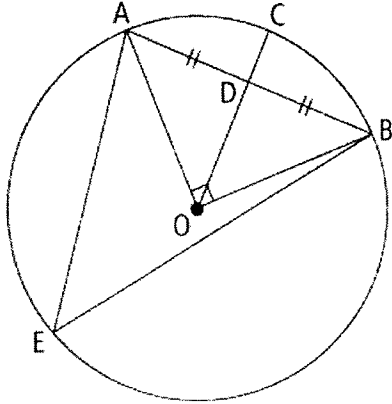


6. Line CD is 60 mm long. Point C is the centre of a circle with a radius of 36 mm. From point D , a tangent extends to the circle. The point of tangency is E . Determine the length of tangent DE .



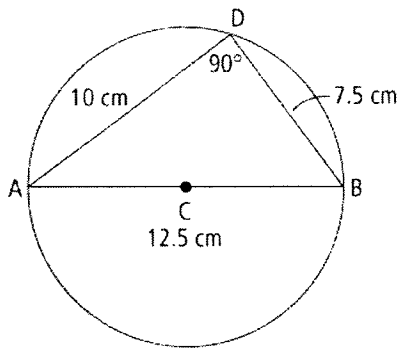
Answer Section

1. ANS:
Example:



$\angle ACB$ is twice the size of $\angle ADB$.

2. ANS:
Example: Not to scale

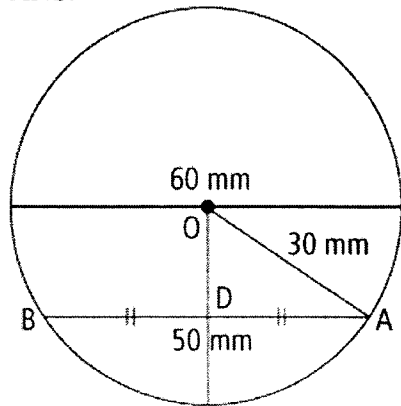


a) $\angle ADB$ is an inscribed angle subtended by a diameter, so it must be 90° .

$$\begin{aligned} \text{b) } DB &= \sqrt{12.5^2 - 10^2} \\ &= \sqrt{156.25 - 100} \\ &= \sqrt{56.25} \\ &= 7.5 \text{ cm} \end{aligned}$$

Line DB is 7.5 cm long.

3. ANS:



$$BA = 50 \text{ mm}$$

$$DA = 25 \text{ mm}$$

$$OA \text{ is a radius} = 30 \text{ mm}$$

$$\begin{aligned} & \sqrt{30^2 - 25^2} \\ &= \sqrt{900 - 625} \\ &= \sqrt{275} \\ &= 16.58 \end{aligned}$$

$$OD = 17 \text{ mm}$$

The length of line OD is approximately 17 mm.

4. ANS:

Radius CM is 35 mm.

Line CP is 21 mm.

The length of chord MN is 56 mm.

$$MN = 2MP$$

$$\begin{aligned} &= 2\sqrt{35^2 - 21^2} \\ &= 2\sqrt{1225 - 441} \\ &= 2\sqrt{784} \\ &= 2(28) \\ &= 56 \text{ mm} \end{aligned}$$

5. ANS:

Example:

$$\begin{aligned}\angle QOP &= 180^\circ - (90^\circ + 56^\circ) \\ &= 34^\circ\end{aligned}$$

$$\begin{aligned}\angle QSP &= \angle OPS = \angle QOP \div 2 \\ &= 17^\circ\end{aligned}$$

$$\angle OPR = 90^\circ \text{ (radius to point of tangency)}$$

$$\begin{aligned}\angle RPS &= \angle OPR - \angle OPS \\ &= 90^\circ - 17^\circ \\ &= 73^\circ\end{aligned}$$

$\angle RPS$ is 73° .

6. ANS:

Triangle DEC is a right triangle.

Tangent DE is 48 mm long.

$$\begin{aligned}DE &= \sqrt{CD^2 - CE^2} \\ &= \sqrt{60^2 - 36^2} \\ &= \sqrt{3600 - 1296} \\ &= \sqrt{2304} \\ &= 48 \text{ mm}\end{aligned}$$