UNIT 2 Powers and Exponent Laws

Imagine folding a piece of paper in half to form 2 layers.

Imagine folding it in half again to form 4 layers.

Try this with a sheet of paper. How many times can you fold the paper before it is impossible to make another fold?

What You'll Learn

- Use powers to represent repeated multiplication.
- Use patterns to understand a power with exponent 0.
- Solve problems involving powers.
- Perform operations with powers.
- Explain and apply the order of operations with exponents.

Why It's Important

Powers provide an efficient way to record our work. The properties of powers lead to even more efficient ways to perform some calculations. Powers are used in many formulas with applications in science, construction, and design.

Key Words

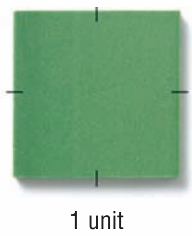
- power
- base
- exponent
- square number
- cube number
- power of a power
- power of a product
- power of a quotient

2.1 What Is a Power?

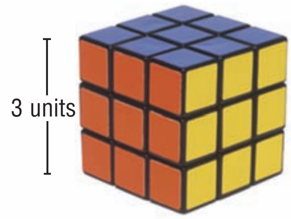
FOCUS

- Use powers to represent repeated multiplication.

What is the area of this square?
Write the area as a product.



What is the volume of this cube?
Write the volume as a product.



Investigate

You will need congruent square tiles and congruent cubes.

➤ Use the tiles to make as many different-sized larger squares as you can.
Write the area of each square as a product. Record your results in a table.

Number of Tiles	Area (square units)	Side Length (units)	Area as a Product
1	1	1	1×1

➤ Use the cubes to make as many different-sized larger cubes as you can.
Write the volume of each cube as a product. Record your results in a table.

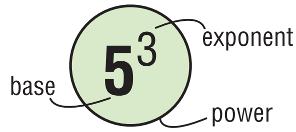
Number of Cubes	Volume (cubic units)	Edge Length (units)	Volume as a Product
1	1	1	$1 \times 1 \times 1$

Reflect & Share

What patterns do you see in the tables?
Use the patterns to predict the areas of the next 3 squares and the volumes of the next 3 cubes.
How are these areas and volumes the same? How are they different?

Connect

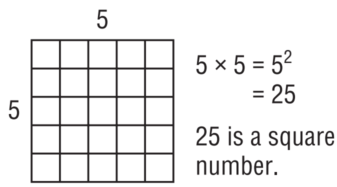
When an integer, other than 0, can be written as a product of equal factors, we can write the integer as a **power**.
For example, $5 \times 5 \times 5$ is 5^3.
5 is the **base**.
3 is the **exponent**.
5^3 is the *power*.
5^3 is a power of 5.



We say: 5 to the 3rd, or 5 cubed

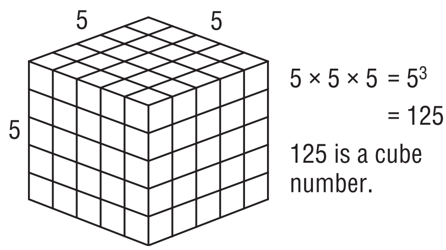
➤ A power with an integer base and exponent 2 is a **square number**.
When the base is a positive integer, we can illustrate a square number.

Here are 3 ways to write 25.
Standard form: 25
As repeated multiplication: 5×5
As a power: 5^2



➤ A power with an integer base and exponent 3 is a **cube number**.
When the base is a positive integer, we can illustrate a cube number.

Here are 3 ways to write 125.
Standard form: 125
As repeated multiplication: $5 \times 5 \times 5$
As a power: 5^3



Example 1 Writing Powers

Write as a power.
a) $3 \times 3 \times 3 \times 3 \times 3 \times 3$
b) 7

A Solution

a) $3 \times 3 \times 3 \times 3 \times 3 \times 3$
The base is 3. There are 6 equal factors, so the exponent is 6.
So, $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

b) 7
The base is 7. There is only 1 factor, so the exponent is 1.
So, $7 = 7^1$

Example 2 Evaluating Powers

Write as repeated multiplication and in standard form.

a) 3^5 b) 7^4

A Solution

a) $3^5 = 3 \times 3 \times 3 \times 3 \times 3$ As repeated multiplication

$= 243$ Standard form

b) $7^4 = 7 \times 7 \times 7 \times 7$ As repeated multiplication

$= 2401$ Standard form

Examples 1 and *2* showed powers with positive integer bases.
A power can also be negative or have a base that is a negative integer.

Example 3 Evaluating Expressions Involving Negative Signs

Identify the base of each power, then evaluate the power.

a) $(-3)^4$ b) -3^4 c) $-(-3^4)$

A Solution

a) The base of the power is -3.

$(-3)^4 = (-3) \times (-3) \times (-3) \times (-3)$ As repeated multiplication

Apply the rules for multiplying integers:
The sign of a product with an even
number of negative factors is positive.

So, $(-3)^4 = 81$ Standard form

b) The base of the power is 3.
The exponent applies only to the base 3,
and not to the negative sign.

$-3^4 = -(3^4)$

$= -(3 \times 3 \times 3 \times 3)$

$= -81$

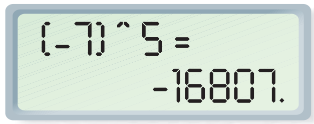
c) From part b, we know that $-3^4 = -81$.

So, $-(-3^4) = -(-81)$ $-(-81)$ is the opposite of -81, which is 81.

$= 81$

We may write the product of integer factors without the multiplication sign.
In *Example 3a*, we may write $(-3) \times (-3) \times (-3) \times (-3)$ as $(-3)(-3)(-3)(-3)$.

A calculator can be used to evaluate a power such as $(-7)^5$ in standard form.



Discuss the ideas

1. Can every integer, other than 0, be written as a power? Explain.

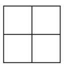
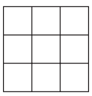
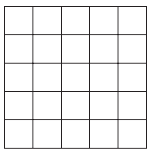
2. Why is -3^4 negative but $(-3)^4$ positive? Give another example like this.

3. Two students compared the calculator key sequences they used to evaluate a power. Why might the sequences be different?

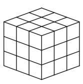
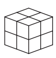
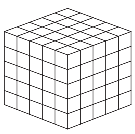
Practice

Check

4. Write the number of unit squares in each large square as a power.

a)  **b)**  **c)** 

5. Write the number of unit cubes in each large cube as a power.

a)  **b)**  **c)** 

6. Use grid paper. Draw a picture to represent each square number.

a) 4^2 **b)** 6×6 **c)** 49
d) 10^2 **e)** 81 **f)** 12×12

7. Write the base of each power.

a) 2^7 **b)** 4^3
c) 8^2 **d)** $(-10)^5$
e) $(-6)^7$ **f)** -8^3

8. Write the exponent of each power.

a) 2^5 **b)** 6^4
c) 9^1 **d)** -3^2
e) $(-2)^9$ **f)** $(-8)^3$

9. Write each power as repeated multiplication.

a) 3^2 **b)** 10^4
c) 8^5 **d)** $(-6)^5$
e) -6^5 **f)** -4^2

10. a) Explain how to build models to show the difference between 3^2 and 2^3.

b) Why is one number called a square number and the other number called a cube number?

11. Use repeated multiplication to show why 6^4 is not the same as 4^6.

12. Write as a power.
a) $4 \times 4 \times 4 \times 4$
b) $2 \times 2 \times 2$
c) $5 \times 5 \times 5 \times 5 \times 5 \times 5$
d) $10 \times 10 \times 10$
e) $(-79)(-79)$
f) $-(-2)(-2)(-2)(-2)(-2)(-2)(-2)(-2)$

Apply

13. Write each product as a power, then evaluate.
a) 5×5
b) $3 \times 3 \times 3 \times 3$
c) $10 \times 10 \times 10 \times 10 \times 10$
d) $-(9 \times 9 \times 9)$
e) $(-2)(-2)(-2)$
f) $-(-4)(-4)(-4)$
g) $(-5)(-5)(-5)(-5)$
h) $-(5)(5)(5)(5)$
i) $-(-5)(-5)(-5)(-5)$

14. Predict whether each answer is positive or negative, then evaluate.
a) 2^3 b) 10^6 c) 3^1 d) -7^3
e) $(-7)^3$ f) $(-2)^8$ g) -2^8 h) -6^4
i) $(-6)^4$ j) $-(-6)^4$ k) $(-5)^3$ l) -4^4

15. Canada Post often creates special postage stamps to celebrate important events and honour famous people.

a) Captain George Vancouver was a Dutch explorer who named almost 400 Canadian places. To commemorate his 250th birthday in 2007, Canada Post created a \$1.55 stamp.
i) How many stamps are in a 3 by 3 block? Write the number of stamps as a power.
ii) What is the value of these stamps?

b) In July 2007, Canada hosted the FIFA U-20 World Cup Soccer Championships. Canada Post issued a 52¢ stamp to honour all the players and fans.
i) How many stamps are in a 4 by 4 block? Write the number of stamps as a power.
ii) What is the value of these stamps?

16. Evaluate.
a) 3^{12} b) -7^7
c) 5^{11} d) $-(-4)^{10}$
e) $(-9)^8$ f) 2^{23}

17. **Assessment Focus**
a) Write as repeated multiplication and in standard form.
i) 4^3 ii) -4^3 iii) $-(-4^3)$ iv) (-4^3)
b) Which products in part a are positive? Why? Which products are negative? Why?
c) Write as repeated multiplication and in standard form.
i) 4^2 ii) -4^2 iii) $-(-4^2)$ iv) (-4^2)
d) Which products in part c are positive? Why? Which products are negative? Why?
e) Write other sets of powers like those in parts a and c. Explain how you know if each product is positive or negative before you write the power in standard form.

18. **a)** Is the value of -3^5 different from the value of $(-3)^5$ or (-3^5)? What purpose do the brackets serve?

b) Is the value of -4^6 different from the value of $(-4)^6$ or (-4^6)? What purpose do the brackets serve?

19. **a)** When does a negative base in a power produce a negative product? Give 3 examples.

b) When does a negative base in a power produce a positive product? Give 3 examples.

Take It Further

20. Write each number as a power with base 2. Explain your method.

a) 4 **b)** 16 **c)** 64

d) 256 **e)** 32 **f)** 128

21. **a)** Write each number as a power in as many ways as possible.

i) 16 **ii)** 81 **iii)** 256

b) Find other numbers that can be written as a power in more than one way. Show your work.

22. **a)** How are the powers in each pair the same? How are they different?

i) 2^3 or 3^2 **ii)** 2^5 or 5^2

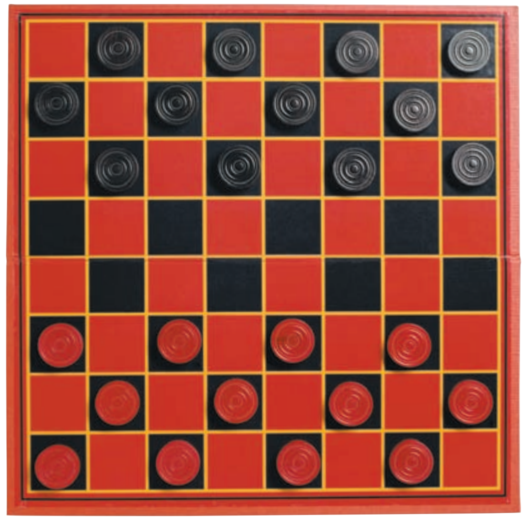
iii) 3^4 or 4^3 **iv)** 5^4 or 4^5

b) In part a, which is the greater power in each pair? Explain how you know.

23. Without evaluating all the powers, write them in order from greatest to least: 3^5, 5^2, 3^4, 6^3

Explain your strategy.

24.



How many squares of each side length are there on a checkerboard? Write each number as a power.

a) 1 unit **b)** 2 units

c) 3 units **d)** 4 units

e) 5 units **f)** 6 units

g) 7 units **h)** 8 units

What patterns do you see in the answers?

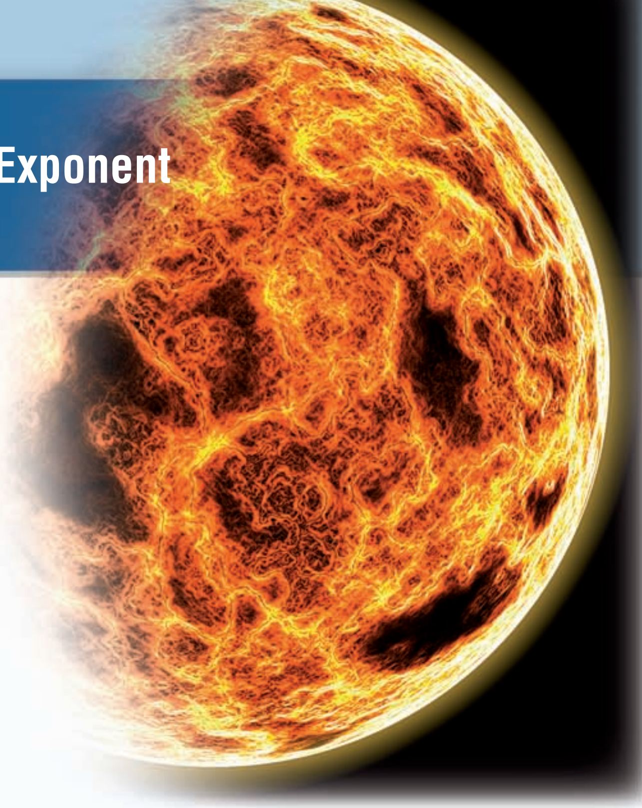
25. Explain how to tell if a number is a square number, or a cube number, or neither. Give examples.

Reflect

What is a power?

Why are brackets used when a power has a negative base?

2.2 Powers of Ten and the Zero Exponent



FOCUS

- Explore patterns and powers of 10 to develop a meaning for the exponent 0.

Nuclear reactions in the core of the sun create solar energy.
For these reactions to take place, extreme temperature and pressure are needed.
The temperature of the sun's core is about 10^7 °C.
What is this temperature in millions of degrees Celsius?

Investigate

Choose a number between 1 and 10 as the base of a power.
Use the exponents 5, 4, 3, 2, and 1.
Use your base and each exponent to write a power.
Then write the power as repeated multiplication and in standard form.
Record your results in a table.

Exponent	Power	Repeated Multiplication	Standard Form
5			
4			
3			
2			
1			

Describe any patterns in your table.
Continue the patterns to complete the entries in the last row.

Reflect & Share

Compare your tables and patterns with those of other pairs of students.
What do you think is the value of a power with exponent 0?
Use a calculator to check your answer for different integer bases.

Connect

This table shows decreasing powers of 10.

Number in Words	Standard Form	Power
One billion	1 000 000 000	10^9
One hundred million	100 000 000	10^8
Ten million	10 000 000	10^7
One million	1 000 000	10^6
One hundred thousand	100 000	10^5
Ten thousand	10 000	10^4
One thousand	1 000	10^3
One hundred	100	10^2
Ten	10	10^1
One	1	10^0

← We use the pattern in the exponents to write 1 as 10^0.

We could make a similar table for the powers of any integer base except 0.
So, 1 can be written as any power with exponent 0.
For example, $1 = 2^0$

$1 = 13^0$

$1 = (-5)^0$

Zero Exponent Law

A power with an integer base, other than 0, and an exponent 0 is equal to 1.

$n^0 = 1, \qquad n \neq 0$

Example 1 Evaluating Powers with Exponent Zero

Evaluate each expression.

a) 4^0 **b)** -4^0 **c)** $(-4)^0$

A Solution

A power with exponent 0 is equal to 1.

a) $4^0 = 1$ **b)** $-4^0 = -1$ **c)** $(-4)^0 = 1$

We can use the zero exponent and powers of 10 to write a number.

Example 2 Writing Numbers Using Powers of Ten

Write 3452 using powers of 10.

A Solution

Use a place-value chart.

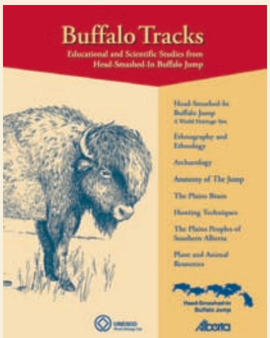
Thousands	Hundreds	Tens	Ones
3	4	5	2

$3452 = 3000 + 400 + 50 + 2$

$= (3 \times 1000) + (4 \times 100) + (5 \times 10) + (2 \times 1)$ We use brackets for clarity.

$= (3 \times 10^3) + (4 \times 10^2) + (5 \times 10^1) + (2 \times 10^0)$

Example 3 Interpreting Numbers in the Media



Head-Smashed-In Buffalo Jump is a UNESCO World Heritage Site in Southern Alberta. This site covers 600 hectares and contains cultural remains used in the communal hunting of buffalo. Head-Smashed-In was first used for hunting bison at least 5700 years ago and perhaps as early as 10 000 years ago. It is estimated that close to sixty million Plains Bison roamed the prairies prior to the Europeans' arrival in Western Canada. Less than one hundred years later, fewer than 1000 animals remained.

Use powers of 10 to write each number in the above paragraph.

A Solution

$600 = 6 \times 100$

$= 6 \times 10^2$

$5700 = 5000 + 700$

$= (5 \times 1000) + (7 \times 100)$

$= (5 \times 10^3) + (7 \times 10^2)$

$10\,000 = 1 \times 10^4$

$60\,000\,000 = 6 \times 10\,000\,000$

$= 6 \times 10^7$

$100 = 1 \times 10^2$

$1000 = 1 \times 10^3$

Discuss the ideas

1. In *Example 1*, why are 4^0 and $(-4)^0$ equal to 1, while -4^0 is equal to -1?

2. What is meant by "a power of 10"? Name 6 numbers that are powers of 10.

3. How would you use patterns to explain that $10^0 = 1$?

Practice

Check

4. Evaluate each power.
   a) 50^0 b) 9^0 c) 1^0 d) 17^0

5. Evaluate each power.
   a) $(-6)^0$ b) -11^0 c) -8^0 d) $(-24)^0$

6. Write each number as a power of 10.
   a) 1000 b) 100 000 c) 1 000 000 000
   d) ten thousand e) one hundred billion

Apply

7. Write 1 as a power in three different ways.

8. Evaluate each power of 10.
   a) 10^7 b) 10^2 c) 10^0
   d) 10^{10} e) 10^1 f) 10^6

9. Use powers of 10 to write each number.
   a) 6 000 000 000 b) 200 c) 51 415
   d) 60 702 008 e) 302 411 f) 2 000 008

10. Write each number in standard form.
    a) 7×10^7
    b) $(3 \times 10^4) + (9 \times 10^3) + (5 \times 10^1) + (7 \times 10^0)$
    c) $(8 \times 10^8) + (5 \times 10^5) + (2 \times 10^2)$
    d) $(9 \times 10^{10}) + (8 \times 10^9) + (1 \times 10^0)$
    e) 1×10^{15}
    f) $(4 \times 10^3) + (1 \times 10^0) + (9 \times 10^5) + (3 \times 10^1)$

11. The data below refer to trees in Vancouver. Use powers of 10 to write each number.
    - Street trees have an estimated value of over $500 million.
    - In the past decade, the Park Board has planted almost 40 000 new street trees.
    - Nearly 3 million ladybugs are released every year to help control aphids on street trees.
    - The most common street tree is the Japanese flowering cherry, with over 17 000 growing on city streets.
    - There are 130 000 trees lining the streets of Vancouver.
    - There are nearly 600 different types of trees.

12. **Assessment Focus** Choose a negative integer as the base of a power. Copy and complete the table below. Use patterns to explain why the power with exponent 0 is equal to 1.

Exponent	Power	Standard Form
5		
4		
3		
2		
1		
0		

13. In each pair, which number is greater? How do you know?

a) $(4 \times 10^3) + (6 \times 10^2) + (6 \times 10^1) + (7 \times 10^0)$ or 4327

b) $(2 \times 10^4) + (4 \times 10^3) + (2 \times 10^2) + (4 \times 10^1)$ or 2432

c) $(7 \times 10^7) + (7 \times 10^3)$ or 777 777

14.

- Worldwide, about one billion people lack access to safe drinking water.
- Glacier ice over 100 000 years old can be found at the base of many Canadian Arctic ice caps.
- Approximately 1000 kg of water is required to grow 1 kg of potatoes.
- Henderson Lake, British Columbia, has the greatest average annual precipitation in Canada of 6655 mm. That is more than 100 times as much as Eureka, in Nunavut, which has the least average annual precipitation of 64 mm.
- In November 2007, at the request of local First Nations, over 10 million hectares of the Mackenzie River Basin were protected from industrial development.

Using this information:

a) Identify the powers of 10 and write them using exponents.

b) Arrange the numbers in order from least to greatest.

c) Explain how writing powers of 10 using exponents can help you to order and compare numbers.

Take It Further

15. What are the meanings of the words trillion, quadrillion, and quintillion? Write these numbers as powers. What strategies did you use?

Reflect

Why is a power with exponent 0 equal to 1?

Math Link

Your World

The amount of data that an MP3 player can store is measured in gigabytes. For example, one MP3 player can store 2 GB (gigabytes) of songs. One song uses about 7000 KB (kilobytes) of space, where 1 GB = 2^{20} KB. About how many songs can the MP3 player hold?

2.3 Order of Operations with Powers

FOCUS

- Explain and apply the order of operations with exponents.

This was a skill-testing question in a competition:
$6 \times (3 + 2) - 10 \div 2$
Which answer is correct: 5, 10, 15, or 25?
How do you know?

Investigate

2

Use each of the digits 2, 3, 4, and 5 once to write an expression.

➤ The expression must have at least one power.
The base of the power can be a positive or negative integer.

➤ The expression can use any of:
addition, subtraction, multiplication, division, and brackets

Evaluate the expression.
Write and evaluate as many different expressions as you can.

Reflect & Share

Share your expressions with another pair of students.
Where does evaluating a power fit in the order of operations?
Why do you think this is?

Connect

To avoid getting different answers when we evaluate an expression, we use this order of operations:

- Evaluate the expression in brackets first.
- Evaluate the powers.
- Multiply and divide, in order, from left to right.
- Add and subtract, in order, from left to right.

Example 1 Adding and Subtracting with Powers

Evaluate.

a) $3^3 + 2^3$ **b)** $3 - 2^3$ **c)** $(3 + 2)^3$

A Solution

a) Evaluate the powers before adding.

$$\begin{aligned} 3^3 + 2^3 &= (3)(3)(3) + (2)(2)(2) \\ &= 27 + 8 \\ &= 35 \end{aligned}$$

b) Evaluate the power, then subtract.

$$\begin{aligned} 3 - 2^3 &= 3 - (2)(2)(2) \\ &= 3 - 8 \\ &= -5 \end{aligned}$$

c) Add first, since this operation is within the brackets. Then evaluate the power.

$$\begin{aligned} (3 + 2)^3 &= 5^3 \\ &= (5)(5)(5) \\ &= 125 \end{aligned}$$

When we need curved brackets for integers, we use square brackets to show the order of operations. When the numbers are too large to use mental math, we use a calculator.

Example 2 Multiplying and Dividing with Powers

Evaluate.

a) $[2 \times (-3)^3 - 6]^2$ **b)** $(18^2 + 5^0)^2 \div (-5)^3$

A Solution

a) Follow the order of operations.

Do the operations in brackets first: evaluate the power $(-3)^3$

$$\begin{aligned} [2 \times (-3)^3 - 6]^2 &= [2 \times (-27) - 6]^2 && \text{Then multiply: } 2 \times (-27) \\ &= [-54 - 6]^2 && \text{Then subtract: } -54 - 6 \\ &= (-60)^2 && \text{Then evaluate the power: } (-60)^2 \\ &= 3600 \end{aligned}$$

b) Use a calculator to evaluate $(18^2 + 5^0)^2 \div (-5)^3$.

For the first bracket:
Use mental math when you can: $5^0 = 1$
Evaluate $18^2 + 1$ to display 325.
Evaluate 325^2 to display 105 625.
For the second bracket:
$(-5)^3$ is negative, so simply evaluate 5^3 to display 125.
To evaluate 105 625 ÷ (−125), the integers have opposite signs, so the quotient is negative.
Evaluate 105 625 ÷ 125 to display 845.
So, $(18^2 + 5^0)^2 \div (-5)^3 = -845$

Example 3 Solving Problems Using Powers

Lyn has a square swimming pool, 2 m deep with side length 4 m. The swimming pool is joined to a circular hot tub, 1 m deep with diameter 2 m. Lyn adds 690 g of chlorine to the pool and hot tub each week. This expression represents how much chlorine is present per 1 m³ of water:

$$\frac{690}{2 \times 4^2 + \pi \times 1^3}$$

The suggested concentration of chlorine is 20 g/m³ of water.
What is the concentration of chlorine in Lyn's pool and hot tub?
Is it close to the suggested concentration?

A Solution

Use a calculator. Since the denominator has a sum, draw brackets around it.
This ensures the entire denominator is divided into the numerator.
Key in the expression as it now appears: $\frac{690}{(2 \times 4^2 + \pi \times 1^3)} \doteq 19.634\ 85$

The concentration is about 19.6 g/m³. This is very close to the suggested concentration.

Discuss the ideas

1. Explain why the answers to $3^3 + 2^3$ and $(3 + 2)^3$ are different.

2. Use the meaning of a power to explain why powers are evaluated before multiplication and division.

Practice

Check

3. Evaluate.

a) $3^2 + 1$ b) $3^2 - 1$
c) $(3 + 1)^2$ d) $(3 - 1)^2$
e) $2^2 + 4$ f) $2^2 - 4$
g) $(2 + 4)^2$ h) $(2 - 4)^2$
i) $2 - 4^2$ j) $2^2 - 4^2$

4. Evaluate. Check using a calculator.

a) $2^3 \times 5$ b) 2×5^2
c) $(2 \times 5)^3$ d) $(2 \times 5)^2$
e) $(-10)^3 \div 5$ f) $(-10) \div 5^0$
g) $[(-10) \div 5]^3$ h) $[(-10) \div 5]^0$

5. Evaluate.

a) $2^3 + (-2)^3$ b) $(2 - 3)^3$
c) $2^3 - (-3)^3$ d) $(2 + 3)^3$
e) $2^3 \div (-1)^3$ f) $(2 \div 2)^3$
g) $2^3 \times (-2)^3$ h) $(2 \times 1)^3$

Apply

6. a) Evaluate. Record your work.

i) $4^2 + 4^3$ ii) $5^3 + 5^6$

b) Evaluate. Record your work.

i) $6^3 - 6^2$ ii) $6^3 - 6^5$

7. Identify, then correct, any errors in the student work below. Explain how you think the errors occurred.

$3^2 + 2^2 \times 2^4 + (-6)^2$
$= 9 + 4 \times 16 - 36$
$= 13 \times 16 - 36$
$= 172$

8. State which operation you will do first, then evaluate.

a) $(7)(4) - (5)^2$ b) $6(2 - 5)^2$
c) $(-3)^2 + (4)(7)$ d) $(-6) + 4^0 \times (-2)$
e) $10^2 \div [10 \div (-2)]^2$ f) $[18 \div (-6)]^3 \times 2$

9. Sometimes it is helpful to use an acronym as a memory trick. Create an acronym to help you remember the order of operations. Share it with your classmates.

An acronym is a word formed from the first letters of other words.

10. Evaluate.

a) $(3 + 4)^2 \times (4 - 6)^3$
b) $(8 \div 2^2 + 1)^3 - 3^5$
c) $4^3 \div [8(6^0 - 2^1)]$
d) $9^2 \div [9 \div (-3)]^2$
e) $(2^2 \times 1^3)^2$
f) $(11^3 + 5^2)^0 + (4^2 - 2^4)$

11. Explain why the brackets are not necessary to evaluate this expression.
$(-4^3 \times 10) - (6 \div 2)$
Evaluate the expression, showing each step.

12. Winona is tiling her 3-m by 3-m kitchen floor. She bought stone tiles at $70/m^2. It costs $60/m^2 to install the tiles. Winona has a coupon for a 25% discount off the installation cost. This expression represents the cost, in dollars, to tile the floor:
$70 \times 3^2 + 60 \times 3^2 \times 0.75$
How much does it cost to tile the floor?

13. Evaluate this expression:
$2^3 + (3 \times 4)^2 - 6$
Change the position of the brackets. Evaluate the new expression. How many different answers can you get by changing only the position of the brackets?

14. Evaluate each pair of expressions. Why are some answers the same? Why are other answers different?

a) $3 + 5 \times 8$ and $5 \times 8 + 3$

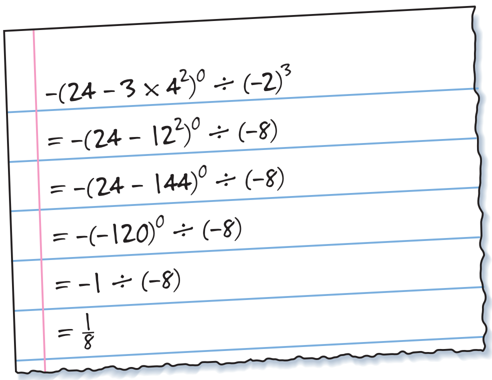
b) $3^2 + 2^2$ and $(3 + 2)^2$

c) $3^3 \times 2^3 - 5^2$ and $(3 \times 2)^3 - 5 \times 5$

d) $2^3 \times 3^2$ and $(2 \times 3)^5$

e) $5 \times 3 - 3^2 \times 4 + 20 \times 7$ and $5 \times (3 - 3^2) \times 4 + 20 \times 7$

15. This student got the correct answer, but she did not earn full marks. Find the mistake this student made. Explain how it is possible she got the correct answer. Write a more efficient solution for this problem.



16. Use a calculator to evaluate. Write the key strokes you used.

a) $(14 + 10)^2 \times (21 - 28)^3$

b) $(36 \div 2^2 + 11)^3 - 10^5$

c) $\dfrac{12^3}{36(12^0 - 13^1)}$

d) $\dfrac{81^2}{9^2 + (-9)^2}$

e) $(14^2 + 6^3)^2$

f) $(11^3 + 25^2)^0 + (27^2 - 33^4)$

17. **Assessment Focus** Predict which expression has a value closest to 0. Explain your strategy for predicting, then verify your prediction.

$(30 + 9 \times 11 \div 3)^0$

$(-3 \times 6) + 4^2$

$1 + (1 \div 1)^2 + 1^0$

18. Robbie, Marcia, and Nick got different answers when they evaluated this expression: $(-6)^2 - 2[(-8) \div 2]^2$
Robbie's answer was 68, Marcia's answer was 4, and Nick's answer was -68.

a) Who had the correct answer?

b) Show and explain how the other two students might have got their answers. Where did they go wrong?

19. A timber supplier manufactures and delivers wood chips. The chips are packaged in boxes that are cubes with edge length 25 cm. The cost of the chips is $14/m³, and delivery costs $10 per 25 km. One customer orders 150 boxes of wood chips and she lives 130 km from the supplier. This expression represents the cost, in dollars:

$\dfrac{10 \times 130}{25} + 25^3 \div 10^6 \times 14 \times 150$

How much does the customer pay?

20. Copy each statement. Insert brackets to make each statement true.

a) $10 + 2 \times 3^2 - 2 = 106$

b) $10 + 2 \times 3^2 - 2 = 24$

c) $10 + 2 \times 3^2 - 2 = 84$

d) $10 + 2 \times 3^2 - 2 = 254$

21. Copy each statement. Insert brackets to make each statement true.

a) $20 \div 2 + 2 \times 2^2 + 6 = 26$

b) $20 \div 2 + 2 \times 2^2 + 6 = 30$

c) $20 \div 2 + 2 \times 2^2 + 6 = 8$

d) $20 \div 2 + 2 \times 2^2 + 6 = 120$

22. Blake answered the following skill-testing question to try to win a prize:

$5 \times 4^2 - (2^3 + 3^3) \div 5$

Blake's answer was 11.

Did Blake win the prize? Show your work.

23. Write an expression that includes integers, powers, brackets, and all four operations. Evaluate the expression. Ask a classmate to evaluate the expression. Did both of you follow the same order of operations? Is it possible to get the same answer if you follow a different order of operations? Explain.

Take It Further

24. Copy and complete each set of equations. Describe any patterns you see. Extend each pattern by 2 more rows.

a) $1^3 = 1^2$

$1^3 + 2^3 = 3^2$

$1^3 + 2^3 + 3^3 = 6^2$

$1^3 + 2^3 + 3^3 + 4^3 =$

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 =$

b) $3^2 - 1^2 = \square^3$

$6^2 - 3^2 = \square^3$

$10^2 - 6^2 = \square^3$

$15^2 - 10^2 = \square^3$

$21^2 - 15^2 = \square^3$

25. Choose two numbers between −5 and +5.

a) Square the numbers, then add the squares. Write this as an expression.

b) Add the numbers, then square the sum. Write this as an expression.

c) Compare the answers to parts a and b. What do you notice?

d) A student said, "The sum of the squares of two numbers is equal to the square of the sum of the numbers." Do you agree with this statement? Justify your answer.

26. Use four 4s and any operations, brackets, or powers to write an expression for each whole number from 1 to 9.

27. a) Write each product as a power of 2 and in standard form.

i) $2 \times 2 \times 2 \times 2$

ii) 2×2

iii) $2 \times 2 \times 2 \times 2 \times 2$

iv) $2 \times 2 \times 2$

b) Write each number as a sum, using only powers of 2.

For example: $27 = 16 + 8 + 2 + 1$

$= 2^4 + 2^3 + 2^1 + 2^0$

i) 28 **ii)** 12

iii) 25 **iv)** 31

v) 50 **vi)** 75

c) Repeat part b with a different base. Share your results with a classmate.

Reflect

Why is the order of operations important? Include examples in your explanation.

Mid-Unit Review

2.1

1. Write each power in standard form.

a) 14^2 b) 5^1 c) -8^3

d) $-(-4)^4$ e) $(-6)^3$ f) $(-2)^8$

2. Copy and complete this table.

	Power	Base	Exponent	Repeated Multiplication	Standard Form
a)	4^3				
b)	2^5				
c)	8^6				
d)		7	2		
e)				3 × 3 × 3 × 3	

3. a) Evaluate the first 8 powers of 7. Copy and complete this table.

Power of 7	Standard Form
7^1	
7^2	
7^3	
7^4	
7^5	
7^6	
7^7	
7^8	

b) What pattern do you see in the ones digits of the numbers in the second column?

c) Verify that the pattern continues by extending the table for as many powers of 7 as your calculator displays.

d) Use the pattern. Predict the ones digit of each power of 7. Explain your strategy.

i) 7^{12} ii) 7^{14}

iii) 7^{17} iv) 7^{22}

2.2

4. Write in standard form.

a) 10^6 b) 10^0 c) 10^8 d) 10^4

5. Write as a power of 10.

a) one billion b) one

c) 100 d) 100 000

6. Evaluate.

a) $(-5)^0$ b) 25^0 c) -6^0 d) 9^0

7. The area of land is measured in hectares (ha). One hectare is the area of a square with side length 100 m. Write the number of square metres in 1 ha as a power.

2.3

8. Evaluate. State which operation you do first.

a) $(-21 - 6)^2 + 14$

b) $6 \div (-2) + (2 \times 3)^2$

c) $[5 - (-4)]^3 - (21 \div 7)^4$

d) $[(6 - 21)^3 \times (2 + 2)^6]^0$

e) $(3 - 5)^5 \div (-4)$

f) $-30 - (7 - 4)^3$

9. Both Sophia and Victor evaluated this expression: $-2^4 \times 5 + 16 \div (-2)^3$
Sophia's answer was −82 and Victor's answer was 78. Who is correct? Find the likely error made by the other student.

10. Identify, then correct, any errors in the student work below. How do you think the errors occurred?

$(-2)^4 - (-3)^3 \div (-9)^0 \times 2^3$

$= 16 - 27 \div (-1) \times 8$

$= -11 \div (-1) \times 8$

$= 11 \times 8$

$= 88$

Start Where You Are

What Strategy Could I Try?

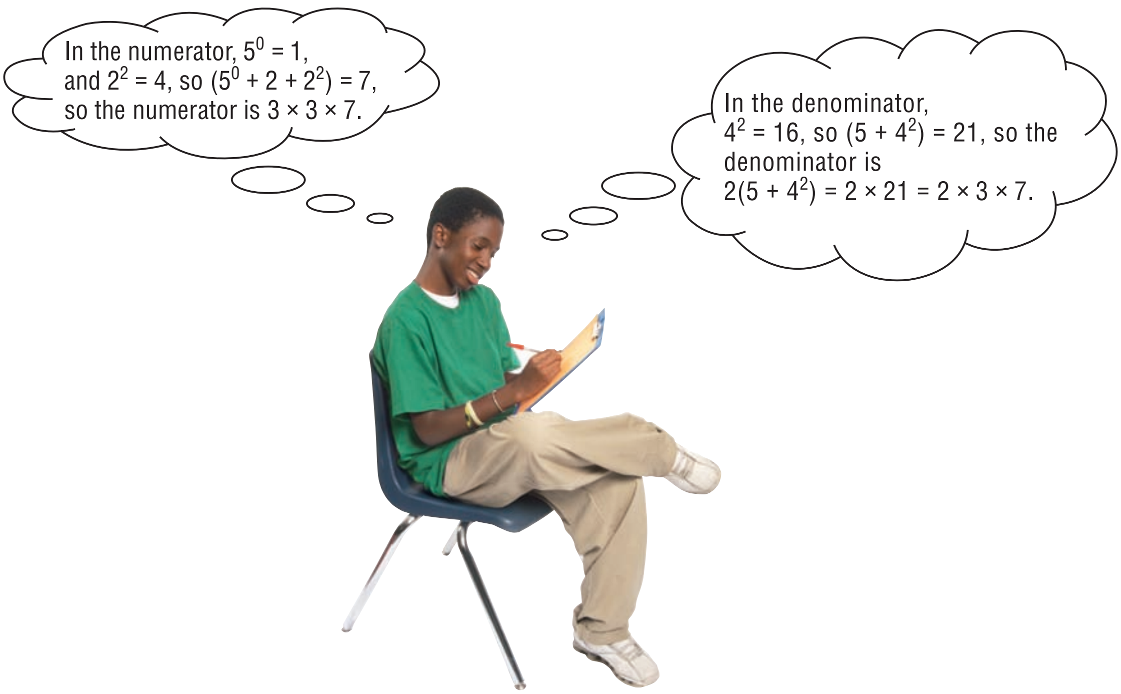
Suppose I have to evaluate this expression:

$$\frac{3^2(5^0 + 2 + 2^2)}{2(5 + 4^2)}$$

➤ What math tools could I use?
- mental math
- mental math, and paper and pencil
- a calculator

If I use only mental math, I might forget the numbers, so I write down the values of the numerator and denominator.

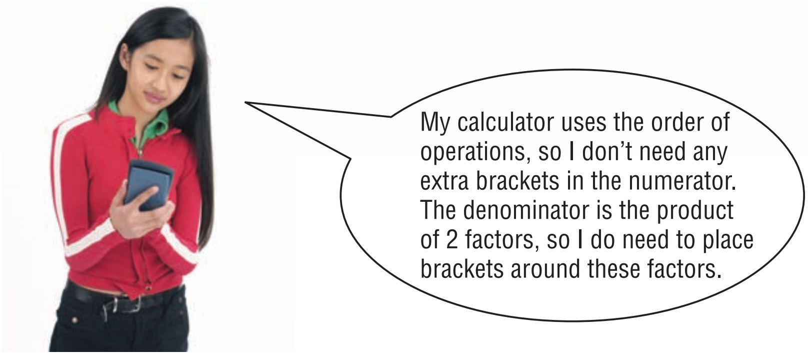
If I use mental math, and paper and pencil,
- I must use the order of operations.
- The fraction bar acts like a bracket, so I work on the numerator and denominator separately.
- I write down the values of the numerator and the denominator.
- I look for friendly numbers to help with the division.

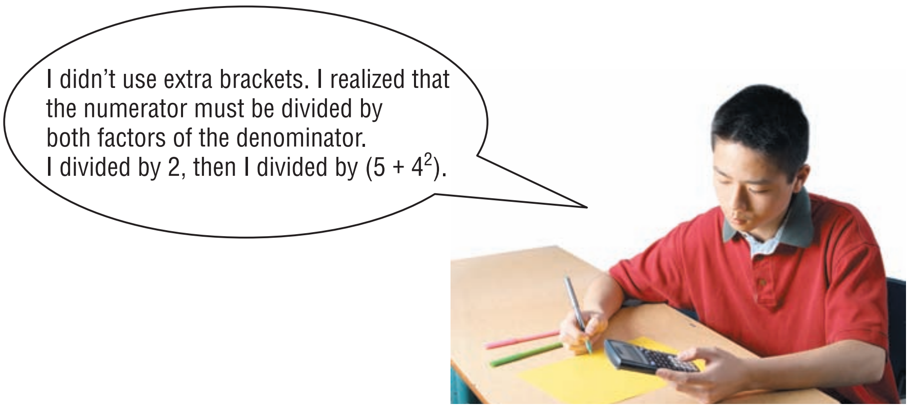


So, $\frac{3^2(5^0 + 2 + 2^2)}{2(5 + 4^2)} = \frac{3 \times 3 \times 7}{2 \times 3 \times 7} = \frac{3}{2}$

If I use a calculator,

- The fraction bar means divide the numerator by the denominator.
- My calculator uses the order of operations.
- Can I enter the expression as it is written?
- Do I need to add extra brackets, or change any operations?





Check

Use any strategies *you* know to evaluate these expressions.

1. a) $\dfrac{3^2 \times 6^2}{2^2 + 1}$ b) $\dfrac{3^2 \times 6^2}{2^3 \div 2^2}$ c) $\dfrac{3^2 + 6^2}{2^2 - 1}$

d) $\dfrac{3^2 - 6^2}{2^2 - 1}$ e) $\dfrac{6^2 \div 3^2}{2^2 \div 2}$

2. a) $\dfrac{3^4 - 2^2}{4^3 + 4^2 - 3^1}$ b) $\dfrac{4^2(3^4 \div 2^0)}{2^4(3^4 - 2^0)}$ c) $\dfrac{2^4(4^3 \div 2^2) - 4^0}{3(3^4 + 2^2)}$

GAME

Operation Target Practice

You will need

- two different-coloured number cubes labelled 1 to 6

Number of Players

- 3 or more

Goal of the Game

- To use the order of operations to write an expression for the target number

How to Play

1. Decide which number cube will represent the tens and which will represent the ones of a 2-digit number.
2. One player rolls the cubes and states the 2-digit target number formed.
3. Use three, four, or five operations.
   Each player writes an expression equal to the target number.
   A power counts as an operation; brackets do not.
4. Score 1 point if you were able to write an expression that equals the target number.
   Score another point if you wrote a correct expression that no one else wrote.
5. The next player repeats Step 2, then all of you repeat Steps 3 and 4.
6. The first player to get 10 points wins.

2.4 Exponent Laws I

FOCUS

- Understand and apply the exponent laws for products and quotients of powers.

When we multiply numbers, the order in which we multiply does not matter.
For example, $(2 \times 2) \times 2 = 2 \times (2 \times 2)$
So, we usually write the product without brackets:
$2 \times 2 \times 2$

Investigate

4

You will need 3 number cubes: 2 of one colour, the other a different colour

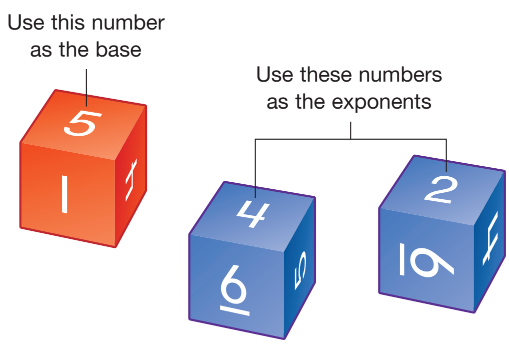
Two of you investigate multiplying powers. Make a table like this:

Product of Powers	Product as Repeated Multiplication	Product as a Power
$5^4 \times 5^2$	$(5 \times 5 \times 5 \times 5) \times (5 \times 5)$	$5^?$

Two of you investigate dividing powers. Make a table like this:

Quotient of Powers	Quotient as Repeated Multiplication	Quotient as a Power
$5^4 \div 5^2 = \frac{5^4}{5^2}$	$\frac{5 \times 5 \times 5 \times 5}{5 \times 5}$	$5^?$

Roll the cubes.
Use the numbers to create powers, as shown.



Record each quotient of powers with the greater exponent in the dividend (the numerator).

Express each power as repeated multiplication, and then as a single power.
Repeat the activity at least five times.

Reflect & Share

Describe the patterns in your table.
Share your patterns with the other pair in your group.
How are your patterns the same? How are they different?
Check your patterns with those of another group.
Use your patterns to describe a way to multiply two powers with the same base, and a way to divide two powers with the same base.

Connect

Patterns arise when we multiply and divide powers with the same base.

➤ To multiply $(-7)^3 \times (-7)^5$:

$$\begin{aligned}(-7)^3 \times (-7)^5 &= (-7)(-7)(-7) \times (-7)(-7)(-7)(-7)(-7)\\ &= (-7)(-7)(-7)(-7)(-7)(-7)(-7)(-7)\\ &= (-7)^8\end{aligned}$$

The base of the product is -7. The exponent is 8.
The sum of the exponents of the powers that were multiplied is $3 + 5 = 8$.
This relationship is true for the product of any two powers with the same base.
We use variables to represent the powers in the relationship:

Exponent Law for a Product of Powers

To multiply powers with the same base, add the exponents.

$a^m \times a^n = a^{m+n}$

The variable a is any integer, except 0.
The variables m and n are any whole numbers.

➤ To divide $8^7 \div 8^4$:

$$\begin{aligned}8^7 \div 8^4 &= \frac{8^7}{8^4}\\ &= \frac{8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8}{8 \times 8 \times 8 \times 8}\\ &= \frac{\cancel{8}^1 \times \cancel{8}^1 \times \cancel{8}^1 \times \cancel{8}^1 \times 8 \times 8 \times 8}{\cancel{8}_1 \times \cancel{8}_1 \times \cancel{8}_1 \times \cancel{8}_1}\\ &= \frac{8 \times 8 \times 8}{1}\\ &= 8 \times 8 \times 8\\ &= 8^3\end{aligned}$$

Divide the numerator and denominator of the fraction by their common factors: $8 \times 8 \times 8 \times 8$

So, $8^7 \div 8^4 = 8^3$

The base of the quotient is 8. The exponent is 3.
The difference of the exponents of the powers that were divided is $7 - 4 = 3$.
This relationship is true for the quotient of any two powers with the same base.

Exponent Law for a Quotient of Powers

To divide powers with the same base, subtract the exponents.

$a^m \div a^n = a^{m-n} \qquad m \geq n$

a is any integer, except 0; m and n are any whole numbers.

Example 1 Simplifying Products and Quotients with the Same Base

Write each expression as a power.

a) $6^5 \times 6^4$

b) $(-9)^{10} \div (-9)^6$

A Solution

a) The powers have the same base.
Use the exponent law for products: add the exponents.

$$6^5 \times 6^4 = 6^{(5+4)} = 6^9$$

b) The powers have the same base.
Use the exponent law for quotients: subtract the exponents.

$$(-9)^{10} \div (-9)^6 = (-9)^{(10-6)} = (-9)^4$$

Example 2 Evaluating Expressions Using Exponent Laws

Evaluate.

a) $(-2)^4 \times (-2)^7$

b) $3^2 \times 3^4 \div 3^3$

Solutions

Method 1

Simplify first using the exponent laws.

a) The bases are the same. Add exponents.

$$(-2)^4 \times (-2)^7 = (-2)^{(4+7)} = (-2)^{11} = -2048$$

b) All the bases are the same so add the exponents of the two powers that are multiplied. Then, subtract the exponent of the power that is divided.

$$3^2 \times 3^4 \div 3^3 = 3^{(2+4)} \div 3^3 = 3^6 \div 3^3 = 3^{(6-3)} = 3^3 = 27$$

Method 2

Use the order of operations.

a) Evaluate each power first.
Then use a calculator.

$$(-2)^4 \times (-2)^7 = 16 \times (-128) = -2048$$

b) Evaluate each power first.
Then use a calculator.

$3^2 \times 3^4 \div 3^3 = 9 \times 81 \div 27$

Multiply and divide in order from left to right.

$3^2 \times 3^4 \div 3^3 = 27$

Example 3 Using Exponent Laws and the Order of Operations

Evaluate.

a) $6^2 + 6^3 \times 6^2$

b) $(-10)^4[(-10)^6 \div (-10)^4] - 10^7$

A Solution

a) Multiply first. Add the exponents.

$$\begin{aligned} 6^2 + 6^3 \times 6^2 &= 6^2 + 6^{(3+2)} \\ &= 6^2 + 6^5 && \text{Evaluate each power.} \\ &= 36 + 7776 && \text{Then add.} \\ &= 7812 \end{aligned}$$

b) Evaluate the expression in the square brackets first.
Divide by subtracting the exponents.

$$\begin{aligned} (-10)^4[(-10)^6 \div (-10)^4] - 10^7 &= (-10)^4[(-10)^{(6-4)}] - 10^7 \\ &= (-10)^4(-10)^2 - 10^7 && \text{Multiply: add the exponents} \\ &= (-10)^{(4+2)} - 10^7 \\ &= (-10)^6 - 10^7 && \text{Evaluate each power.} \\ &= 1\,000\,000 - 10\,000\,000 && \text{Then subtract.} \\ &= -9\,000\,000 \end{aligned}$$

Discuss the ideas

1. Use your own words to explain how to:
   a) multiply two powers with the same base
   b) divide two powers with the same base

2. Do you think it makes sense to simplify an expression as much as possible before using a calculator? Explain.

3. When can you not add or subtract exponents to multiply or divide powers?

Practice

Check

4. Write each product as a single power.
   a) $5^5 \times 5^4$
   b) $10^2 \times 10^{11}$
   c) $(-3)^3 \times (-3)^3$
   d) $21^6 \times 21^4$
   e) $(-4)^1 \times (-4)^3$
   f) $6^{12} \times 6^3$
   g) $2^0 \times 2^4$
   h) $(-7)^3 \times (-7)^0$

5. Write each quotient as a power.
   a) $4^5 \div 4^3$
   b) $8^9 \div 8^6$
   c) $15^{10} \div 15^0$
   d) $(-6)^8 \div (-6)^3$
   e) $\frac{2^{12}}{2^{10}}$
   f) $\frac{(-10)^{12}}{(-10)^6}$
   g) $\frac{6^5}{6^1}$
   h) $\frac{(-1)^5}{(-1)^4}$

Apply

6. a) Evaluate.

i) $3^4 \div 3^4$ ii) $(-4)^6 \div (-4)^6$

iii) $\frac{5^8}{5^8}$ iv) $\frac{(-6)^3}{(-6)^3}$

b) Use the results of part a. Explain how the exponent law for the quotient of powers can be used to verify that a power with exponent 0 is 1.

7. a) Compare these products.

i) $3^4 \times 3^9$ ii) $3^9 \times 3^4$

b) Explain the results in part a.

8. Express as a single power.

a) $3^4 \times 3^9 \div 3^{11}$

b) $(-4)^3 \div (-4)^2 \times (-4)^{10}$

c) $6^0 \times 6^3 \div 6^2$

d) $\frac{4^3 \times 4^5}{4^2 \times 4^6}$ e) $\frac{(-3)^4 \times (-3)^4}{(-3)^4}$

9. a) Express as a single power, then evaluate.

i) $(-6)^1 \times (-6)^7 \div (-6)^7$

ii) $(-6)^7 \div (-6)^7 \times (-6)^1$

b) Explain why changing the order of the terms in the expressions in part a does not affect the answer.

10. Simplify, then evaluate.

a) $10^2 \times 10^2 + 10^4$ b) $10^3 \times 10^3 - 10^3$

c) $10^{11} - 10^3 \times 10^6$ d) $10^1 + 10^5 \times 10^2$

e) $10^6 \div 10^2 \times 10^2$ f) $10^9 \div 10^9$

g) $\frac{10^{12}}{10^6}$ h) $\frac{10^4 \times 10^3}{10^2}$

i) $\frac{10^{11}}{10^4 \times 10^2}$ j) $\frac{10^5}{10^3} + 10^2$

11. a) Evaluate: $2^6 - 2^2 \times 2^3$

Describe the steps you used.

b) Evaluate: $2^6 \times 2^2 - 2^3$

Describe the steps you used.

c) Were the steps for parts a and b different? Explain.

12. Assessment Focus An alfalfa field is a rectangle 10^4 m long and 10^3 m wide.

a) Write an expression for the area of the field, then evaluate the expression.

b) Write an expression for the perimeter of the field, then evaluate the expression.

c) i) Use the area in part a. Find all possible dimensions for a rectangular field with side lengths that are powers of 10.

ii) Find the perimeter of each field in part i.

d) Explain why the exponent laws are helpful for solving area problems, but not for perimeter problems.

13. Evaluate.

a) $2^3 \times 2^2 - 2^5 \times 2$

b) $3^2 \times 3 + 2^2 \times 2^4$

c) $4^2 - 3^0 \times 3 + 2^3$

d) $(-3)^6 \div (-3)^5 - (-3)^5 \div (-3)^3$

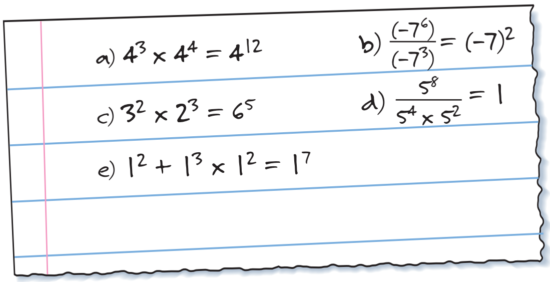
e) $(-2)^4[(-2)^5 \div (-2)^3] + (-2)^4$

f) $-2^4(2^6 \div 2^2) - 2^4$

g) $(-5)^3 \div (-5)^2 \times (-5)^0 + (-5)^2 \div (-5)$

14. Provide examples to show why the exponent laws for products and quotients cannot be applied when the powers have different bases.

15. Identify, then correct any errors in the student work below. Explain how you think the errors occurred.



16. Muguet uses a microscope to view bacteria. The bacteria are first magnified 10^2 times. This image is then magnified 10^1 times.

a) Use powers to write an expression for the total magnification.

b) How many times as large as the actual bacteria does the image appear?

17. a) Evaluate.

i) $5^2 + 5^3$ ii) $5^2 \times 5^3$

b) In part a, explain why you could use an exponent law to simplify one expression, but not the other.

18. a) Evaluate.

i) $4^3 - 4^2$ ii) $4^3 \div 4^2$

b) In part a, explain why you could use an exponent law to simplify one expression, but not the other.

19. Simplify, then evaluate only the expressions with a positive value. Explain how you know the sign of each answer without evaluating.

a) $(-2)^2 \times (-2)^3$

b) $(-2)^0 \times (-2)^5$

c) $(-2)^5 \div (-2)^3$

d) $(-2)^6 \div (-2)^6$

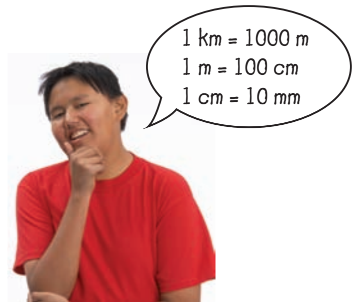
e) $\frac{(-2)^3 \times (-2)^4}{(-2)^3 \times (-2)^2}$

f) $\frac{(-2)^6}{(-2)^3 \times (-2)^2}$

Take It Further

20. Find two powers that have a product of 64. How many different pairs of powers can you find?

21. Write a product or quotient, then use the exponent laws to find the number of:



a) centimetres in 1 km

b) millimetres in 1 km

c) kilometres in 10^5 m

d) metres in 10^9 mm

22. Write a product or quotient, then use the exponent laws to find the number of:

a) square metres in 10^2 km^2

b) square metres in 10^6 cm^2

c) square millimetres in 10^6 cm^2

d) square centimetres in 1 km^2

23. Explain how the exponent laws help you to convert among units of measure.

Reflect

When can you use the exponent laws to evaluate an expression with powers? When can you *not* use these laws? Include examples in your explanation.

2.5 Exponent Laws II

FOCUS

- Understand and apply exponent laws for powers of: products; quotients; and powers.

A power indicates repeated multiplication.
What is the standard form of $(2^3)^2$? How did you find out?
$(2^3)^2$ is a **power of a power**.

The base of a power may be a product; for example, $(2 \times 3)^4$.
$(2 \times 3)^4$ is a **power of a product**.

Investigate

2

Copy and complete this table.
Choose your own power of a power to complete the 5th and 6th rows.
Choose your own power of a product to complete the 11th and 12th rows.

Power	As Repeated Multiplication	As a Product of Factors	As a Power	As a Product of Powers
$(2^4)^3$	$2^4 \times 2^4 \times 2^4$	(2)(2)(2)(2) × (2)(2)(2)(2) × (2)(2)(2)(2)	$2^?$	
$(3^2)^4$				
$[(-4)^3]^2$				
$[(-5)^3]^5$				
$(2 \times 5)^3$	$(2 \times 5) \times (2 \times 5) \times (2 \times 5)$	$2 \times 2 \times 2 \times 5 \times 5 \times 5$		$2^? \times 5^?$
$(3 \times 4)^2$				
$(4 \times 2)^5$				
$(5 \times 3)^4$				

Reflect & Share

What patterns do you see in the rows of the table?
Compare your patterns with those of another pair of classmates.
Use these patterns to record a rule for:

- writing the power of a power as a single power
- writing the power of a product as a product of two powers

How can you check your rules?

Connect

We can use the exponent laws from Lesson 2.4 to simplify powers written in other forms.

➤ Power of a power

We can raise a power to a power.
For example, 3^2 raised to the power 4 is written as $(3^2)^4$.
$(3^2)^4$ is a *power of a power*.
$(3^2)^4$ means $3^2 \times 3^2 \times 3^2 \times 3^2$.
So, $3^2 \times 3^2 \times 3^2 \times 3^2 = 3^{2+2+2+2}$ Using the exponent law for the product of powers
$= 3^8$
The exponent of 3^8 is the product of the exponents in $(3^2)^4$.
That is, $(3^2)^4 = 3^{2 \times 4}$
$= 3^8$
We can use this result to write an exponent law for the power of a power.

Exponent Law for a Power of a Power

To raise a power to a power, multiply the exponents.
$(a^m)^n = a^{mn}$
a is any integer, except 0.
m and n are any whole numbers.

mn means $m \times n$

➤ Power of a product

The base of a power may be a product; for example, $(3 \times 4)^5$.
$(3 \times 4)^5$ is a *power of a product*.
$(3 \times 4)^5$ means $(3 \times 4) \times (3 \times 4) \times (3 \times 4) \times (3 \times 4) \times (3 \times 4)$
So, $(3 \times 4) \times (3 \times 4) \times (3 \times 4) \times (3 \times 4) \times (3 \times 4)$
$= 3 \times 4 \times 3 \times 4 \times 3 \times 4 \times 3 \times 4 \times 3 \times 4$ Removing the brackets
$= (3 \times 3 \times 3 \times 3 \times 3) \times (4 \times 4 \times 4 \times 4 \times 4)$ Grouping equal factors
$= 3^5 \times 4^5$ Writing repeated multiplications as powers

We can use this result to write an exponent law for the power of a product.

Exponent Law for a Power of a Product

$(ab)^m = a^m b^m$

a and b are any integers, except 0.

m is any whole number.

➤ Power of a quotient

The base of a power may be a quotient; for example, $\left(\frac{5}{6}\right)^3$.

$\left(\frac{5}{6}\right)^3$ is a **power of a quotient**.

$\left(\frac{5}{6}\right)^3$ means $\left(\frac{5}{6}\right) \times \left(\frac{5}{6}\right) \times \left(\frac{5}{6}\right)$

So, $\left(\frac{5}{6}\right) \times \left(\frac{5}{6}\right) \times \left(\frac{5}{6}\right) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$

$= \frac{5 \times 5 \times 5}{6 \times 6 \times 6}$ Multiplying the fractions

$= \frac{5^3}{6^3}$ Writing repeated multiplications as powers

We can use this result to write an exponent law for the power of a quotient.

Exponent Law for a Power of a Quotient

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $\quad b \neq 0$

a and b are any integers, except 0.

n is any whole number.

We can use these exponent laws to simplify or evaluate an expression.

Example 1 Simplifying a Power of a Power

Write as a power.

a) $[(-7)^3]^2$ **b)** $-(2^4)^5$ **c)** $(6^2)^7$

A Solution

Use the exponent law for a power of a power.

a) $[(-7)^3]^2 = (-7)^{3 \times 2}$
$= (-7)^6$

b) $-(2^4)^5 = -(2^{4 \times 5})$
$= -2^{20}$

c) $(6^2)^7 = 6^{2 \times 7}$
$= 6^{14}$

Example 2 Evaluating Powers of Products and Quotients

Evaluate.

a) $[(-7) \times 5]^2$ b) $[24 \div (-6)]^4$ c) $-(3 \times 2)^2$ d) $\left(\frac{78}{13}\right)^3$

Solutions

Method 1	Method 2
a) Use the exponent law for a power of a product. $[(-7) \times 5]^2 = (-7)^2 \times 5^2 = 49 \times 25 = 1225$	Use the order of operations. a) $[(-7) \times 5]^2 = (-35)^2 = 1225$
b) Use the exponent law for a power of a quotient. Write the quotient in fraction form. $[24 \div (-6)]^4 = \left(\frac{24}{-6}\right)^4 = \frac{24^4}{(-6)^4} = \frac{331\ 776}{1296} = 256$	b) $[24 \div (-6)]^4 = (-4)^4 = 256$
c) Use the exponent law for a power of a product. $-(3 \times 2)^2 = -(3^2 \times 2^2) = -(9 \times 4) = -36$	c) $-(3 \times 2)^2 = -(6)^2 = -6^2 = -36$
d) Use the exponent law for a power of a quotient. $\left(\frac{78}{13}\right)^3 = \frac{78^3}{13^3} = \frac{474\ 552}{2197} = 216$	d) $\left(\frac{78}{13}\right)^3 = 6^3 = 216$

We can use the order of operations with the exponent laws when an expression involves the sum or difference of powers.

Example 3 Applying Exponent Laws and Order of Operations

Simplify, then evaluate each expression.

a) $(3^2 \times 3^3)^3 - (4^3 \times 4^2)^2$ **b)** $(6 \times 7)^2 + (3^8 \div 3^6)^3$ **c)** $[(-5)^3 + (-5)^4]^0$

A Solution

Use the exponent laws to simplify first, where appropriate.

a) In each set of brackets, the bases are the same, so use the exponent law for products.

$(3^2 \times 3^3)^3 - (4^3 \times 4^2)^2$	
$= (3^{2+3})^3 - (4^{3+2})^2$	Add the exponents in each set of brackets.
$= (3^5)^3 - (4^5)^2$	Use the power of a power law.
$= 3^{5 \times 3} - 4^{5 \times 2}$	Multiply the exponents.
$= 3^{15} - 4^{10}$	Use a calculator.
$= 14\ 348\ 907 - 1\ 048\ 576$	
$= 13\ 300\ 331$	

b) Multiply in the first set of brackets. Use the exponent law for the quotient of powers in the second set of brackets.

$(6 \times 7)^2 + (3^8 \div 3^6)^3$	
$= (42)^2 + (3^{8-6})^3$	
$= 42^2 + (3^2)^3$	Use the power of a power law.
$= 42^2 + 3^6$	Use a calculator.
$= 1764 + 729$	
$= 2493$	

c) The expression is a power with exponent 0, so its value is 1.

$[(-5)^3 + (-5)^4]^0 = 1$

Discuss the ideas

1. Why do you add the exponents to simplify $3^2 \times 3^4$, but multiply the exponents to simplify the expression $(3^2)^4$?

2. a) What is the difference between a quotient of powers and a power of a quotient?

b) What is the difference between a product of powers and a power of a product?

3. In *Example 3*, is it easier to key the original expressions in a calculator or use the exponent laws to simplify first? Justify your answer.

Practice

Check

4. Write each expression as a product of powers.

a) $(6 \times 4)^3$ b) $(2 \times 5)^4$ c) $[(-2) \times 3]^5$

d) $(25 \times 4)^2$ e) $(11 \times 3)^1$ f) $[(-3) \times (-2)]^3$

5. Write each expression as a quotient of powers.

a) $(8 \div 5)^3$ b) $(21 \div 5)^4$ c) $[(-12) \div (-7)]^5$

d) $\left(\frac{10}{3}\right)^3$ e) $\left(\frac{1}{3}\right)^2$ f) $\left(\frac{27}{100}\right)^4$

6. Write as a power.

a) $(3^2)^4$ b) $(6^3)^3$ c) $(5^3)^1$

d) $(7^0)^6$ e) $-(8^2)^2$ f) $[(-3)^4]^2$

7. Simplify $(2^4)^2$ and $(2^2)^4$. What do you notice? Explain the results.

8. Write each expression as a product or quotient of powers.

a) $[3 \times (-5)]^3$ b) $-(2 \times 4)^5$

c) $\left(\frac{2}{3}\right)^4$ d) $\left(\frac{-7}{-2}\right)^2$

e) $-[(-10) \times 3]^3$ f) $(16 \div 9)^2$

Apply

9. Why is the value of $(-5^2)^3$ negative?

10. Simplify each expression, then evaluate it. For each expression, state the strategy you used and why.

a) $(3 \times 2)^3$ b) $[(-2) \times 4]^2$ c) $\left(\frac{9}{-3}\right)^3$

d) $\left(\frac{8}{2}\right)^2$ e) $(12^8)^0$ f) $[(-4)^2]^2$

11. Why is the value of $[(-2)^3]^4$ positive but the value of $[(-2)^3]^5$ is negative?

12. Compare the values of $-(4^2)^3$, $(-4^2)^3$, and $[(-4)^2]^3$.
What do you notice? Explain the results.

13. **Assessment Focus** For each expression below:

i) Evaluate it in two different ways:
- do the operation in brackets first
- use the exponent laws

ii) Compare the results.
Which method do you prefer?
Was it always the same method each time? Explain.

a) $(4 \times 3)^3$ b) $[(-2) \times (-5)]^2$ c) $\left(\frac{6}{2}\right)^4$

d) $\left(\frac{14}{2}\right)^0$ e) $[(-5)^2]^2$ f) $(2^5)^3$

14. Simplify, then evaluate. Show your work.

a) $(3^2 \times 3^1)^2$ b) $(4^6 \div 4^4)^2$

c) $[(-2)^0 \times (-2)^3]^2$ d) $(10^6 \div 10^4)^3$

e) $(10^3)^2 \times (10^2)^3$ f) $(12^2)^4 \div (12^3)^2$

g) $(5^2)^6 \div (5^3)^4$ h) $[(-2)^2]^3 \times (-2)^3$

15. Find any errors in this student's work.
Copy the solution and correct the errors.

a) $(3^2 \times 2^2)^3 = (6^4)^3$
$= 6^{12}$
$= 2\,176\,782\,336$

b) $[(-3)^2]^3 = (-3)^5$
$= -243$

c) $\left(\frac{6^2}{6^1}\right)^2 = 6^4$
$= 1296$

d) $(2^6 \times 2^2 \div 2^4)^3 = (2^3)^3$
$= 2^9$
$= 512$

e) $(10^2 + 10^3)^2 = (10^5)^2$
$= 10^{10}$
$= 10\,000\,000\,000$

16. Simplify, then evaluate each expression.

a) $(4^2 \times 4^3)^2 - (5^4 \div 5^2)^2$

b) $(3^3 \div 3^2)^3 + (8^4 \times 8^3)^0$

c) $(2^3)^4 + (2^4 \div 2^3)^2$

d) $(6^2 \times 6^0)^3 + (2^6 \div 2^4)^3$

e) $(5^3 \times 5^3)^0 - (4^2)^2$

f) $(10^5 \div 10^2)^2 + (3^3 \div 3^1)^4$

17. Simplify, then evaluate each expression.

a) $[(-2)^3 \times (-2)^2]^2 - [(-3)^3 \div (-3)^2]^2$

b) $[(-2)^3 \div (-2)^2]^2 - [(-3)^3 \times (-3)^2]^2$

c) $[(-2)^3 \times (-2)^2]^2 + [(-3)^3 \div (-3)^2]^2$

d) $[(-2)^3 \div (-2)^2]^2 + [(-3)^3 \times (-3)^2]^2$

e) $[(-2)^3 \div (-2)^2]^2 - [(-3)^3 \div (-3)^2]^2$

f) $[(-2)^3 \times (-2)^2]^2 + [(-3)^3 \times (-3)^2]^2$

18. Use grid paper. For each expression below:

i) Draw a rectangle to represent the expression.

ii) Use the exponent laws to write the expression as a product of squares.

iii) Draw a rectangle to represent the new form of the expression.

iv) Compare the two rectangles for each expression.
How are the rectangles the same?
How are they different?
Use these rectangles to explain how the square of a product and the product of squares are related.

a) $(2 \times 3)^2$

b) $(2 \times 4)^2$

c) $(3 \times 4)^2$

d) $(1 \times 4)^2$

19. Simplify, then evaluate each expression.

a) $(2^3 \times 2^6)^2 - (3^7 \div 3^5)^4$

b) $(6 \times 8)^5 + (5^3)^2$

c) $[(-4)^3 \times (-4)^2]^2 + (4^3 \times 4^2)^2$

d) $[(-2)^4]^3 + [(-4)^3]^2 - [(-3)^2]^4$

e) $[(-3)^4]^2 \times [(-4)^0]^2 - [(-3)^3]^0$

f) $[(-5) \times (-4)]^3 + [(-6)^3]^2 - [(-3)^9 \div (-3)^8]^5$

Take It Further

20. a) Write 81:

i) as a power of 9

ii) as a power of a product

iii) as a power of 3

b) Write 64:

i) as a power of 8

ii) as a power of a product

iii) as a power of 2

c) Find other numbers for which you can follow steps similar to those in parts a and b.

21. a) List the powers of 2 from 2^0 to 2^{12} in standard form.

b) Use your list from part a to write each number in the expressions below as a power of 2. Evaluate each expression using the exponent laws and the list in part a.

i) 32×64

ii) $16 \times 8 \times 32$

iii) $1024 \div 128$

iv) $\frac{16 \times 256}{1024}$

v) $(8 \times 4)^3$

vi) $\left(\frac{256}{64}\right)^4$

Reflect

Design and create a poster that summarizes all the exponent laws you have learned. Provide an example of each law.

Study Guide

- A power represents repeated multiplication.
  $2^5 = 2 \times 2 \times 2 \times 2 \times 2$
  $= 32$
  $(-3)^4 = (-3)(-3)(-3)(-3)$
  $= 81$
  $-3^4 = -(3)(3)(3)(3)$
  $= -81$

- A power with an integer base, other than 0, and an exponent 0 is equal to 1.
  $2^0 = 1$
  $(-4)^0 = 1$
  $-4^0 = -1$

- To evaluate an expression, follow this order of operations:
  Evaluate inside brackets.
  Evaluate powers.
  Multiply and divide, in order, from left to right.
  Add and subtract, in order, from left to right.

Exponent Laws

m and n are whole numbers.
a and b are any integers, except 0.

- Product of Powers
  $a^m \times a^n = a^{m+n}$

- Quotient of Powers
  $a^m \div a^n = a^{m-n} \quad m \geq n$

- Power of a Power
  $(a^m)^n = a^{mn}$

- Power of a Product
  $(ab)^m = a^m b^m$

- Power of a Quotient
  $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad b \neq 0$

Review

2.1 **1.** Write as repeated multiplication, then in standard form.

a) 4^3 **b)** 7^2 **c)** $-(-2)^5$
d) -3^4 **e)** -1^8 **f)** $(-1)^8$

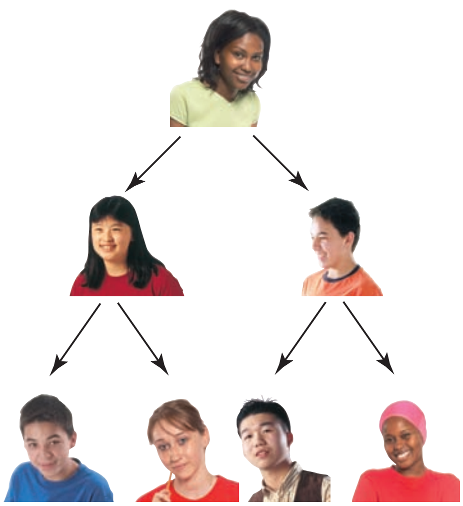
2. Use tiles and cubes to explain the difference between 2^2 and 2^3.

3. Write as a power, then in standard form.

a) $3 \times 3 \times 3 \times 3 \times 3 \times 3$
b) $(-8)(-8)(-8)$
c) $-(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)$
d) 12×12
e) $4 \times 4 \times 4 \times 4 \times 4$
f) $(-5)(-5)(-5)(-5)$

4. Explain the difference between 5^8 and 8^5.

5. A telephone tree is used to send messages. The person at the top calls 2 people. Each person calls 2 more people. Suppose it takes 1 min to call someone. A message is relayed until the bottom row of the tree has 256 people. How long does this take? How do you know?



6. a) Is the value of -4^2 different from the value of $(-4)^2$? What purpose do the brackets serve?
b) Is the value of -2^3 different from the value of $(-2)^3$? What purpose do the brackets serve?

7. a) Evaluate each expression.
i) -3^2 **ii)** $-(3)^2$ **iii)** $-(-3)^2$ **iv)** $(-3)^2$
b) For each expression in part a that includes brackets, explain what the brackets show.

2.2 **8.** Write as a power of 10.

a) one hundred million
b) $10 \times 10 \times 10 \times 10$
c) 1
d) 1 000 000 000
e) one thousand

9. Use powers of 10 to write each number.

a) 700 000 000
b) 345
c) 80 027

10. a) Copy and complete this table.

Power	Repeated Multiplication	Standard Form
3^5	3 × 3 × 3 × 3 × 3	243
3^4		
	3 × 3 × 3	
3^2		
		3

b) Describe the patterns in the table.
c) Extend the pattern to show why any number with an exponent of 0 is equal to 1.

11. a) The tallest tree in the world, Hyperion in California, is about 10^2 m tall. The highest mountain, Mount Everest, is about 10^4 m high. About how many times as high as the tree is the mountain?

b) Earth's diameter is about 10^7 m. The largest known star has a diameter of about 10^{12} m. About how many times as great as the diameter of Earth is the diameter of the largest known star?

12. Write each number in standard form.

a) $(4 \times 10^3) + (7 \times 10^2) + (2 \times 10^1) + (9 \times 10^0)$

b) $(3 \times 10^5) + (2 \times 10^2) + (8 \times 10^0)$

2.3 **13.** Evaluate.

a) $3^4 + 3^2$ **b)** $(-4)^2 + (-4)^3$

c) $10^3 - 10^2$ **d)** $(-5)^4 - (-5)^2$

14. Evaluate.

a) $2^3 + (5 - 2)^4$

b) $100 \div 2 + (4 + 1)^3$

c) $(6^2 + 7^2)^0 - (8^4 + 2^4)^0$

d) $3 \times 2^3 + 8 \div 4$

e) $(21 \div 7)^4 - 2^3$

f) $[(-4)^0 \times 10]^6 \div (15 - 10)^2$

15. Scientists grow bacteria.
This table shows how the number of bacteria doubles every hour.

Time	Elapsed Time After Noon (h)	Number of Bacteria
noon	0	1000×2^0
1:00 P.M.	1	1000×2^1
2:00 P.M.	2	1000×2^2
3:00 P.M.	3	1000×2^3

a) Evaluate the expressions in the table to find the number of bacteria at each time.

i) noon **ii)** 1:00 P.M.

iii) 2:00 P.M. **iv)** 3:00 P.M.

b) The pattern continues. Write an expression, then evaluate it, to find the number of bacteria at each time.

i) 4:00 P.M. **ii)** 6:00 P.M.

iii) 9:00 P.M. **iv)** midnight

16. Use a calculator to evaluate this expression:
$4^3 - (2 \times 3)^4 + 11$
Change the position of the brackets. Evaluate the new expression. How many different answers can you get by changing only the position of the brackets?

17. Identify, then correct, any errors in the student work below. Explain how you think the errors occurred.

$(-2)^2 \times 2^3 - 3^2 \div (-3) + (-4)^2$
$= (-2)^5 - 9 \div (-3) + 16$
$= -32 - 3 + 16$
$= -35 + 16$
$= -19$

2.4

18. Write each product as a power, then evaluate the power.

a) $5^3 \times 5^4$
b) $(-2)^3 \times (-2)^2$
c) $3^2 \times 3^3 \times 3^1$
d) $-10^4 \times 10^0$

19. There are about 10^{11} galaxies in the universe. Each galaxy contains about 10^{11} stars. About how many stars are in the universe?

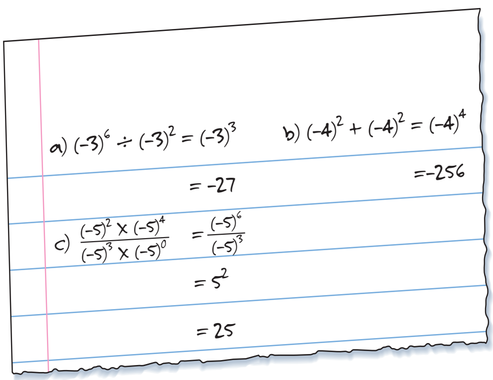
20. Write each quotient as a power, then evaluate the power.

a) $7^5 \div 7^3$
b) $(-10)^9 \div (-10)^3$
c) $\frac{8^4}{8^2}$
d) $-\frac{6^7}{6^4}$

21. a) Can you use the laws of exponents to simplify $6^3 \times 5^5$? Explain.

b) Can you use the laws of exponents to simplify $27^2 \div 9^2$? Explain.

22. Find and correct any errors in the student work below.
Explain how you think the errors occurred.



a) $(-3)^6 \div (-3)^2 = (-3)^3$
$= -27$

b) $(-4)^2 + (-4)^2 = (-4)^4$
$= -256$

c) $\frac{(-5)^2 \times (-5)^4}{(-5)^3 \times (-5)^0} = \frac{(-5)^6}{(-5)^3}$
$= 5^2$
$= 25$

2.5

23. Write each expression as a product or quotient of powers, then evaluate it.

a) $(3 \times 5)^3$
b) $(12 \div 3)^5$
c) $[(-4) \times 2]^4$
d) $(63 \times 44)^0$
e) $\left(\frac{3}{2}\right)^5$
f) $\left(\frac{15}{2}\right)^2$

24. Write each expression as a power.

a) $(3^2)^3$
b) $(4^0)^6$
c) $[(-2)^3]^3$
d) $(5^5)^2$

25. For each expression below:
Evaluate it in two different ways:

i) do the operation in brackets first
ii) use the exponent laws

In each case, which method is more efficient? Explain why.

a) $(5 \times 3)^3$
b) $(3 \times 3)^4$
c) $(8 \div 2)^5$
d) $\left(\frac{9}{3}\right)^2$
e) $(2^3)^4$
f) $(6^2)^0$

26. Write each expression as a power, then evaluate.

a) $6^4 \times 6^3$
b) $(-11)^7 \div (-11)^5$
c) $\frac{3^4 \times 3^5}{3^3}$
d) $\frac{5^5}{5^3 \times 5^2}$
e) $\frac{(-4)^3 \times (-4)^6}{(-4)^2 \times (-4)^4}$
f) $\frac{10^6 \times 10^0}{10^3 \times 10^2}$

27. Simplify, then evaluate each expression.

a) $2^3 \times 2^2 - 2^0 + 2^4 \div 2^3$
b) $\frac{(-2)^3 \times (-2)^2}{(-2)^3 - (-2)^2}$
c) $12^2 \times 12^4 \div (-2)^4 - 12^0$
d) $\frac{(-12)^2 \times (-12)^4}{(-2)^4 - 12^0}$

Practice Test

1. Write as a product or quotient of powers.
   a) $(3 \times 4)^3$ b) $[(-5) \times 2]^4$ c) $\left(\frac{1}{4}\right)^4$ d) $-\left(\frac{9}{3}\right)^3$

2. Simplify.
   a) $-(2^3)^3$ b) $(6^2)^0$ c) $[(-5)^2]^3$ d) $-[(-3)^2]^4$

3. Simplify each expression, then evaluate it.
   a) $[(-3) \times (-2)]^4$ b) $\left(\frac{1}{2}\right)^5$ c) $(6^0)^4$ d) $[(-3)^2]^3$

4. Is the value of a power with a negative base always negative?
   Or, is it always positive? Or, is it sometimes negative and sometimes positive?
   Illustrate your answer with some examples.

5. A baseball diamond is a square with side length about 27 m.
   Is the area of the baseball diamond greater or less than 10^3 m^2?
   How do you know?

6. Explain why the brackets are not necessary in this expression:
   $(-3^5 \times 10) - (9 \div 3)$
   Evaluate the expression, showing each step.

7. Identify the correct answer for $(2^3 + 4)^2 \times (-10)^3 \div (5 + 5)^2$.
   a) -240 b) -1440 c) 1440 d) $-28\ 825$
   Explain how each of the other incorrect answers could have been determined.

8. Evaluate only the expressions with a positive value. Explain how you know the sign of each expression before you evaluate it.
   a) $(-5)^3 \times (-5)^2 \div (-5)^1$ b) $[(-9)^6 - (-9)^3]^0$
   c) $\dfrac{(-1)^2 \times (-1)^4}{(-1)^3 \times (-1)^2}$ d) $(-4)^6 + (-4)^4 \times (-4)^0$

Unit Problem — How Thick Is a Pile of Paper?

You will need a sheet of paper and a ruler.

➤ Fold the paper in half to form 2 layers. Fold it in half again.
Keep folding until you cannot make the next fold.

➤ Create a table to show how many layers of paper there are after each fold.

Number of Folds	Number of Layers
0	1
1	2

Complete the table for the number of folds you were able to make.

➤ Look for a pattern in the numbers of layers.
How can you express the pattern using powers?
Draw another column on your table to show the *Number of Layers as Powers.*
Suppose you could make 25 folds.
Use patterns in the table to predict a power for the number of layers after 25 folds. Evaluate the power.

➤ Measure the thickness of 100 sheets (200 pages) in your math textbook.
Use this measure to calculate the thickness of 1 sheet of paper in millimetres.
How high would the layers be if you could make 25 folds? Give your answer in as many different units as you can.
What do you know that is approximately this height or length?

Your work should show:

- a completed table showing the numbers of layers
- the calculations of the thickness of 1 layer and the height after 25 folds
- an example of something with the same height

Reflect on Your Learning

What have you learned about powers and their exponent laws?
What ways can you think of to remember the laws and how to use them?