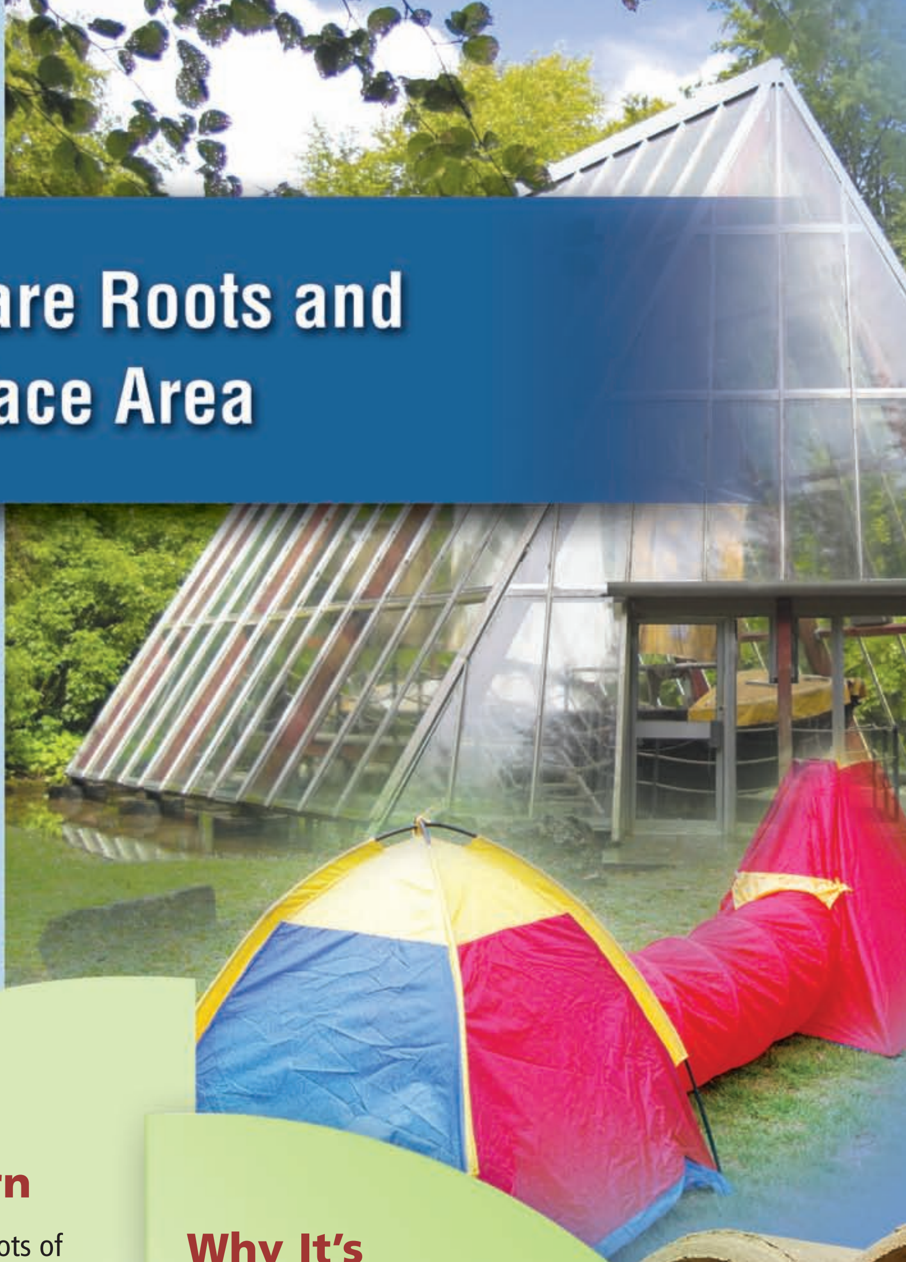
UNIT 1

Square Roots and Surface Area



Which geometric objects can you name?

How could you determine their surface areas?

What You'll Learn

- Determine the square roots of fractions and decimals that are perfect squares.
- Approximate the square roots of fractions and decimals that are non-perfect squares.
- Determine the surface areas of composite 3-D objects to solve problems.

Why It's Important

Real-world measures are often expressed as fractions or decimals.
We use the square roots of these measures when we work with formulas such as the Pythagorean Theorem.
An understanding of surface area allows us to solve practical problems such as calculating:
the amount of paper needed to wrap a gift;
the number of cans of paint needed to paint a room;
and the amount of siding needed to cover a building

Key Words

- perfect square
- non-perfect square
- composite object

1.1 Square Roots of Perfect Squares

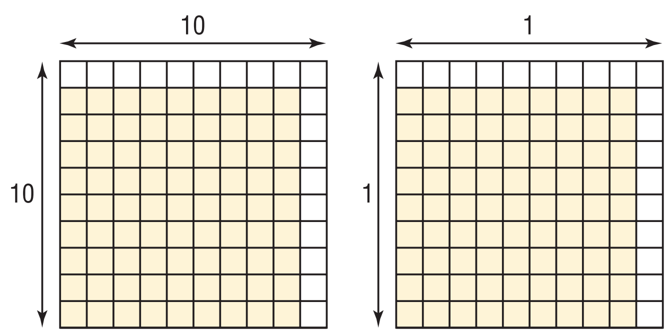
FOCUS

- Determine the square roots of decimals and fractions that are perfect squares.

A children's playground is a square with area 400 m^2.
What is the side length of the square?
How much fencing is needed to go around the playground?

Investigate

Each square below has been divided into 100 equal parts.
In each diagram, what is the area of one small square?



For the shaded square on the left:
- ➤ What is its area?
- ➤ Write this area as a product.
- ➤ How can you use a square root to relate the side length and area?

For the shaded square on the right:
- ➤ What is its area?
- ➤ Write this area as a product of fractions.
- ➤ How can you use a square root to relate the side length and area?

For the area of each square in the table:

- Write the area as a product.
- Write the side length as a square root.

Area as a Product	Side Length as a Square Root
49 =	
$\frac{49}{100}$ =	
64 =	
$\frac{64}{100}$ =	
121 =	
$\frac{121}{100}$ =	
144 =	
$\frac{144}{100}$ =	

Reflect & Share

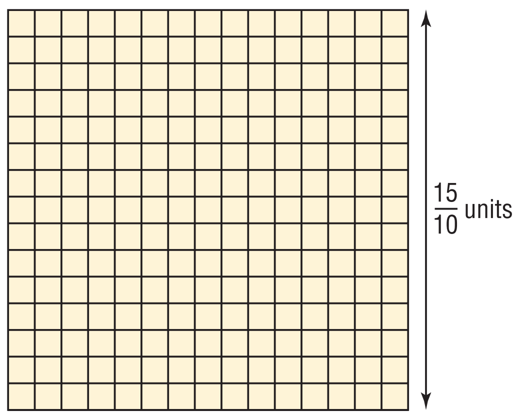
Compare your results with those of your classmates.
How can you use the square roots of whole numbers to determine the square roots of fractions?
Suppose each fraction in the table is written as a decimal.
How can you use the square roots of whole numbers to determine the square roots of decimals?

Connect

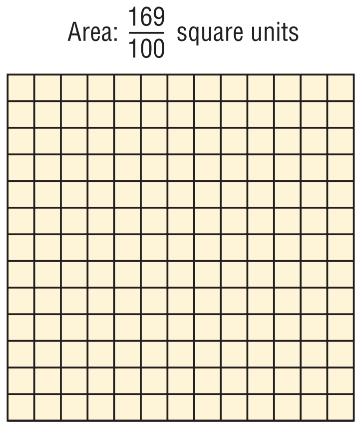
To determine the area of a square, we multiply the side length by itself.
That is, we *square* the side length.

$$\text{Area} = \left(\frac{15}{10}\right)^2$$
$$= \frac{15}{10} \times \frac{15}{10}$$
$$= \frac{225}{100}$$

The area is $\frac{225}{100}$ square units.



To determine the side length of a square, we calculate the square root of its area.



$$\text{Side length} = \sqrt{\frac{169}{100}}$$
$$= \sqrt{\frac{13}{10} \times \frac{13}{10}}$$
$$= \frac{13}{10}$$

The side length is $\frac{13}{10}$ units.

Squaring and taking the square root are opposite, or inverse, operations.
The side length of a square is the square root of its area.

That is, $\sqrt{\frac{225}{100}} = \frac{15}{10}$ and $\sqrt{\frac{169}{100}} = \frac{13}{10}$

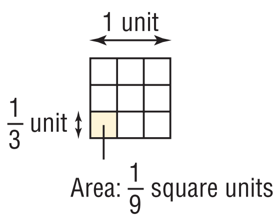
We can rewrite these equations using decimals:

$$\sqrt{2.25} = 1.5 \text{ and } \sqrt{1.69} = 1.3$$

1.5 and 1.3 are terminating decimals.

The square roots of some fractions are repeating decimals.
To determine the side length of the shaded square, take the square root of $\frac{1}{9}$:

$$\sqrt{\frac{1}{9}} = \sqrt{\frac{1}{3} \times \frac{1}{3}}$$
$$= \frac{1}{3}$$
$$= 0.333\ 333\ 333\ \ldots$$
$$= 0.\overline{3}$$



When the area of a square is $\frac{1}{9}$ square units, its side length is $\frac{1}{3}$, or $0.\overline{3}$ of a unit.

A fraction in simplest form is a **perfect square** if it can be written as a product of two equal fractions.

When a decimal can be written as a fraction that is a perfect square, then the decimal is also a perfect square. The square root is a terminating or repeating decimal.

Example 1 Determining a Perfect Square Given its Square Root

Calculate the number whose square root is:

a) $\frac{3}{8}$ b) 1.8

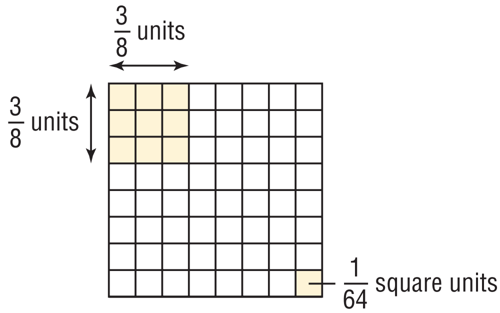
A Solution

a) Visualize $\frac{3}{8}$ as the side length of a square.

The area of the square is: $\left(\frac{3}{8}\right)^2 = \frac{3}{8} \times \frac{3}{8}$

$= \frac{9}{64}$

So, $\frac{3}{8}$ is a square root of $\frac{9}{64}$.

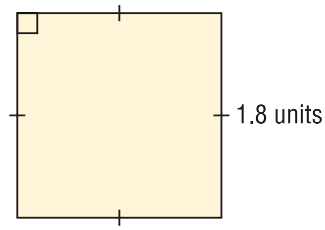


b) Visualize 1.8 as the side length of a square.

The area of the square is: $1.8^2 = 1.8 \times 1.8$

$= 3.24$

So, 1.8 is a square root of 3.24.



Example 2 Identifying Fractions that Are Perfect Squares

Is each fraction a perfect square? Explain your reasoning.

a) $\frac{8}{18}$ b) $\frac{16}{5}$ c) $\frac{2}{9}$

A Solution

a) $\frac{8}{18}$

Simplify the fraction first. Divide the numerator and denominator by 2.

$\frac{8}{18} = \frac{4}{9}$

Since $4 = 2 \times 2$ and $9 = 3 \times 3$, we can write:

$\frac{4}{9} = \frac{2}{3} \times \frac{2}{3}$

Since $\frac{4}{9}$ can be written as a product of two equal fractions, it is a perfect square.

So, $\frac{8}{18}$ is also a perfect square.

b) $\frac{16}{5}$

The fraction is in simplest form.

So, look for a fraction that when multiplied by itself gives $\frac{16}{5}$.

The numerator can be written as $16 = 4 \times 4$, but the denominator cannot be written as a product of equal factors.

So, $\frac{16}{5}$ is not a perfect square.

c) $\frac{2}{9}$

The fraction is in simplest form.

So, look for a fraction that when multiplied by itself gives $\frac{2}{9}$.

The denominator can be written as $9 = 3 \times 3$, but the numerator cannot be written as a product of equal factors.

So, $\frac{2}{9}$ is not a perfect square.

Example 3 Identifying Decimals that Are Perfect Squares

Is each decimal a perfect square? Explain your reasoning.

a) 6.25 **b)** 0.627

Solutions

Method 1

a) Write 6.25 as a fraction.

$6.25 = \frac{625}{100}$

Simplify the fraction. Divide the numerator and denominator by 25.

$6.25 = \frac{25}{4}$

$\frac{25}{4}$ can be written as $\frac{5}{2} \times \frac{5}{2}$.

So, $\frac{25}{4}$, or 6.25 is a perfect square.

b) Write 0.627 as a fraction.

$0.627 = \frac{627}{1000}$

This fraction is in simplest form.

Neither 627 nor 1000 can be written as a product of equal factors, so 0.627 is not a perfect square.

Method 2

Use a calculator.

Use the square root function.

a) $\sqrt{6.25} = 2.5$

The square root is a terminating decimal, so 6.25 is a perfect square.

b) $\sqrt{0.627} \doteq 0.791\ 833\ 316$

The square root appears to be a decimal that neither terminates nor repeats, so 0.627 is not a perfect square. To be sure, write the decimal as a fraction, then determine if the fraction is a perfect square, as shown in *Method 1*.

Discuss the ideas

1. How can you tell if a decimal is a perfect square?

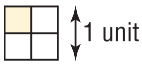
2. How can you tell if a fraction is a perfect square?

Practice

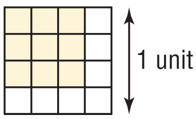
Check

3. Use each diagram to determine the value of the square root.

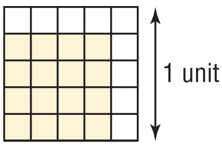
a) $\sqrt{0.25}$



b) $\sqrt{\frac{9}{16}}$



c) $\sqrt{\frac{16}{25}}$



4. a) List all the whole numbers from 1 to 100 that are perfect squares.

b) Write a square root of each number you listed in part a.

5. Use your answers to question 4. Determine the value of each square root.

a) $\sqrt{0.36}$ **b)** $\sqrt{0.49}$

c) $\sqrt{0.81}$ **d)** $\sqrt{0.16}$

e) $\sqrt{\frac{1}{36}}$ **f)** $\sqrt{\frac{25}{9}}$

g) $\sqrt{\frac{64}{100}}$ **h)** $\sqrt{\frac{36}{16}}$

6. a) List all the whole numbers from 101 to 400 that are perfect squares.

b) Write a square root of each number you listed in part a.

7. Use your answers to questions 4 and 6. Determine the value of each square root.

a) $\sqrt{\frac{169}{16}}$ **b)** $\sqrt{\frac{400}{196}}$

c) $\sqrt{\frac{256}{361}}$ **d)** $\sqrt{\frac{225}{289}}$

e) $\sqrt{144}$ **f)** $\sqrt{0.0225}$

g) $\sqrt{0.0121}$ **h)** $\sqrt{3.24}$

i) $\sqrt{0.0324}$ **j)** $\sqrt{0.0169}$

Apply

8. Which decimals and fractions are perfect squares? Explain your reasoning.

a) 0.12 **b)** 0.81 **c)** 0.25

d) 1.69 **e)** $\frac{9}{12}$ **f)** $\frac{36}{81}$

g) $\frac{81}{49}$ **h)** $\frac{75}{27}$ **i)** 0.081

j) $\frac{25}{10}$ **k)** 2.5 **l)** $\frac{8}{50}$

9. Calculate the number whose square root is:

a) 0.3 **b)** 0.12

c) 1.9 **d)** 3.1

e) $\frac{2}{3}$ **f)** $\frac{5}{6}$

g) $\frac{1}{7}$ **h)** $\frac{2}{5}$

10. Determine the value of each square root.

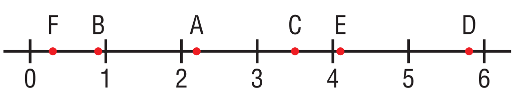
a) $\sqrt{12.25}$ **b)** $\sqrt{30.25}$

c) $\sqrt{20.25}$ **d)** $\sqrt{56.25}$

11. a) Write each decimal as a fraction. Which fractions are perfect squares?
  i) 36.0 ii) 3.6 iii) 0.36
  iv) 0.036 v) 0.0036 vi) 0.000 36
b) To check your answers to part a, use a calculator to determine a square root of each decimal.
c) What patterns do you see in your answers to parts a and b?
d) When can you use the square roots of perfect squares to determine the square roots of decimals?

12. a) Use the fact that $\sqrt{9} = 3$ to write the value of each square root.
  i) $\sqrt{90\ 000}$ ii) $\sqrt{900}$
  iii) $\sqrt{0.09}$ iv) $\sqrt{0.0009}$
b) Use the fact that $\sqrt{25} = 5$ to write the value of each square root.
  i) $\sqrt{0.0025}$ ii) $\sqrt{0.25}$
  iii) $\sqrt{2500}$ iv) $\sqrt{250\ 000}$
c) Use the patterns in parts a and b. Choose a whole number whose square root you know. Use that number and its square root to write 3 decimals and their square roots. How do you know the square roots are correct?

13. **Assessment Focus**
a) Which letter on the number line below corresponds to each square root? Justify your answers.
  i) $\sqrt{12.25}$ ii) $\sqrt{\frac{121}{25}}$ iii) $\sqrt{16.81}$
  iv) $\sqrt{\frac{81}{100}}$ v) $\sqrt{0.09}$ vi) $\sqrt{\frac{841}{25}}$



b) Sketch the number line in part a. Write 3 different decimals, then use the letters G, H, and J to represent their square roots. Place each letter on the number line. Justify its placement.

14. A square has area 5.76 cm^2.
a) What is the side length of the square?
b) What is the perimeter of the square? How do you know?

15. A square piece of land has an area not less than 6.25 km^2 and not greater than 10.24 km^2.
a) What is the least possible side length of the square?
b) What is the greatest possible side length of the square?
c) A surveyor determined that the side length is 2.8 km. What is the area of the square?

16. A student said that $\sqrt{0.04} = 0.02$.
Is the student correct?
If your answer is yes, how could you check that the square root is correct?
If your answer is no, what is the correct square root? Justify your answer.

17. Look at the perfect squares you wrote for questions 4 and 6.
The numbers 36, 64, and 100 are related:
$36 + 64 = 100$, or $6^2 + 8^2 = 10^2$
These numbers form a *Pythagorean triple.*

a) Why do you think this name is appropriate?

b) How many other Pythagorean triples can you find? List each triple.

Take It Further

18. Are there any perfect squares between 0.64 and 0.81? Justify your answer.

19. A student has a rectangular piece of paper 7.2 cm by 1.8 cm. She cuts the paper into parts that can be rearranged and taped to form a square.

a) What is the side length of the square?

b) What are the fewest cuts the student could have made? Justify your answer.

Reflect

Explain the term *perfect square*. List some whole numbers, fractions, and decimals that are perfect squares. Determine a square root of each number.

Math Link

History

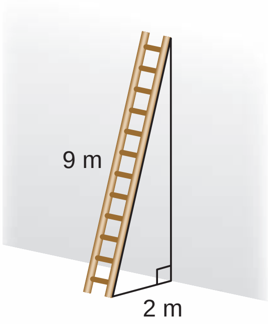
The Pythagorean Theorem is named for the Greek philosopher, Pythagoras, because he was the first person to record a proof for the theorem, around 540 BCE. However, clay tablets from around 1700 BCE show that the Babylonians knew how to calculate the length of the diagonal of a square. And, around 2000 BCE, it is believed that the Egyptians may have used a knotted rope that formed a triangle with side lengths 3, 4, and 5 to help design the pyramids.

1.2 Square Roots of Non-Perfect Squares

FOCUS

- Approximate the square roots of decimals and fractions that are non-perfect squares.

A ladder is leaning against a wall.
For safety, the distance from the base of a ladder to the wall must be about $\frac{1}{4}$ of the height up the wall.
How could you check if the ladder is safe?



Investigate

2

A ladder is 6.1 m long.
The distance from the base of the ladder to the wall is 1.5 m.
Estimate how far up the wall the ladder will reach.

Reflect & Share

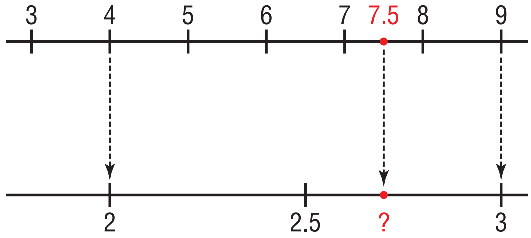
Compare your strategy for estimating the height with that of another pair of classmates. Did you use a scale drawing? Did you calculate?
Which method gives the closer estimate?

Connect

Many fractions and decimals are not perfect squares.
That is, they cannot be written as a product of two equal fractions.
A fraction or decimal that is not a perfect square is called a **non-perfect square**.

Here are two strategies for estimating a square root of a decimal that is a non-perfect square.

➤ Using benchmarks,
To estimate $\sqrt{7.5}$, visualize a number line and the closest perfect square on each side of 7.5.



$\sqrt{4} = 2$ and $\sqrt{9} = 3$
7.5 is closer to 9 than to 4, so
$\sqrt{7.5}$ is closer to 3 than to 2.
From the diagram, an approximate value for $\sqrt{7.5}$ is 2.7.
We write $\sqrt{7.5} \doteq 2.7$

➤ Using a calculator
$\sqrt{7.5} \doteq 2.738\ 612\ 788$
This decimal does not appear to terminate or repeat.
There may be many more numbers after the decimal point that cannot be displayed on the calculator.
To check, determine: $2.738\ 612\ 788^2 = 7.500\ 000\ 003$
Since this number is not equal to 7.5, the square root is an approximation.

Example 1 illustrates 4 different strategies for determining the square root of a fraction that is a non-perfect square.

Example 1 Estimating a Square Root of a Fraction

Determine an approximate value of each square root.

a) $\sqrt{\frac{8}{5}}$ **b)** $\sqrt{\frac{3}{10}}$ **c)** $\sqrt{\frac{3}{7}}$ **d)** $\sqrt{\frac{19}{6}}$

A Solution

a) Use benchmarks. Think about the perfect squares closest to the numerator and denominator. In the fraction $\frac{8}{5}$, 8 is close to the perfect square 9, and 5 is close to the perfect square 4.

So, $\sqrt{\frac{8}{5}} \doteq \sqrt{\frac{9}{4}}$

$\sqrt{\frac{9}{4}} = \frac{3}{2}$

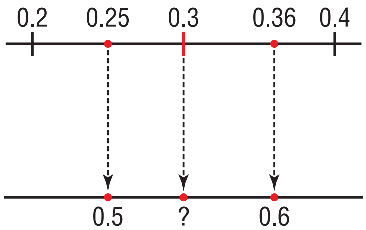
So, $\sqrt{\frac{8}{5}} \doteq \frac{3}{2}$

b) Write the fraction as a decimal, then think about benchmarks.

Write $\frac{3}{10}$ as a decimal: 0.3

Think of the closest perfect squares on either side of 0.3.

$\sqrt{0.25} = 0.5$ and $\sqrt{0.36} = 0.6$



0.3 is approximately halfway between 0.25 and 0.36, so choose 0.55 as a possible estimate for a square root.

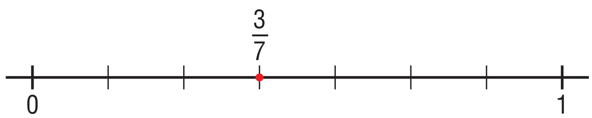
To check, evaluate:

$0.55^2 = 0.3025$

0.3025 is close to 0.3, so 0.55 is a reasonable estimate.

So, $\sqrt{\frac{3}{10}} \doteq 0.55$

c) Choose a fraction close to $\frac{3}{7}$ that is easier to work with.



$\frac{3}{7}$ is a little less than $\frac{1}{2}$.

$\frac{1}{2} = 0.5$

$\sqrt{0.5} \doteq \sqrt{0.49}$

And, $\sqrt{0.49} = 0.7$

So, $\sqrt{\frac{3}{7}} \doteq 0.7$

d) Use the square root function on a calculator.

$\sqrt{\frac{19}{6}} \doteq 1.779\ 513\ 042$

To the nearest hundredth, $\sqrt{\frac{19}{6}} \doteq 1.78$

Example 2 Finding a Number with a Square Root between Two Given Numbers

Identify a decimal that has a square root between 10 and 11. Check the answer.

Solutions

Method 1	Method 2
The number with a square root of 10 is: $10^2 = 100$ The number with a square root of 11 is: $11^2 = 121$ So, any number between 100 and 121 has a square root between 10 and 11. A decimal between 100 and 121 is 105.6. So, $\sqrt{105.6}$ is between 10 and 11. Use a calculator to check. $\sqrt{105.6} \doteq 10.276\ 186\ 06$ So, the decimal 105.6 is one correct answer.	One decimal between 10 and 11 is 10.4. To determine the number whose square root is 10.4, evaluate: $10.4^2 = 108.16$ So, $\sqrt{108.16}$ is between 10 and 11. Use a calculator to check. $\sqrt{108.16} = 10.4$ So, the decimal 108.16 is one correct answer.

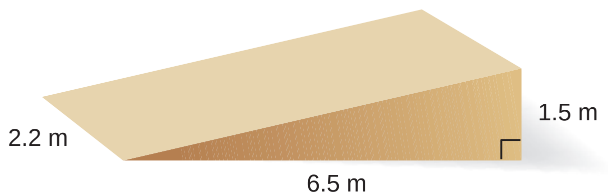
Example 3 Applying the Pythagorean Theorem

The sloping face of this ramp is to be covered in carpet.

a) Estimate the length of the ramp to the nearest tenth of a metre.

b) Use a calculator to check the answer.

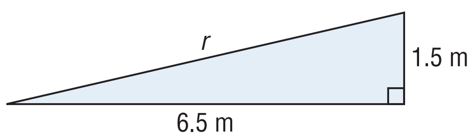
c) Calculate the area of carpet needed.



A Solution

a) The ramp is a right triangular prism with a base that is a right triangle.
The base of the prism is its side view.
To calculate the length of the ramp, r, use the Pythagorean Theorem.

$$r^2 = 6.5^2 + 1.5^2$$
$$= 42.25 + 2.25$$
$$= 44.5$$
$$r = \sqrt{44.5}$$



44.5 is between the perfect squares 36 and 49, and closer to 49.
So, $\sqrt{44.5}$ is between 6 and 7, and closer to 7.
Estimate $\sqrt{44.5}$ as 6.7.
To check, evaluate: $6.7^2 = 44.89$
This is very close to 44.5, so $r \doteq 6.7$
The ramp is about 6.7 m long.

Since the dimensions of the ramp were given to the nearest tenth, the answer is also written in this form.

b) Use a calculator to check: $\sqrt{44.5} \doteq 6.670\,832\,032$
This number is 6.7 to the nearest tenth, so the answer is correct.

c) The sloping face of the ramp is a rectangle with dimensions 6.7 m by 2.2 m.
The area of the rectangle is about: $6.7 \times 2.2 = 14.74$
Round the answer up to the nearest square metre to ensure there is enough carpet.
So, about 15 m^2 of carpet are needed.

Discuss the ideas

1. Explain the term *non-perfect square.*

2. Name 3 perfect squares and 3 non-perfect squares between the numbers 0 and 10. Justify your answers.

3. Why might the square root shown on a calculator be an approximation?

Practice

Check

4. For each square root, name the two closest perfect squares and their square roots.

a) $\sqrt{3.5}$ **b)** $\sqrt{13.5}$
c) $\sqrt{53.5}$ **d)** $\sqrt{73.5}$
e) $\sqrt{93.5}$ **f)** $\sqrt{113.5}$

5. For each square root, name the two closest perfect squares and their square roots.

a) $\sqrt{\frac{5}{10}}$ **b)** $\sqrt{\frac{55}{10}}$
c) $\sqrt{\frac{95}{10}}$ **d)** $\sqrt{\frac{595}{10}}$
e) $\sqrt{\frac{795}{10}}$ **f)** $\sqrt{\frac{1095}{10}}$

Apply

6. Use benchmarks to estimate a fraction for each square root.
State the benchmarks you used.

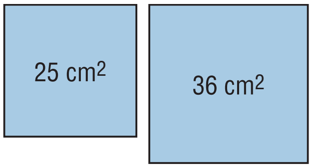
a) $\sqrt{\frac{8}{10}}$ **b)** $\sqrt{\frac{17}{5}}$
c) $\sqrt{\frac{7}{13}}$ **d)** $\sqrt{\frac{29}{6}}$

7. Use benchmarks to approximate each square root to the nearest tenth.
State the benchmarks you used.

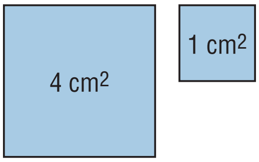
a) $\sqrt{4.5}$ **b)** $\sqrt{14.5}$
c) $\sqrt{84.5}$ **d)** $\sqrt{145.5}$
e) $\sqrt{284.5}$ **f)** $\sqrt{304.5}$

8. Use each pair of squares to approximate each square root.
Explain your strategy.

a) $\sqrt{29.5}$



b) $\sqrt{\frac{5}{2}}$



9. Which of the following square roots are correct to the nearest tenth?
How do you know? Correct the square roots that are incorrect.

a) $\sqrt{4.4} \doteq 2.2$ b) $\sqrt{0.6} \doteq 0.3$
c) $\sqrt{6.6} \doteq 2.6$ d) $\sqrt{0.4} \doteq 0.2$

10. Find 2 decimals that have square roots between each pair of numbers.
Justify your answers.

a) 3 and 4
b) 7 and 8
c) 12 and 13
d) 1.5 and 2.5
e) 4.5 and 5.5

11. Use any strategy you wish to estimate the value of each square root. Explain why you used the strategy you did.

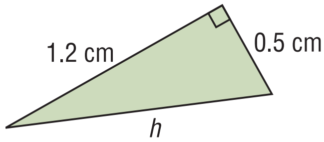
a) $\sqrt{4.5}$ b) $\sqrt{\frac{17}{2}}$ c) $\sqrt{0.15}$ d) $\sqrt{\frac{10}{41}}$
e) $\sqrt{0.7}$ f) $\sqrt{\frac{8}{45}}$ g) $\sqrt{0.05}$ h) $\sqrt{\frac{90}{19}}$

12. Approximate each square root to the nearest tenth. Explain your strategy.

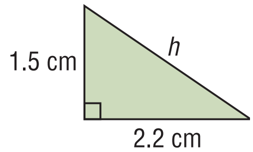
a) $\sqrt{\frac{3}{8}}$ b) $\sqrt{\frac{5}{12}}$ c) $\sqrt{\frac{13}{4}}$ d) $\sqrt{\frac{25}{3}}$

13. In each triangle, determine the unknown length.

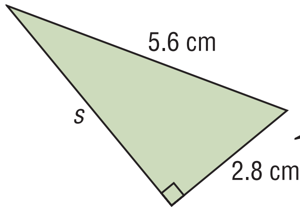
a)



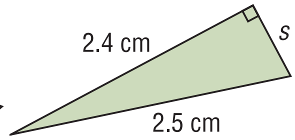
b)



c)



d)

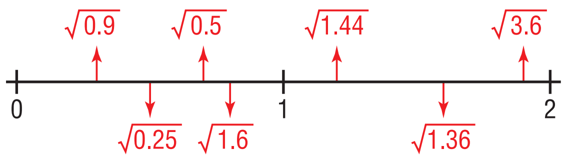


14. **Assessment Focus** How many decimals and fractions can you find with square roots between 0.5 and 0.6?
List the decimals and fractions.
Justify your answers. Show your work.

15. Sketch a number line from 0 to 10.
Place each square root on the number line to show its approximate value.

a) $\sqrt{0.1}$ b) $\sqrt{56.3}$
c) $\sqrt{0.6}$ d) $\sqrt{0.03}$

16. a) Which square roots are correctly placed on the number line below?
How do you know?



b) Sketch a number line from 0 to 2. On the number line, correctly place the square roots that were incorrectly placed in part a.

17. Use a calculator to determine each square root. Which square roots are approximate? How do you know?

a) $\sqrt{52.9}$ **b)** $\sqrt{5.29}$ **c)** $\sqrt{2.25}$ **d)** $\sqrt{22.5}$

18. Look at the numbers and their square roots you have determined in this lesson.
How would you describe the numbers whose square roots are:

a) less than the number?
b) equal to the number?
c) greater than the number?

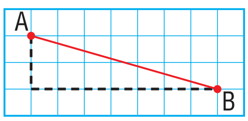
Justify your answer.

19. Determine a decimal or a fraction whose square root is between each pair of numbers.

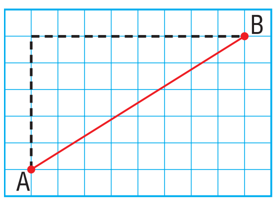
a) 0 and 1 **b)** 1.5 and 2
c) $\frac{1}{2}$ and $\frac{3}{4}$ **d)** $3\frac{3}{4}$ and 4

20. On each grid below, the side length of each square represents 0.25 km.
Determine the length of AB to the nearest hundredth of a kilometre.

a)



b)



21. a) Use a calculator to approximate each square root.

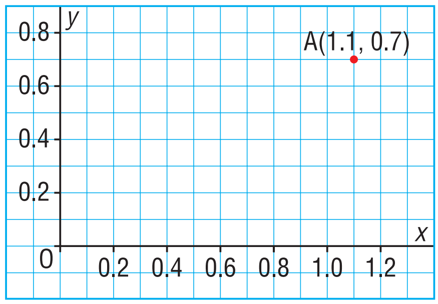
i) $\sqrt{0.005}$ **ii)** $\sqrt{0.5}$ **iii)** $\sqrt{50}$
iv) $\sqrt{5000}$ **v)** $\sqrt{500\ 000}$

b) What patterns do you see in the square roots in part a? Use the patterns to write the previous two square roots less than $\sqrt{0.005}$ and the next two square roots greater than $\sqrt{500\ 000}$.

Take It Further

22. Are there any square numbers between 0.6 and 0.61? How do you know?

23. The grid below shows point A(1.1, 0.7) that is one vertex of a square with area 0.25 square units. What are the coordinates of the other three vertices of the square? Justify your answer.



24. The side length of a square photograph is 5.5 cm. An enlargement of the photograph is a square with an area that is twice the area of the smaller photograph.

a) Estimate the side length of the larger photograph. Justify your answer.
b) Why is the side length of the larger photograph not twice the side length of the smaller photograph?

Reflect

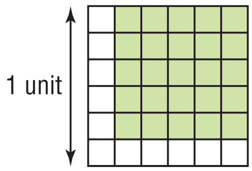
Explain why the square root of a non-perfect square displayed on a calculator is only an approximation. Include examples in your explanation.

Mid-Unit Review

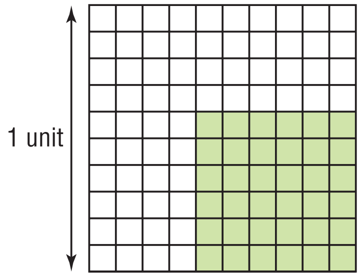
1.1

1. Explain how you can use each diagram to determine the square root.

a) $\sqrt{\frac{25}{36}}$



b) $\sqrt{0.36}$



2. Calculate the number whose square root is:

a) 1.4 b) $\frac{3}{8}$ c) $\frac{7}{4}$ d) 0.5

3. Determine the value of each square root.

a) $\sqrt{0.04}$ b) $\sqrt{\frac{1}{16}}$ c) $\sqrt{1.96}$ d) $\sqrt{\frac{4}{81}}$

e) $\sqrt{1.69}$ f) $\sqrt{\frac{121}{49}}$ g) $\sqrt{0.09}$ h) $\sqrt{\frac{289}{100}}$

4. Determine the value of each square root.

a) $\sqrt{3.24}$ b) $\sqrt{90.25}$ c) $\sqrt{2.56}$

5. A square has area 148.84 cm^2.

a) What is the side length of the square?

b) What is the perimeter of the square?

6. A student said that $\sqrt{0.16} = 0.04$.
Is the student correct?
If your answer is yes, how could you check that the square root is correct?
If your answer is no, explain how to get the correct square root.

7. Which decimals and fractions are perfect squares? Explain your reasoning.

a) $\frac{9}{64}$ b) 3.6 c) $\frac{6}{9}$ d) 5.76

1.2

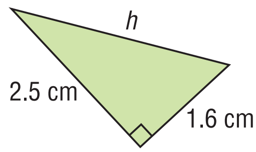
8. Use benchmarks to estimate each square root.

a) $\sqrt{5.6}$ b) $\sqrt{\frac{9}{10}}$ c) $\sqrt{42.8}$

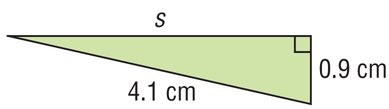
d) $\sqrt{\frac{356}{10}}$ e) $\sqrt{0.056}$ f) $\sqrt{\frac{9}{100}}$

9. In each triangle, determine the unknown length.

a)



b)



10. Which of the following square roots are correct to the nearest tenth?
How do you know? Correct the square roots that are incorrect.

a) $\sqrt{0.09} \doteq 0.3$ b) $\sqrt{1.7} \doteq 0.4$

c) $\sqrt{8.5} \doteq 2.9$ d) $\sqrt{27.5} \doteq 5.2$

11. Find 2 decimals that have square roots between each pair of numbers.
Justify your answers.

a) 4 and 8 b) 0.7 and 0.9

c) 1.25 and 1.35 d) 0.25 and 0.35

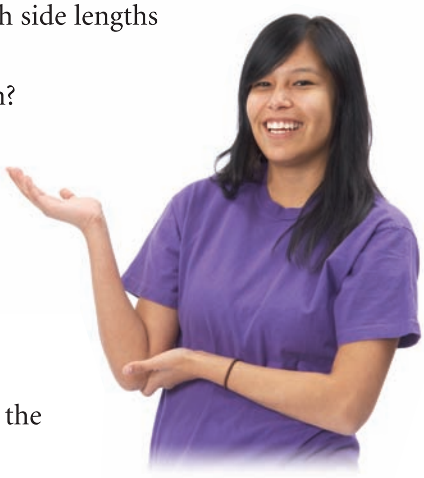
e) 4.5 and 5.5 f) 0.05 and 0.1

Start Where You Are

How Can I Begin?

Suppose I have to solve this problem:

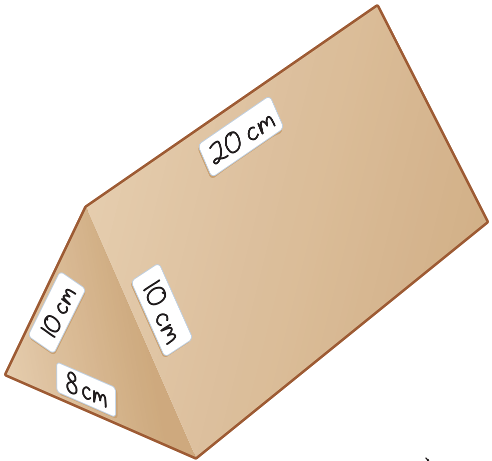
A right triangular prism is 20 cm long.
Each base is an isosceles triangle with side lengths 10 cm, 10 cm, and 8 cm.
What is the surface area of the prism?



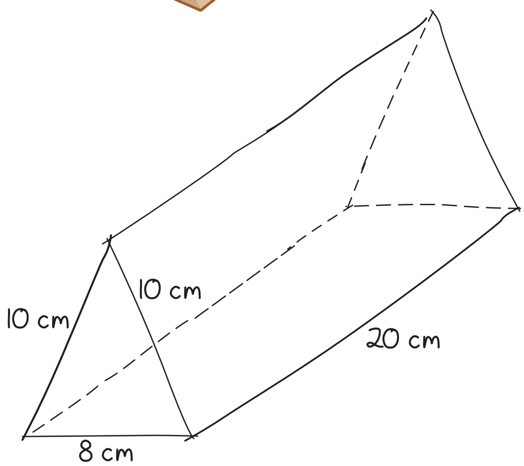
➤ What is my first step?

- I could use a model.
- I could sketch a diagram.
- I could visualize the prism in my mind.

If I use a model, I can place stickers on the prism to label its dimensions.



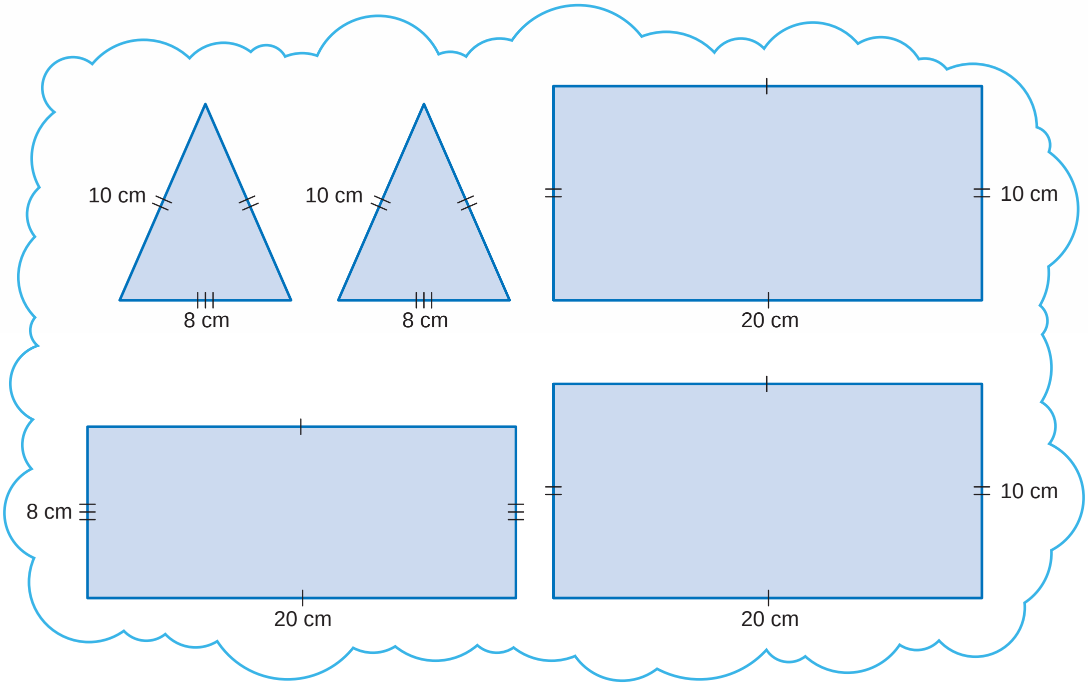
The model should have the shape of a triangular prism, but the dimensions of the prism do not have to match the given dimensions.



If I sketch a diagram, I label it with the given dimensions.

The diagram does not have to be drawn to scale.

If I visualize the prism, I picture its faces.



➤ What do I already know?

- a strategy to find the area of a rectangle
- a strategy to find the height of an isosceles triangle when the lengths of its sides are known
- a strategy to find the area of an isosceles triangle

Use strategies *you* know to find the surface area of the right triangular prism.

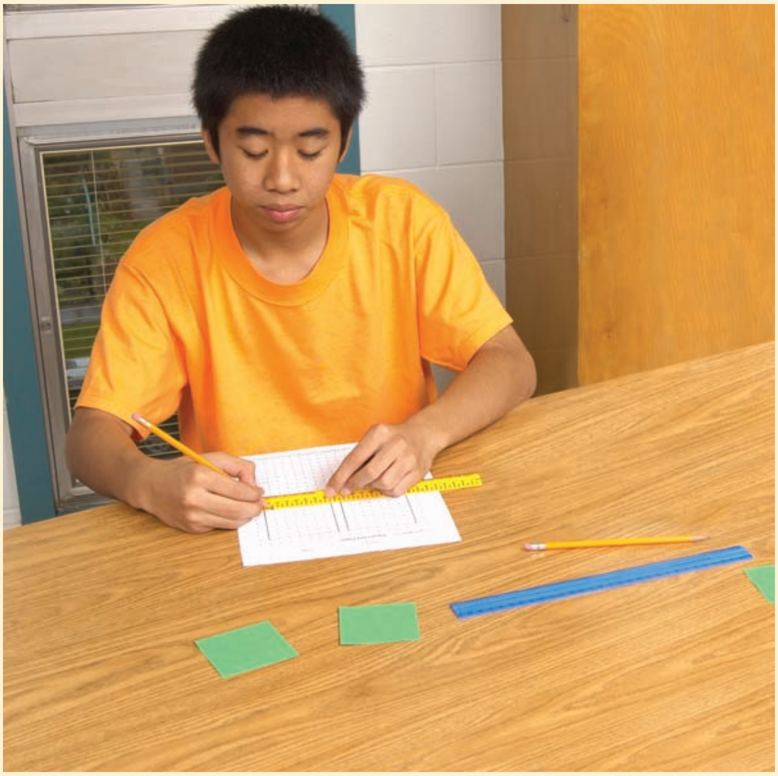
Check

1. A right triangular prism is 35 cm high. Its bases are equilateral triangles, with side lengths 12 cm. What is the surface area of the prism?

2. A right cylinder is 35 cm long. Its diameter is 12 cm. What is the surface area of the cylinder?

GAME

Making a Larger Square from Two Smaller Squares



You will need

- 2 congruent square pieces of paper
- scissors
- square dot paper

Number of Players

- 2

Goal of the Game

- To cut two congruent squares and rearrange the pieces to form one larger square

Before you cut the squares, sketch them on square dot paper.
Draw possible cuts you could make.
Imagine joining all the pieces with no overlap.
Do the pieces form a larger square?

Check your prediction by cutting the squares and arranging the pieces to form a larger square. If your prediction did not work, try again using another two congruent squares.

Share your method with another pair of students.
Are there other possible ways of forming the larger square?
How could you do this by making the fewest cuts possible?

Suppose the area of each congruent square is 1 square unit.
- What is the area of the larger square?
- What is the side length of the larger square, to the nearest tenth?

Suppose the area of each congruent square is 2 square units.
Determine the area and side length of the larger square.

1.3 Surface Areas of Objects Made from Right Rectangular Prisms

FOCUS

- Determine the surface areas of composite objects made from cubes and other right rectangular prisms.

These cube houses were built in Rotterdam, Netherlands.
Suppose you wanted to determine the surface area of one of these houses.
What would you need to know?

Investigate

2

Each of you needs 5 linking cubes.
Assume each face of a linking cube has area 1 unit^2.

➤ What is the surface area of 1 cube?
Put 2 cubes together to make a "train."
What is the surface area of the train?
Place another cube at one end of your train.
What is its surface area now?
Continue to place cubes at one end of the train, and determine its surface area.
Copy and complete this table.

Number of Cubes	Surface Area (square units)
1	
2	
3	
4	
5	

What patterns do you see in the table?
What happens to the surface area each time you place another cube on the train?
Explain why the surface area changes this way.

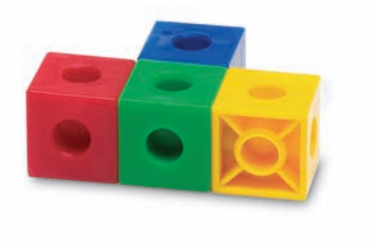
➤ With the 5 cubes, build an object that is different from the train and different from your partner's object.
Determine its surface area.
Compare the surface area of your object with that of your partner's object.

Reflect & Share

Compare your objects with those of another pair of students who made different objects. Are any of the surface areas different?
If your answer is yes, explain how they can be different when all the objects are made with 5 cubes.

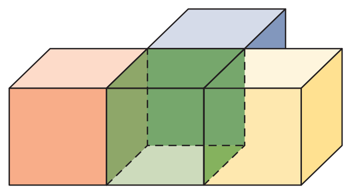
Connect

Here is an object made from 4 unit cubes.
Each face of a cube is a square with area 1 $unit^2$.



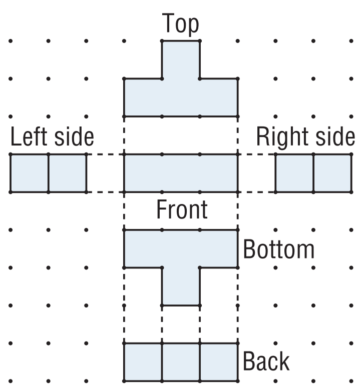
Here are 2 strategies for determining the surface area of the object.

➤ Count the square faces of all the cubes, then subtract 2 faces for each surface where the cubes are joined.
We say the faces *overlap*.



The object has 4 cubes. Each cube has 6 faces.
So, the number of faces is: $6 \times 4 = 24$
There are 3 places where the faces overlap,
so subtract: 3×2, or 6 faces
The surface area, in square units, is: $24 - 6 = 18$

➤ Count the squares on each of the 6 views.
There are:
4 squares on the top,
4 squares on the bottom,
3 squares on the front,
3 squares on the back,
2 squares at the right,
and 2 squares at the left.
The surface area, in square units, is:
$4 + 4 + 3 + 3 + 2 + 2 = 18$



An object like that on page 26 is called a **composite object** because it is made up, or *composed*, of other objects.

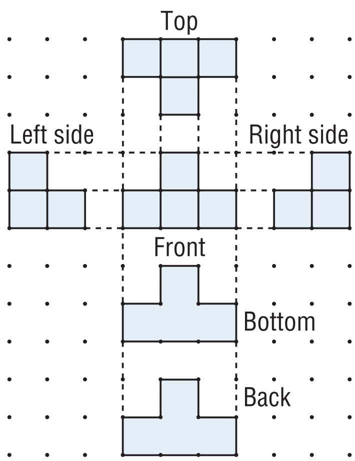
Example 1 Determining the Surface Area of a Composite Object Made from Cubes

Determine the surface area of this composite object.
Each cube has edge length 2 cm.

Solutions

Method 1

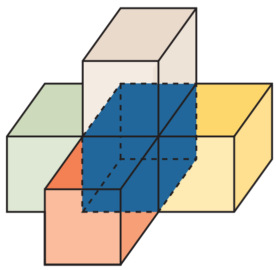
Count the squares on each of the 6 views:



Each of the front, back, top, and bottom views has 4 squares.
Each of the right and left views has 3 squares.
The surface area, in squares, is:
$(4 \times 4) + (3 \times 2) = 22$
Each square has area: $2 \text{ cm} \times 2 \text{ cm} = 4 \text{ cm}^2$
So, the surface area is: $22 \times 4 \text{ cm}^2 = 88 \text{ cm}^2$

Method 2

The composite object has 5 cubes.
Each cube has 6 square faces.
So, the total number of squares is: $5 \times 6 = 30$

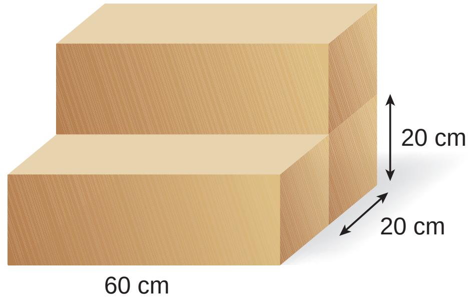


The cubes overlap at 4 places, so there are 4×2, or 8 squares that are not part of the surface area.
The surface area, in squares, is: $30 - 8 = 22$
Each square has area: $2 \text{ cm} \times 2 \text{ cm} = 4 \text{ cm}^2$
So, the surface area is: $22 \times 4 \text{ cm}^2 = 88 \text{ cm}^2$

We can use the surface area of composite objects to solve problems outside the classroom.

Example 2 Determining the Surface Area of a Composite Object Made from Right Rectangular Prisms

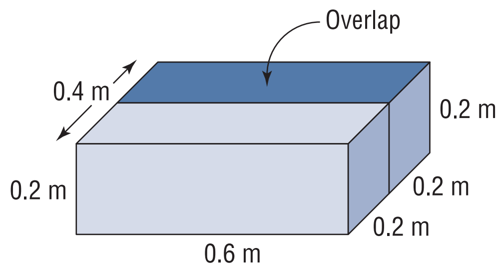
Renee uses 3 pieces of foam to make this chair.
Each piece of foam is a right rectangular prism with dimensions 60 cm by 20 cm by 20 cm.
Can Renee cover the chair with 2 m^2 of fabric?
Explain.



A Solution

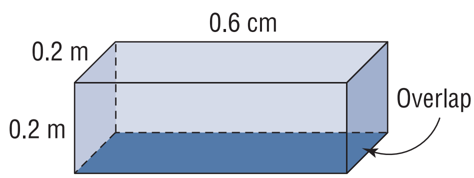
Convert each measurement to metres, then the surface area is measured in square metres.
60 cm = 0.6 m 20 cm = 0.2 m

Determine the surface area of the rectangular prism that is the base of the chair.



Area of top and bottom faces: $2(0.6 \times 0.4) = 0.48$
Area of front and back faces: $2(0.6 \times 0.2) = 0.24$
Area of left and right faces: $2(0.2 \times 0.4) = 0.16$
Surface area of the base of the chair:
$0.48 + 0.24 + 0.16 = 0.88$

Determine the surface area of the rectangular prism that is the back rest.



Area of top, bottom, front, and back:
$4(0.6 \times 0.2) = 0.48$
Area of left and right faces: $2(0.2 \times 0.2) = 0.08$
Surface area of back rest: $0.48 + 0.08 = 0.56$

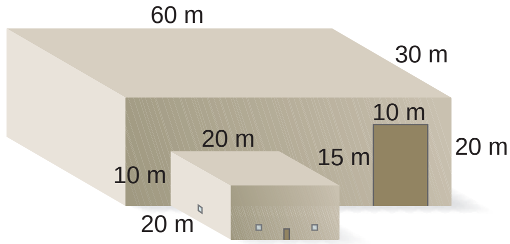
Add the two surface areas, then subtract twice the area of the overlap because neither of these areas is part of the surface area of the chair:
$$0.88 + 0.56 - 2(0.6 \times 0.2) = 1.44 - 0.24$$
$$= 1.2$$
The surface area that is to be covered in fabric is 1.2 m^2.
Since $2\text{ m}^2 > 1.2\text{ m}^2$, Renee can cover the chair with 2 m^2 of fabric.

Example 3 Solving Problems Involving the Surface Area of a Composite Object

A warehouse measures 60 m by 30 m by 20 m. An office attached to one wall of the warehouse measures 20 m by 20 m by 10 m.

a) Determine the surface area of the building.

b) A contractor quotes to paint the exterior of the building at a rate of $\$2.50/m^2$.
These parts of the building are not to be painted: the 2 roofs; the office door with area 2 m^2; 3 loading doors, each measuring 10 m by 15 m; and 4 windows on the office, each with area 1 m^2.
How much would it cost to paint the building?



A Solution

The surface area is measured in square metres.

a) The 4 walls and roof of the warehouse form its surface area.
Area of roof: $60 \times 30 = 1800$
Area of left and right side walls: $2(30 \times 20) = 1200$
Area of the front and back walls: $2(60 \times 20) = 2400$
So, the surface area of the warehouse is: $1800 + 1200 + 2400 = 5400$

The 3 walls and roof of the office form its surface area.
Area of roof: $20 \times 20 = 400$
Area of front, left, and right side walls: $3(20 \times 10) = 600$
So, the surface area of the office is: $400 + 600 = 1000$

For the surface area of the building, add the surface areas of the warehouse and the office, then subtract the area of the overlap.
The area of the overlap, which is the back of the office, is: $20 \times 10 = 200$
So, the surface area of the building is: $5400\ m^2 + 1000\ m^2 - 200\ m^2 = 6200\ m^2$

b) To calculate the area to be painted, subtract the areas of the roofs, doors, and windows from the surface area of the building.
Area of roofs: $1800 + 400 = 2200$
Area of loading doors: $3(10 \times 15) = 450$
Area of office door and windows: $2 + 4(1) = 6$
So, the area to be painted is: $6200\ m^2 - 2200\ m^2 - 450\ m^2 - 6\ m^2 = 3544\ m^2$
The cost to paint the building is: $3544 \times \$2.50 = \$8860.00$

Discuss the ideas

1. When a composite object is made from right rectangular prisms, why is the surface area of the object not the sum of the surface areas of the individual prisms?

2. The surface area of an object is the area of a net of the object. How would drawing a net help you determine the surface area of a composite object?

3. In *Example 3*, why are the bases of the warehouse and office not included in the surface area?

Practice

Check

4. Make each composite object with cubes. Assume each face of a cube has area 1 unit^2. Determine the surface area of each composite object.

a) 

b) 

c) 

d) 

e) 

f) 

Apply

5. These are 1-cm cubes.

a) Determine the surface area of the composite object formed by placing cube 4 on top of each indicated cube.
   i) cube 1 ii) cube 2 iii) cube 3

b) Why are the surface areas in part a equal?

6. These are 1-cm cubes.

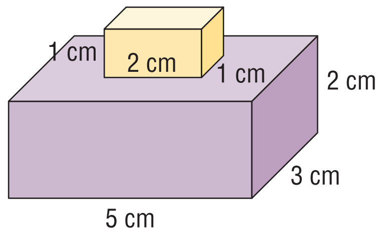
a) Determine the surface area of the composite object formed by placing cube 5 on top of each indicated cube.
   i) cube 1 ii) cube 2 iii) cube 3

b) Why are all the surface areas in part a not equal?

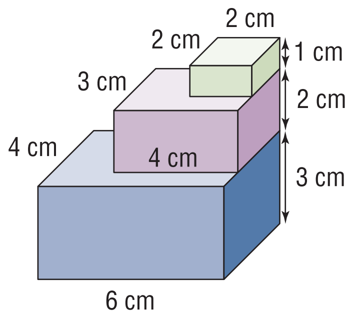
7. Why could you not use 6 views to determine the surface area of this composite object?

8. Determine the surface area of each composite object.
What effect does the overlap have on the calculation of the surface area?

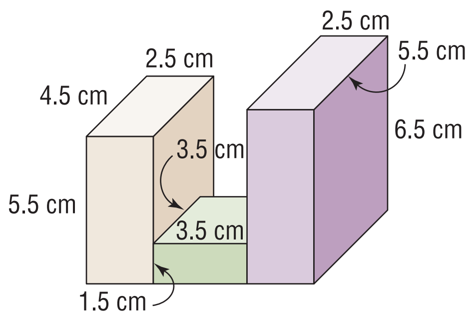
a)



b)



c)

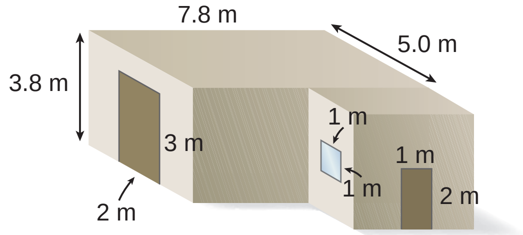


9. Work with a partner. Tape a tissue box on a shoebox to form a composite object.

a) What is the area of the overlap? How did you calculate it?

b) Determine the surface area of the object. How did you use the area of the overlap in your calculation?

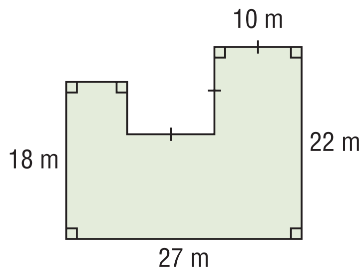
10. Assessment Focus A garage has the dimensions shown. The attached shed has the same height as the garage, but is one-half as long and one-half as wide.



a) What is the surface area of the building?

b) Vinyl siding costs $15/m^2$. The doors, windows, and roof will not be covered with siding. How much will it cost to cover this building with siding?

11. This is a floor plan of a building that is 8 m tall. It has a flat roof. What is the surface area of the building, including its roof?



12. Use 27 small cubes to build a large cube.

a) Determine and record its surface area.

b) How many ways can you remove one cube without changing the surface area? Explain your work.

c) Suppose you painted the large cube. How many small cubes would have paint on:
i) exactly 1 face? ii) exactly 2 faces?
iii) exactly 3 faces? iv) 0 faces?
v) more than 3 faces?
How could you check your answers?

13. Every January, the Ice Magic Festival is held at Chateau Lake Louise in Banff National Park. An ice castle is constructed from huge blocks of ice.

a) Suppose you have 30 blocks of ice measuring 25 cm by 50 cm by 100 cm. Sketch a castle with no roof that could be built with some or all of these blocks.
b) Determine the surface area of your castle, inside and out.

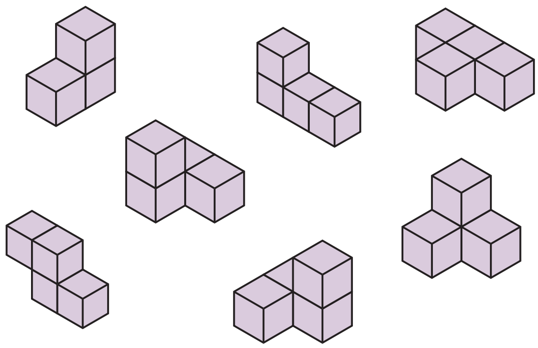
Take It Further

14. Use 6 centimetre cubes.
a) Build a composite object. Sketch the object, then determine and record its surface area.
b) Use the cubes to build other objects with different surface areas. Sketch each object and record its surface area.
c) Determine all the different surface areas for a composite object of 6 cubes.
d) Describe the object with the greatest surface area. Describe the object with the least surface area.

15. Use centimetre cubes. Build, then sketch all possible composite objects that have a surface area of 16 cm^2.

16. A pyramid-like structure is made with 1-m^3 wooden cubes. The bottom layer of the structure is a rectangular prism with a square base and a volume of 25 m^3. The next layer has a volume of 16 m^3. The pattern of layers continues until the top layer, which has a volume of 1 m^3. Determine the surface area of the structure. Describe any patterns you find.

17. The SOMA Puzzle was invented by a Danish poet and scientist named Piet Hein in 1936. The object of the puzzle is to arrange these 7 pieces to form one large cube:



a) Determine the surface area of each piece.
b) Use linking cubes to make your own pieces and arrange them to form a large cube.
c) Suppose you painted the large cube. How many faces of the original 7 pieces would not be painted? How do you know?

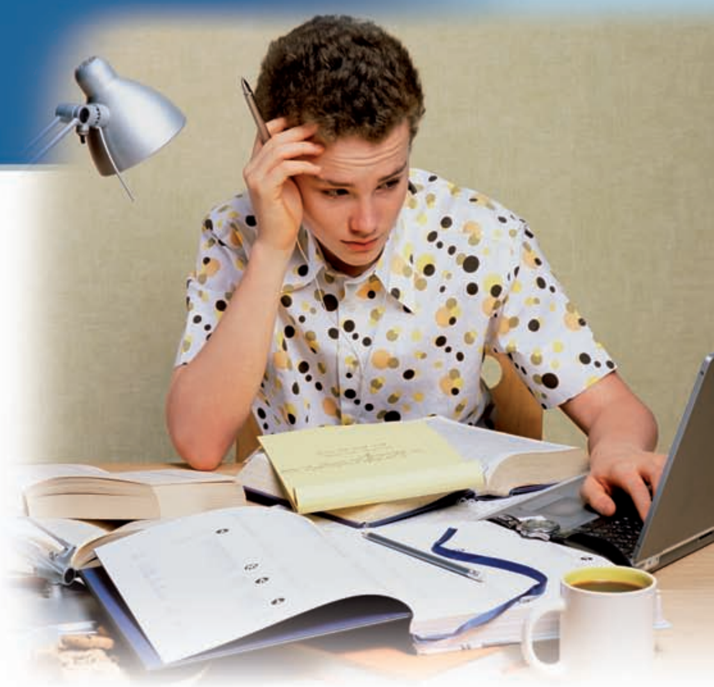
Reflect

Why is it important to consider the areas of overlap when determining the surface area of a composite object? Include an example in your explanation.

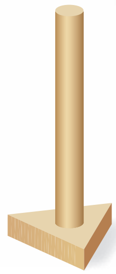
1.4 Surface Areas of Other Composite Objects

FOCUS

- Determine the surface areas of composite objects made from right prisms and right cylinders.



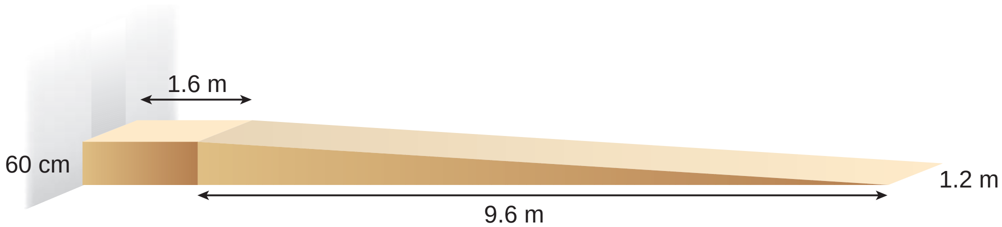
A student designed this stand for a table lamp. How could the student determine the surface area of this stand? What would he need to know?



Investigate

2

To meet safety regulations, a wheelchair ramp must be followed by a landing. This wheelchair ramp and landing lead into an office building. Calculate the surface area of the ramp and landing.



Reflect & Share

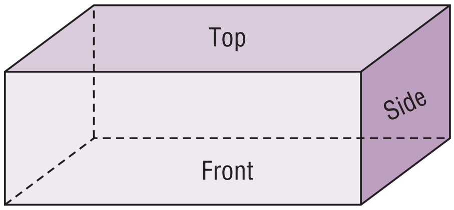
What strategies did you use to determine the surface area?
What assumptions did you make?
Compare your strategy and calculations with those of another pair of students.
How many different ways can you determine the surface area? Explain.

Connect

We use the strategies from Lesson 1.3 to determine the surface area of a composite object made from right cylinders and right triangular prisms. That is, consider each prism or cylinder separately, add their surface areas, then account for the overlap.

For composite objects involving right prisms, we can use word formulas to determine the surface areas of the prisms.

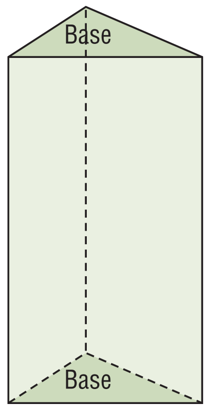
➤ A right rectangular prism has 3 pairs of congruent faces:
- the top and bottom faces
- the front and back faces
- the left side and right side faces



The surface area is the sum of the areas of the faces:
Surface area = 2 × area of top face + 2 × area of front face + 2 × area of side face

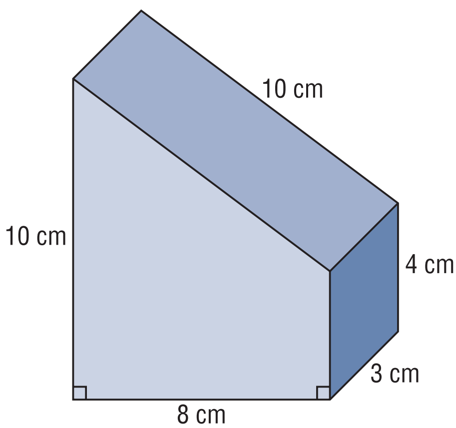
➤ A right triangular prism has 5 faces:
- 2 congruent triangular bases
- 3 rectangular faces

The surface area is the sum of the areas of the triangular bases and the rectangular faces:
Surface area = 2 × area of base + areas of 3 rectangular faces



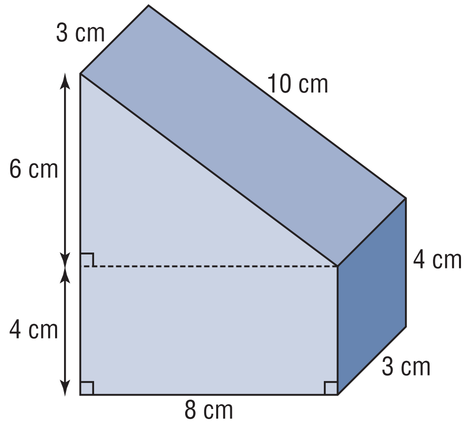
Example 1 Determining the Surface Area of a Composite Object Made from Two Prisms

Determine the surface area of this object.

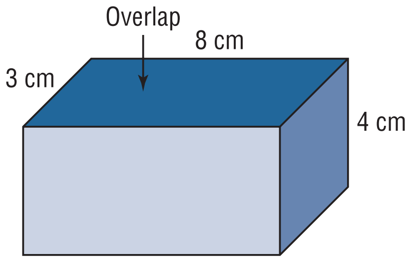


A Solution

The object is composed of a right triangular prism on top of a right rectangular prism. The surface area is measured in square centimetres.



For the surface area of the rectangular prism:



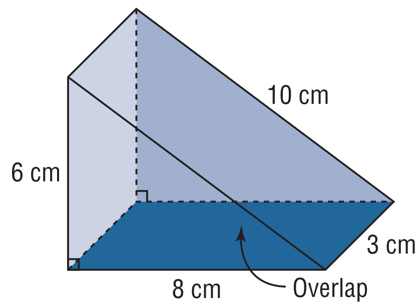
Surface area $= 2 \times$ area of top face $+ 2 \times$ area of front face $+ 2 \times$ area of side face

$= (2 \times 8 \times 3) + (2 \times 8 \times 4) + (2 \times 3 \times 4)$ Use the order of operations.

$= 48 + 64 + 24$

$= 136$

The surface area of the right rectangular prism is 136 cm^2.



For the surface area of the triangular prism:
Each base of the prism is a right triangle, with base 8 cm and height 6 cm.

Surface area $= 2 \times$ area of base $+$ areas of 3 rectangular faces

$= (2 \times \frac{1}{2} \times 8 \times 6) + (3 \times 6) + (3 \times 8) + (3 \times 10)$ Use the fact that $2 \times \frac{1}{2} = 1$.

$= (1 \times 8 \times 6) + (3 \times 6) + (3 \times 8) + (3 \times 10)$ Use the order of operations.

$= 48 + 18 + 24 + 30$

$= 120$

The surface area of the right triangular prism is 120 cm^2.

Add the two surface areas, then subtract twice the area of the overlap.

Surface area $= 136 + 120 - (2 \times 8 \times 3)$

$= 136 + 120 - 48$

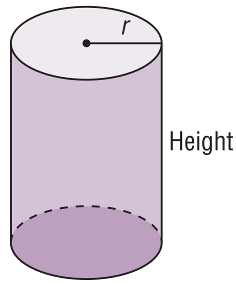
$= 208$

The surface area of the object is 208 cm^2.

When a composite object includes a right cylinder, we can use a formula to determine its surface area.
A cylinder has 2 congruent bases and a curved surface.
Each base is a circle, with radius r and area πr^2.
The curved surface is formed from a rectangle with:

- one side equal to the circumference of the circular base, and
- one side equal to the height of the cylinder

The circumference of the circular base is $2\pi r$.



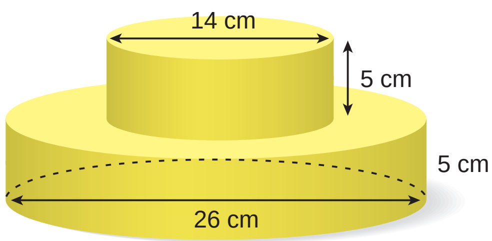
Surface area = area of two circular bases + curved surface area
= 2 × area of one circular base + circumference of base × height of cylinder
$= 2 \times \pi r^2 + 2\pi r \times$ height

Sometimes, one base of the cylinder is not included in the surface area calculation because the cylinder is sitting on its base. Then,

Surface area = area of one base + circumference of base × height of cylinder
$= \pi r^2 + 2\pi r \times$ height

Example 2 Determining the Surface Area of a Composite Object Made from Two Cylinders

Two round cakes have diameters of 14 cm and 26 cm, and are 5 cm tall.
They are arranged as shown. The cakes are covered in frosting. What is the area of frosting?



Solutions

Method 1

Calculate the surface area of each cake.
Do not include the base it sits on because this will not be frosted.
The surface area is measured in square centimetres.

For the smaller cake:
The diameter is 14 cm, so the radius, r, is 7 cm. The height is 5 cm.

Surface area = area of one base + circumference of base × height of cylinder
$= \pi r^2 + 2\pi r \times \text{height}$
$= (\pi \times 7^2) + (2 \times \pi \times 7 \times 5)$ Use a calculator and the order of operations.
$\doteq 373.85$

For the larger cake:
The diameter is 26 cm, so the radius, r, is 13 cm. The height is 5 cm.

Surface area = area of one base + circumference of base × height of cylinder
$= \pi r^2 + 2\pi r \times \text{height}$
$= (\pi \times 13^2) + (2 \times \pi \times 13 \times 5)$ Use a calculator.
$\doteq 939.34$

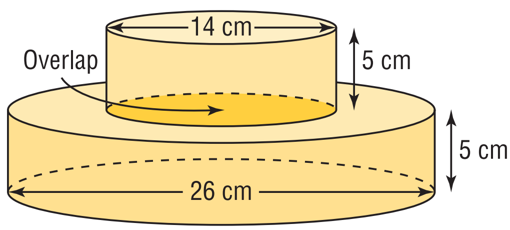
To calculate the area of frosting, add the two surface areas, then subtract the area of the overlap; that is, the area of the base of the smaller cake: $\pi \times 7^2$

Area of frosting $\doteq 373.85 + 939.34 - (\pi \times 7^2)$ Use a calculator.
$\doteq 1159.25$

The area of frosting is about 1159 cm^2.
Since the dimensions were given to the nearest centimetre, the surface area is given to the nearest square centimetre.

Method 2

Calculate the surface area directly.
The overlap is the area of the base of the smaller cake. So, instead of calculating the area of the top of the smaller cake, then subtracting that area as the overlap, we calculate only the curved surface area of the smaller cake.



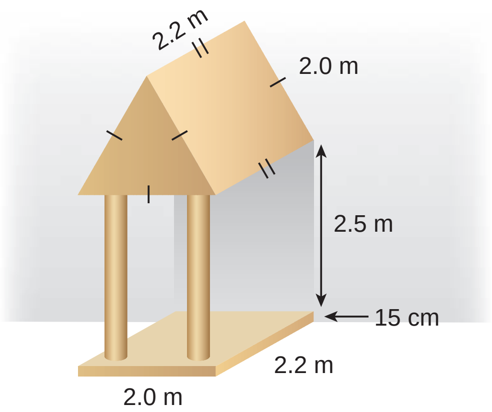
Area of frosting = curved surface area of smaller cake
+ surface area of larger cake, without one base
$= (2 \times \pi \times 7 \times 5) + [(\pi \times 13^2) + (2 \times \pi \times 13 \times 5)]$ Use a calculator.
$\doteq 1159.25$

The area of frosting is about 1159 cm^2.

When some of the lengths on a right triangular prism are not given, we may need to use the Pythagorean Theorem to calculate them.

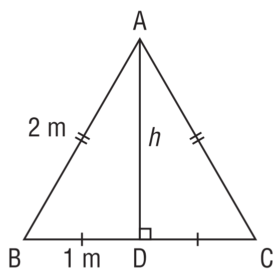
Example 3 Using the Pythagorean Theorem in Surface Area Calculations

The roof, columns, and base of this porch are to be painted.
The radius of each column is 20 cm.
What is the area to be painted, to the nearest square metre?



A Solution

The roof is a triangular prism with its base an equilateral triangle.
To determine the area of the triangular base, we need to know the height of the triangle.
Let the height of the triangle be h.



The height, AD, bisects the base, BC.
Use the Pythagorean Theorem in ΔABD.

$h^2 + 1^2 = 2^2$

$h^2 + 1 = 4$ Solve for h^2.

$h^2 = 4 - 1$

$= 3$

$h = \sqrt{3}$ Determine the square root.

$\doteq 1.732$

The height of the equilateral triangle is about 1.7 m.

Since one base of the triangular prism is against the house, it will not be painted.
The rectangular faces are congruent because they have the same length and width.

So, for the roof:

Surface area = area of one triangular base + areas of 3 congruent rectangular faces

$\doteq \left(\frac{1}{2} \times 2.0 \times 1.732\right) + [3 \times (2.0 \times 2.2)]$

$= 1.732 + 13.2$

$= 14.932$

The base of the porch is a right rectangular prism with only the front, top, and 2 side faces to be painted. The units must match, so convert 15 cm to 0.15 m.

Surface area = area of front face + area of top face + 2 × area of side face

$= (2.0 \times 0.15) + (2.0 \times 2.2) + [2 \times (2.2 \times 0.15)]$

$= 0.3 + 4.4 + 0.66$

$= 5.36$

The two columns are cylinders. Only the curved surfaces need to be painted. The radius is 20 cm, which is 0.2 m.

Surface area = 2 × (circumference of base × height of cylinder)

$= 2 \times (2\pi r \times \text{height})$

$= 2 \times (2 \times \pi \times 0.2 \times 2.5)$

$\doteq 6.283$

To calculate the area to be painted, add the surface areas of the roof, base, and columns, then subtract the area of overlap at the top and bottom of the columns. The area of overlap is 4 times the area of the base of one column.

The area of each circular base is:

$\pi r^2 = \pi \times 0.2^2$

$\doteq 0.126$

Surface area = area of roof + area of base + area of cylinders − 4 × area of circular base of column

$\doteq 14.932 + 5.36 + 6.283 - (4 \times 0.126)$

$= 26.071$

The area to be painted is about 26 m^2.

Discuss the ideas

1. What can you use to calculate an unknown length when the base of a right prism is a right triangle? Explain why.

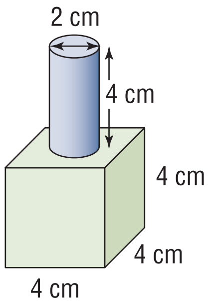
2. When do you think it is not helpful to draw a net to calculate the surface area of a composite object?

Practice

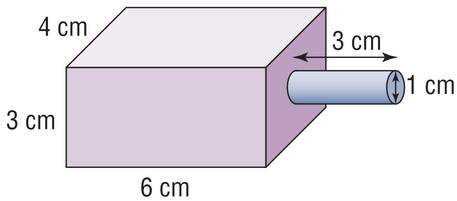
Check

3. Determine the surface area of each composite object. Give the answers to the nearest whole number.

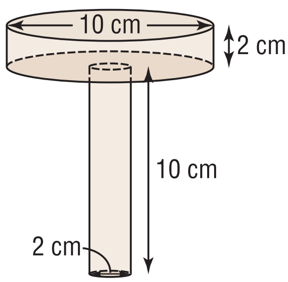
a) cylinder on a cube



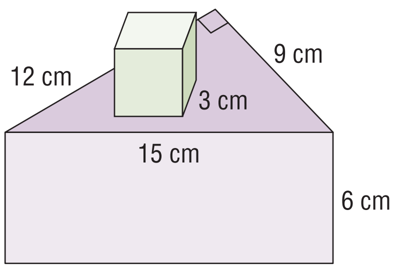
b) cylinder on a rectangular prism



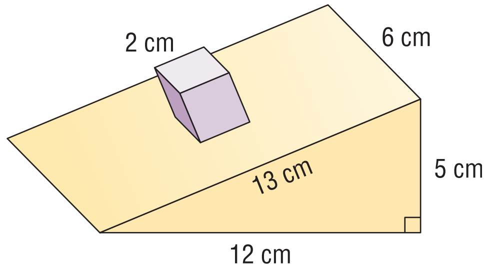
c) cylinder on a cylinder



d) cube on a triangular prism

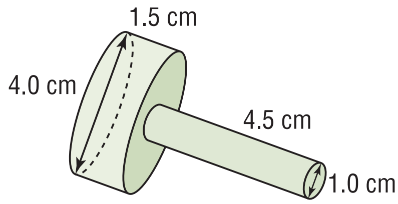


e) cube on a triangular prism

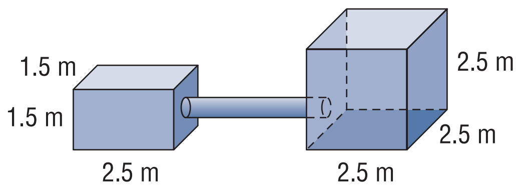


4. Determine the surface area of each composite object. Give the answers to the nearest tenth.

a)

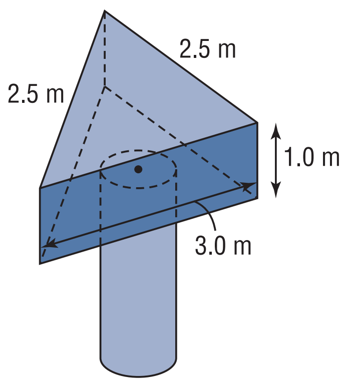


b) The cylinder is 3.5 m long with diameter 0.5 m.

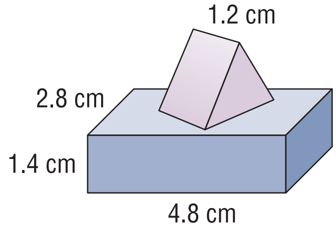


5. Determine the surface area of each composite object.

a) The cylinder is 2.5 m long with radius 0.5 m.

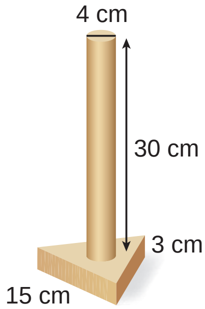


b) The base of the triangular prism is an equilateral triangle with side length 2.8 cm.



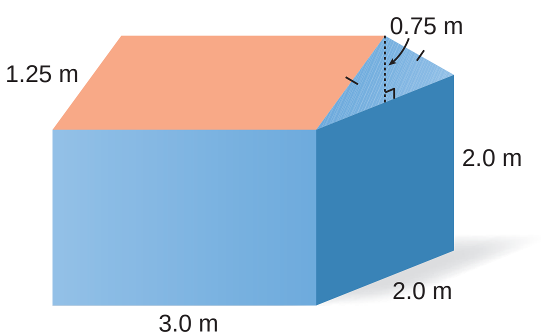
Apply

6. Here is the lamp stand from the top of page 33. The base of the lamp is a triangular prism with an equilateral triangle base. The surface of the stand is to be painted. What is the area that will be painted? Give the answer to the nearest whole number.



7. **Assessment Focus**

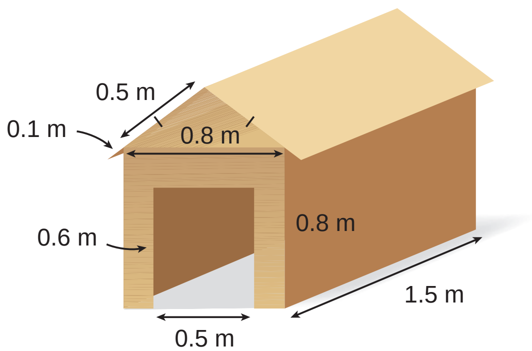
a) A playhouse has the shape of a rectangular prism with a triangular prism roof. Determine the surface area of the playhouse.



b) What are possible dimensions for a door and 2 windows? Explain how including these features will affect the surface area of the playhouse.

c) Determine the surface area of the playhouse not including its doors and windows.

8. Jemma has built this doghouse. The roof is a triangular prism with an isosceles triangle base. There is an overhang of 0.1 m. There is an opening for the doorway.



a) Determine the surface area of the doghouse.

b) The doghouse is to be covered with 2 coats of wood stain. Wood stain can be bought in 1-L or 4-L cans. One litre of stain covers 6 m^2. How many cans of either size are needed? Explain your thinking.

9. Each layer of a three-layer cake is a cylinder with height 7.5 cm.
The bottom layer has diameter 25 cm.
The middle layer has diameter 22.5 cm.
The top layer has diameter 20 cm.
The surface of the cake is frosted.

a) Sketch the cake.

b) What area of the cake is frosted?

10. In question 9, you determined the surface area of a three-layer cake.

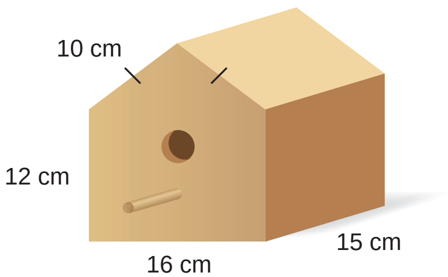
a) Suppose a fourth layer, with diameter 27.5 cm, is added to the bottom of the cake. What is the surface area of cake that will be frosted now?

b) Suppose a fifth layer, with diameter 30 cm, is added to the bottom of the cake. What is the surface area of cake that will be frosted now?

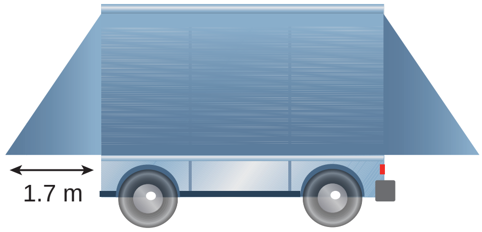
c) How does the surface area change when each new layer is added?

Give all the answers to the nearest tenth.

11. Rory will paint this birdhouse he built for his backyard. The perch is a cylinder with length 7 cm and diameter 1 cm. The diameter of the entrance is 3 cm. What is the area that needs to be painted? Give the answer to the nearest whole number.

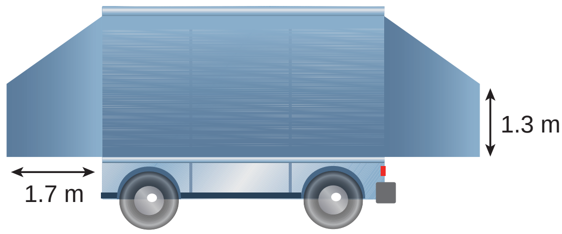


12. Shael and Keely are camping with their parents at Waskesiu Lake in Prince Albert National Park. Their tent trailer is 5 m long and 2.5 m wide. When the trailer is set up, the canvas expands to a height of 2.5 m. At each end, there is a fold out bed that is 1.7 m wide, in a space that is shaped like a triangular prism. The diagram shows a side view of the trailer.



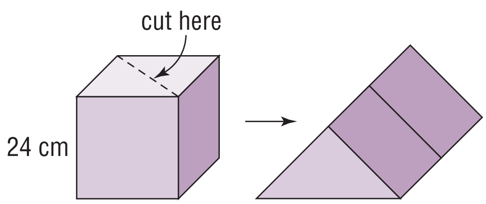
a) Determine the surface area of the canvas on the trailer.

b) Two parallel bars, 1.3 m high, are placed vertically at each end to support the canvas and provide more space in the beds. Does the surface area of the canvas change when the bars are inserted? Explain how you know.



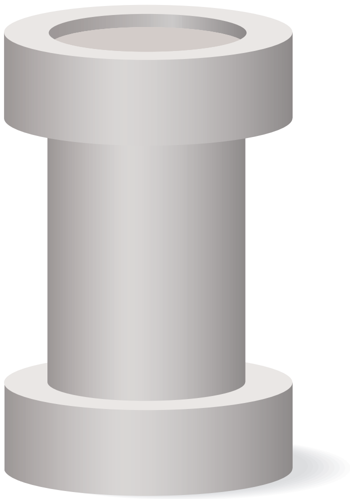
13. a) What is the surface area of a cube with edge length 24 cm?

b) The cube is cut along a diagonal of one face to form two triangular prisms. These prisms are glued together to form a longer triangular prism. What is the surface area of this prism? Give the answer to the nearest whole number.



c) Why do the cube and the triangular prism have different surface areas?

14. A birdbath and stand are made from 3 cylinders. The top and bottom cylinders have radius 22 cm and height 13 cm. The middle cylinder has radius 15 cm and height 40 cm. The "bath" has radius 15 cm and depth 2 cm. The birdbath and stand are to be tiled. Calculate the area to be tiled.



Take It Further

15. **a)** What is the surface area of a cylinder that is 50 cm long and has diameter 18 cm?

b) The cylinder is cut in half along its length and the two pieces are glued together end to end.

i) Sketch the composite object.

ii) What is its surface area?

Give the answers to the nearest whole number.

16. Grise Fiord, Nunavut, is Canada's northernmost Inuit community and it is home to 150 residents. In Inuktitut, this hamlet is called Aujuittuq, which means "the place that never thaws." Although the ground is frozen most of the year, it softens in the summer. The freezing and thawing of the ground would ruin a house foundation. The houses are made of wood, and are built on platforms. The homes are compact and have few windows.

a) Design and sketch the exterior of a home that could fit on a platform that is 10 m wide and 20 m long.

b) Determine the surface area of this home.

c) Every outside face needs to be insulated. Insulation costs $\$4.25/m^2$. How much will it cost to insulate this home?

Reflect

Sketch a building or structure in your community that is made up of two or more prisms or cylinders. Explain how you would determine its surface area.

Study Guide

Perfect Squares

When a fraction can be written as a product of two equal fractions, the fraction is a perfect square.

For example, $\frac{144}{25}$ is a perfect square because $\frac{144}{25} = \frac{12}{5} \times \frac{12}{5}$; and $\sqrt{\frac{144}{25}} = \frac{12}{5}$

When a decimal can be written as a fraction that is a perfect square, then the decimal is also a perfect square.

The square root is a terminating or repeating decimal.

For example, 12.25 is a perfect square because $12.25 = \frac{1225}{100}$, and $\sqrt{\frac{1225}{100}} = \frac{35}{10}$, or 3.5

Non-Perfect Squares

A fraction or decimal that is not a perfect square is a non-perfect square.

To estimate the square roots of a non-perfect square, use perfect squares as benchmarks or use a calculator.

For example, $\sqrt{\frac{143}{25}} \doteq \sqrt{\frac{144}{25}}$, which is $\frac{12}{5}$, or 2.4

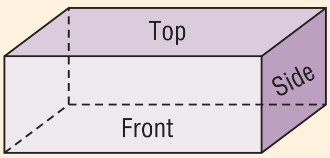
And, $\sqrt{6.4} \doteq 2.5$ to the nearest tenth

Surface Area of a Composite Object

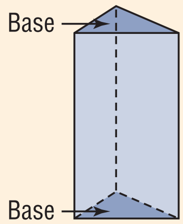
This is the sum of the surface areas of the objects that make up the composite object, minus the overlap.

The objects that make up the composite object can be:

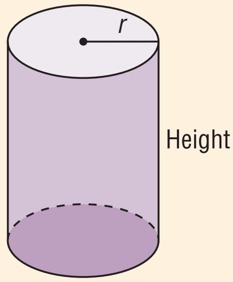
- A right rectangular prism with
  Surface area = 2 × area of top face + 2 × area of front face + 2 × area of side face



- A right triangular prism with
  Surface area = 2 × area of base + areas of 3 rectangular faces



- A right cylinder, radius r, with
  Surface area = 2 × area of one circular base + circumference of base × height of cylinder
  $= 2\pi r^2 + 2\pi r \times$ height



Review

1.1

1. Use grid paper to illustrate each square root as the side length of a square, then determine the value of the square root.

a) $\sqrt{1.21}$ b) $\sqrt{\frac{9}{25}}$ c) $\sqrt{0.64}$

d) $\sqrt{\frac{81}{16}}$ e) $\sqrt{2.56}$ f) $\sqrt{\frac{1}{36}}$

g) $\sqrt{0.25}$ h) $\sqrt{\frac{100}{64}}$ i) $\sqrt{3.61}$

j) $\sqrt{\frac{4}{121}}$ k) $\sqrt{2.89}$ l) $\sqrt{\frac{36}{49}}$

2. Determine each square root.

a) $\sqrt{\frac{144}{25}}$ b) $\sqrt{\frac{225}{64}}$

c) $\sqrt{\frac{196}{81}}$ d) $\sqrt{\frac{324}{121}}$

e) $\sqrt{0.0196}$ f) $\sqrt{0.0289}$

g) $\sqrt{1.69}$ h) $\sqrt{4.41}$

3. Which fractions and decimals are perfect squares? Explain your reasoning.

a) $\frac{48}{120}$ b) 1.6 c) $\frac{49}{100}$

d) 0.04 e) $\frac{144}{24}$ f) 2.5

g) $\frac{50}{225}$ h) 1.96 i) $\frac{63}{28}$

4. Calculate the number whose square root is:

a) $\frac{3}{5}$ b) 1.6 c) $\frac{9}{7}$ d) 0.8

5. Determine the side length of a square with each area below. Explain your strategy.

a) 0.81 m^2 b) 0.01 m^2

c) 4.84 cm^2 d) 6.25 cm^2

e) 0.16 km^2 f) 1.44 km^2

1.2

6. Use benchmarks to approximate each square root to the nearest tenth. State the benchmarks you used.

a) $\sqrt{3.8}$ b) $\sqrt{33.8}$

c) $\sqrt{133.8}$ d) $\sqrt{233.8}$

7. Use benchmarks to estimate a fraction for each square root. State the benchmarks you used.

a) $\sqrt{\frac{77}{10}}$ b) $\sqrt{\frac{18}{11}}$ c) $\sqrt{\frac{15}{39}}$

d) $\sqrt{\frac{83}{19}}$ e) $\sqrt{\frac{28}{103}}$ f) $\sqrt{\frac{50}{63}}$

8. Use any strategy you wish to estimate the value of each square root. Explain why you used the strategy you did.

a) $\sqrt{5.9}$ b) $\sqrt{\frac{7}{20}}$ c) $\sqrt{0.65}$

d) $\sqrt{\frac{21}{51}}$ e) $\sqrt{23.2}$ f) $\sqrt{\frac{88}{10}}$

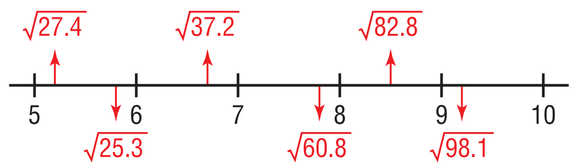
9. Which of the following square roots are correct to the nearest tenth? How do you know? Correct the square roots that are incorrect.

a) $\sqrt{2.4} \doteq 1.5$ b) $\sqrt{1.6} \doteq 0.4$

c) $\sqrt{156.8} \doteq 15.6$ d) $\sqrt{47.8} \doteq 6.9$

e) $\sqrt{0.5} \doteq 0.7$ f) $\sqrt{0.7} \doteq 0.5$

10. Which square roots are correctly placed on the number line below? How do you know?

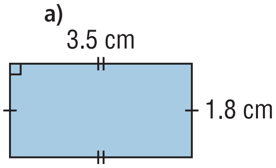


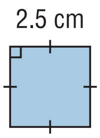
11. Use the square roots listed below. Which square roots are between each pair of numbers? Justify your answers.

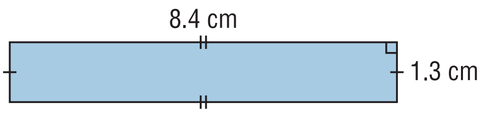
a) 1 and 2 **b)** 11 and 12
c) 3.5 and 4.5 **d)** 1.5 and 2.5
e) 4.5 and 5.5 **f)** 14.5 and 15.5

$\sqrt{12.9}$	$\sqrt{4.8}$	$\sqrt{134.5}$	$\sqrt{1.2}$
$\sqrt{21.2}$	$\sqrt{15.2}$	$\sqrt{222.1}$	$\sqrt{9.6}$
$\sqrt{3.2}$	$\sqrt{237.1}$	$\sqrt{2.3}$	$\sqrt{213.1}$
$\sqrt{125.4}$	$\sqrt{23.1}$	$\sqrt{129.9}$	$\sqrt{2.8}$
$\sqrt{5.7}$	$\sqrt{29.1}$		

12. Determine the length of a diagonal of each rectangle.

a)


b)


c)


13. Determine a decimal or a fraction whose square root is between each pair of numbers.

a) $\frac{1}{3}$ and 1 **b)** 0.2 and 0.3
c) 1.4 and 1.41 **d)** $\frac{1}{10}$ and $\frac{3}{10}$

14. a) Use a calculator to approximate each square root.

i) $\sqrt{0.0015}$ **ii)** $\sqrt{0.15}$ **iii)** $\sqrt{15}$
iv) $\sqrt{1500}$ **v)** $\sqrt{150\ 000}$

b) What patterns do you see in the square roots in part a? Use the patterns to write the previous two square roots less than $\sqrt{0.0015}$ and the next two square roots greater than $\sqrt{150\ 000}$.

1.3

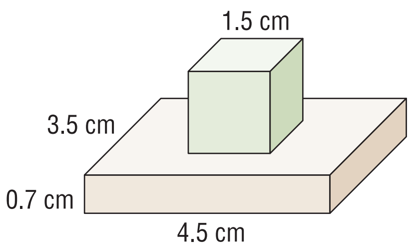
15. Each object is built with 1-cm cubes. Determine its surface area.

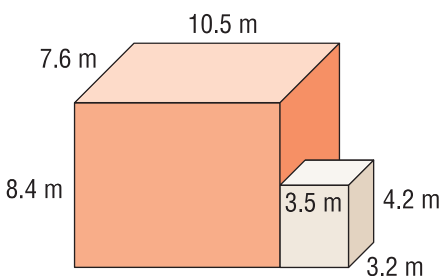
a)


b)

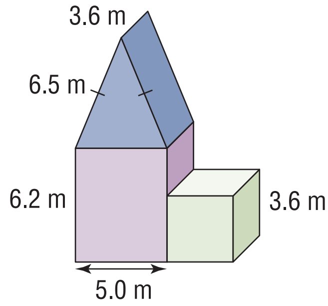

c)


16. Determine the surface area of each composite object. What effect does the overlap have on the surface area?

a) rectangular prism and cube


b) two rectangular prisms


c) triangular prism, rectangular prism, and cube



17. A desk top is a rectangular prism with dimensions 106 cm by 50 cm by 2 cm. Each of 4 legs of the desk is a rectangular prism with dimensions 75 cm by 3 cm by 3 cm.

a) Sketch the desk.

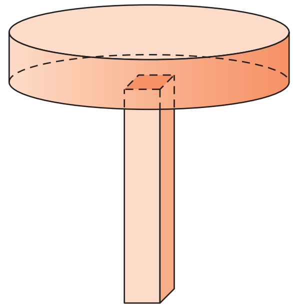
b) Determine the surface area of the desk.

18. An Inukshuk is a human-like object constructed from stone by Canada's Inuit People. Traditionally, Inukshuks were used as markers during the Caribou hunt. The Inukshuk is now a symbol of leadership, cooperation, and human spirit. Each stone is separate; the stones are balanced to make the Inukshuk. This giant Inukshuk in Igloolik, Nunavut was built to commemorate the new millenium.

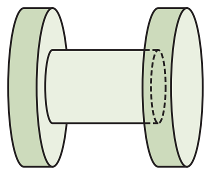
Construct an Inukshuk of cardboard boxes or other materials. Determine its surface area.

1.4 **19.** Determine the surface area of each composite object. Give the answers to the nearest tenth.

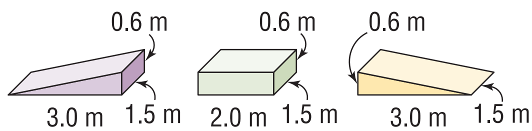
a) The rectangular prism has dimensions 2.5 cm by 2.5 cm by 15.0 cm. The cylinder is 3.5 cm high with radius 9.6 cm.



b) Each of the two congruent cylinders is 2.8 cm long, with radius 7.8 cm. The middle cylinder is 10.4 cm long, with radius 3.6 cm.



20. There are 2 wooden ramps, each of which is a triangular prism with a right triangle base; and a platform that is a rectangular prism. The ramps are joined to the platform to make one larger ramp for a BMX bike. This ramp will be painted completely.



a) Calculate the surface area to be painted.

b) The paint costs $19.95 for one 3.78-L container. This will cover 35 m^2. The surface area needs 2 coats of paint. How much paint is needed and how much will it cost?

Practice Test

1. Sketch this number line.

a) Do *not* use a calculator. Determine or estimate each square root. Where necessary, write the square root to the nearest tenth. Place each square root on the number line.

i) $\sqrt{\frac{49}{4}}$ **ii)** $\sqrt{6.25}$ **iii)** $\sqrt{\frac{64}{9}}$ **iv)** $\sqrt{98.5}$ **v)** $\sqrt{\frac{9}{100}}$ **vi)** $\sqrt{\frac{9}{10}}$

b) How can you use benchmarks to determine or estimate square roots?

2. a) Use a calculator to determine or estimate each square root. Where necessary, write the square root to the nearest hundredth.

i) $\sqrt{\frac{3}{7}}$ **ii)** $\sqrt{52.5625}$ **iii)** $\sqrt{\frac{576}{25}}$ **iv)** $\sqrt{213.16}$ **v)** $\sqrt{135.4}$

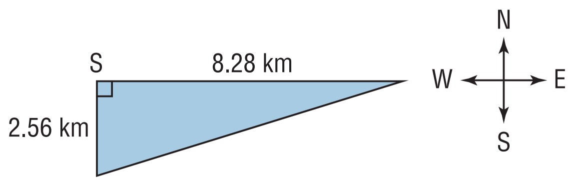
b) Which square roots in part a are exact? Which are approximate?

c) Explain why a square root shown on a calculator display may be approximate.

3. a) Identify a perfect square between 0 and 0.5.
How do you know the number is a perfect square?

b) Identify a number whose square root is between 0 and 0.5.
How can you check your answer?

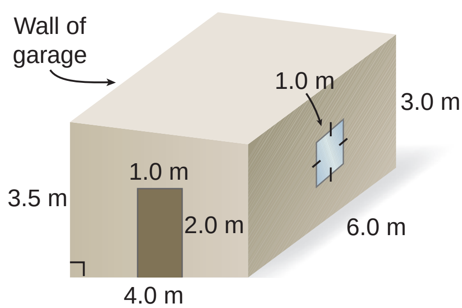
4. One canoe is 2.56 km due south of a small island, S. Another canoe is 8.28 km due east of the island. How far apart are the canoes? How do you know?



5. A garden shed is built against one wall of a garage. The shed has a sloping roof.

a) What is the surface area of the shed, not including its door and window?

b) The shed is to be painted with 2 coats of paint. Paint costs \$3.56/L. One litre covers 10 m^2. What will it cost to paint the shed?



6. Each of two congruent cubes has volume 64 cm^3. The cubes are joined at their faces to a cylinder that is 5 cm long and has radius 2 cm.

a) Sketch the object. **b)** What is the surface area of this object?

Unit Problem — Design a Play Structure

You will design a play structure for young children, constructed of light-weight nylon fabric and fibreglass poles.

Your budget for this project is $800. A student has offered to sew the fabric for a donation of $125 toward upgrading the school sewing machines.

The design can only include cylinders, rectangular prisms, and triangular prisms.
There should be between 6 and 8 objects, with at least one of each type.
The objects can be connected face to face.
Keep in mind that cylinders and openings need to allow enough movement space to safely accommodate a small child.

The fabric is available in three different colours:
Red costs $10/m^2$. Yellow costs $11/m^2$. Blue costs $12/m^2$.

The skeleton of the structure is made from fibreglass poles that cost $3/m.
A fabric cylinder needs flexible circular supports every 1 m for reinforcement.
These cost $4/m.
Joiners are included at no cost.

Your work should show:

- models or sketches of your design
- the surface area of each object
- the cost of each object
- how you calculated the total surface area and the cost of the project
- an explanation of any unique features of your structure and why you included them

Reflect on Your Learning

What have you learned about perfect squares, non-perfect squares, and square roots?
How are square roots used when you calculate surface area?