June 5th , 2015

Circle Geometry

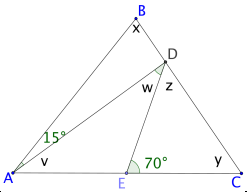
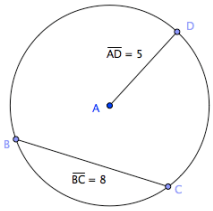
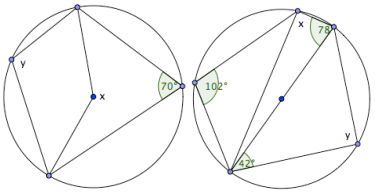
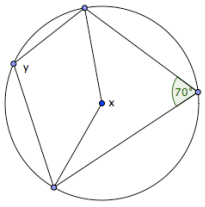
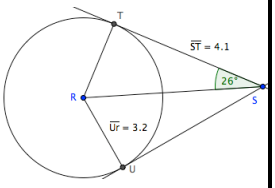
This booklet belongs to:_____

LESSON #	DATE	QUESTIONS FROM NOTES	Questions that I find difficult
1.		Pg.	
2.		Pg.	
3.		Pg.	
4.		Pg.	
5.		Pg.	
6.		Pg.	
7.		Pg.	
8.		Pg.	
9.		Pg.	
10.		Pg.	
11.		REVIEW	
12.		**TEST**	

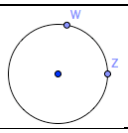
☺ ☺ ☺ Find detailed homework solutions at www.mathbeacon.ca/guidebooks/#math9 ☺ ☺ ☺

Your teacher has important instructions for you to write down below.

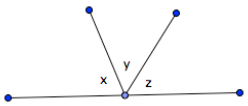
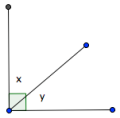
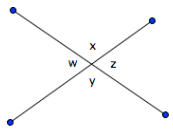
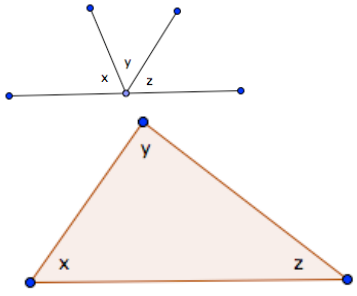
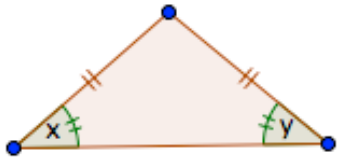
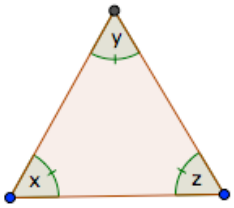
Circle Geometry

IRP	#	Daily Topic	Key Ideas
This section is not part of the WNCP IRPS. This section reviews the important geometric language needed to be successful in this chapter.	1.	Geometry Review: Angles & Triangles (pg. 5-7) • Review terms, supplementary, complementary, angles on a line, Angles in a triangle…	Given: AE= ED; ∠BDA=90°. Find ∠v, ∠x, & ∠z. Name ΔDEC type. 
C1: Solve problems and justify the solution strategy using circle properties, including: The perpendicular from the center of a circle to a chord bisects the chord. The measure of the central angle is equal to twice the measure of the inscribed angle coming from the same arc The inscribed angles coming from the same arc are congruent A tangent to a circle is perpendicular to the radius at the point of tangency [C, CN, PS, R, T, V]	2.	Perpendicular Chord Theorem (pg. 8-12) • Solve problems and justify the solution strategy using the perpendicular from the center of a circle to a chord bisects the chord. • Explain the relationship among the center of a circle, a chord, and the perpendicular bisector of the chord. • Provide an example that illustrates the perpendicular from the center of a circle to a chord bisects the chord.	Determine the shortest distance between the center of the circle and the chord BC. 
	3.	Inscribed & Central Angles (pg. 13-18) • Solve problems and justify the solution strategy using the inscribed angles coming from the same arc are congruent. • Provide an example that illustrates the measure of the central angle is equal to twice the measure of the inscribed angle coming from the same arc. • Provide an example that illustrates the inscribed angles coming from the same arc are congruent.	Find x and y. 
	4.	(This is left blank to give extra time to lesson 2 or 3.	
	5.	Inscribed Quadrilateral Properties (Pg. 19-21) • Solve a given problem involving application of one or more of the circle properties.	Find x and y. 
	6.	**Tangent Properties** (Pg. 22-26) • Solve problems and justify the solution strategy using a tangent to a circle is perpendicular to the radius at the point of tangency. • Provide an example that illustrates a tangent to a circle is perpendicular to the radius at the point of tangency.	U and T are points of tangency. Determine the length of RS and ∠RTS. 
	7.	Extra Practice (Pg. 27-28) • Solve a given problem involving application of one or more of the circle properties.	
	8.	Chapter Review and Practice Test • Help students develop sound study habits. • Many students will graduate high school saying they do not know how to study for math tests.	
	9.	Go over Practice Test	
	10.	Unit Evaluation	

Key Terms

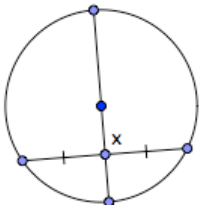
Angle bisector	A line that cuts an angle in half
Arc	A segment of the circumference of the circle. The curved line segment is an arc. Moving clockwise, a minor arc travels from W to Z and a major arc travels from Z to W. 
Bisector	A line that cuts something in half.
Central Angle	An angle whose arms are both radii of a circle
Chord	A line segment with endpoints on the circumference of the circle
Complementary	Angles are complementary if they have a sum of 90°.
Equilateral Triangle Property	If a triangle has 3 equal sides, then it has 3 equal angles. Or if a triangle has 3 equal angles then it has 3 equal sides.
Inscribed angle	An angle where both end points and the vertex are on the circle
Inscribed polygon	A closed shape with every vertex on the circle.
Midpoint	A point in the middle of a line segment.
Perpendicular	Two lines are perpendicular if they meet at 90°.
Perpendicular Bisector	A line that meets another line at the midpoint and 90°
Point of tangency	The exact point at which a line is tangent to a circle.
Radius ⊥ Tangent	Radius and tangent create a 90° angle at the point of tangency
Secant	A line that intersects a circle in exactly 2 places.
Subtend	To be across from. An angle subtends a chord if it is opposite to it.
Supplementary angles	Two angles are supplementary if they have a sum of 180°.
Tangent	A line is tangent to a circle if it touches in exactly one place.

Geometric Properties you should know.

Angles on a line add to 180°.  $x + y + z = 180°$	Complementary Angles  $x + y = 90°$	Vertically Opposite Angles Are congruent.  $x = y$ & $w = z$
Angles in a triangle add to 180°.  $x + y + z = 180°$	Isosceles Triangle Property.  If a triangle has 2 equal sides, then it has 2 equal angles. Or if a triangle has 2 equal angles then it has 2 equal sides.	Equilateral Triangle Property.  If a triangle has 3 equal sides, then it has 3 equal angles. Or if a triangle has 3 equal angles then it has 3 equal sides.

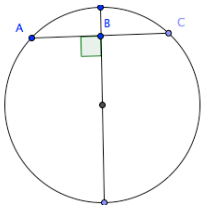
Circle Geometry Properties

If a line goes through the center and bisects a chord **then** the line is perpendicular to the chord.



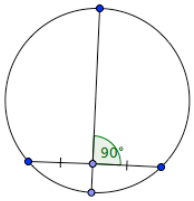
x=90°

If a line goes through the center and is perpendicular to a chord **then** that chord is bisected.

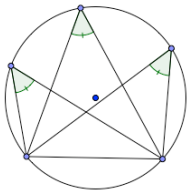


AB=BC

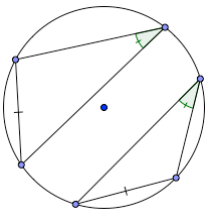
The perpendicular bisector of any chord goes through center.



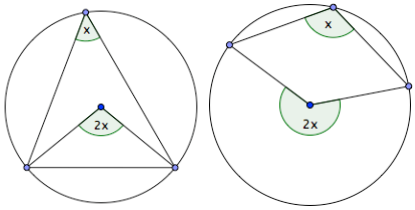
Incribed angles coming from the same chord or arc are equal.



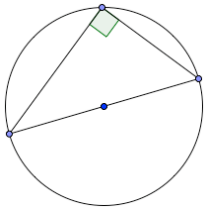
Incribed angles coming from the same size chord or arc are equal.



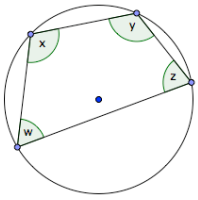
Central angles coming from the same chord or equal chords are twice as big as inscribed angle.



Inscribed angles in a semi circle equal 90°.



Opposite Angles in Inscribed quadrilaterals are supplementary

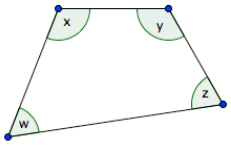


If WXYZ is an inscribed quadrilateral then

$\angle X + \angle Z = 180°$

$\angle W + \angle Y = 180°$

If opposite sides of a quad are supplementary **then** it is an inscribed quadrilateral.

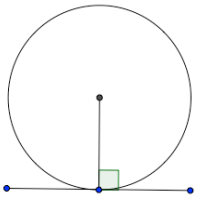


If $\angle X + \angle Z = 180°$

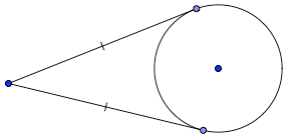
$\angle W + \angle Y = 180°$

Then WXYZ is an inscribed quadrilateral.

A tangent is perpendicular to the radius at the point of tangency.



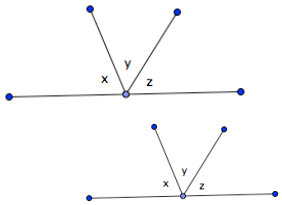
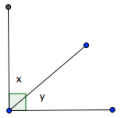
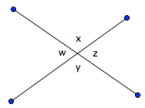
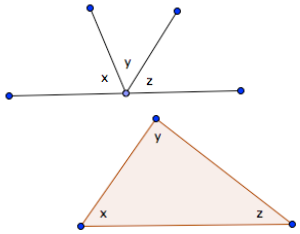
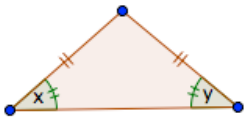
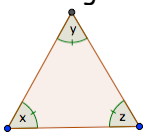
Tangents from an external point are equal.



Geometry Review: Angles & Triangles

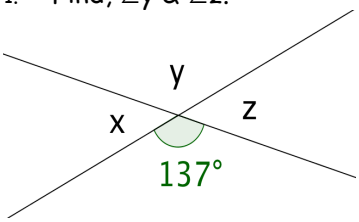
Math is a language that is spoken all over the world to solve problems. For this reason it is important that we all use the same language so what we can understand one another.

Take a moment to refresh your memory of the following terms.

Angles on a line add to 180°.  $x + y + z = 180°$	Complementary Angles  $x + y = 90°$	Vertically Opposite Angles Are congruent.  $x = y$ & $w = z$
Angles in a triangle add to 180°.  $x + y + z = 180°$	Isosceles Triangle Property.  If a triangle has 2 equal sides, then it has 2 equal angles. The reverse is also true	Equilateral Triangle Property.  If a triangle has 3 equal sides, then it has 3 equal angles. The reverse is also true

Challenge #1:

1. Find, ∠y & ∠z.

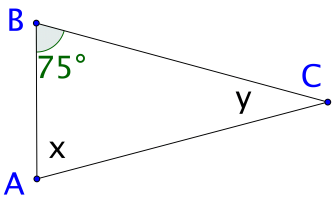


State a geometric reason

∠y= °

∠z= °

2. Given BC = AC. Find ∠x & ∠y.

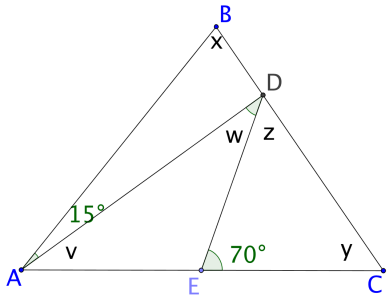


State a geometric reason

∠x= °

∠y= °

3. Given: AE= ED; ∠BDA=90°. Find ∠v, ∠x, & ∠z. What kind of triangle is ΔDEC?

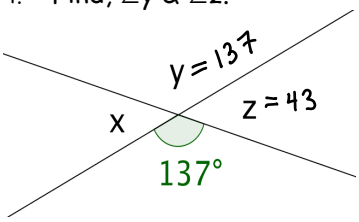


∠v= °, ∠x= °

∠z= °, ΔDEC:

For **ALL** of the following, solve & give an **explanation** for each answer given.

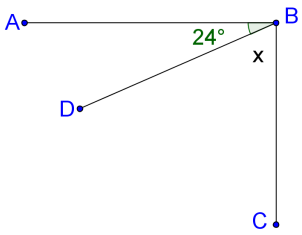
4. Find, ∠y & ∠z.



∠y=137°, Vertically opposite angles

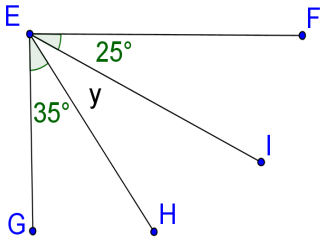
∠z=43°, Angles on a line are supplementary.

5. Given: ∠ABC = 90°. Find ∠x.



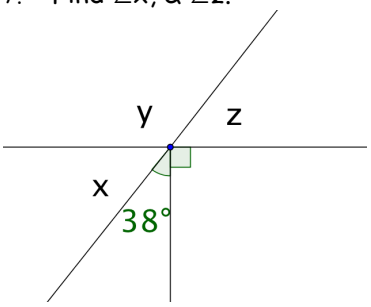
∠x= °

6. Given: ∠FEG = 90°. Find ∠y.



∠y= °

7. Find ∠x, & ∠z.



∠x= °

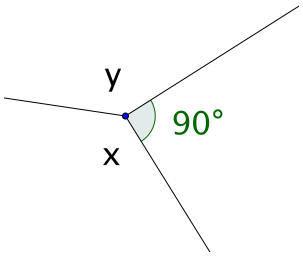
∠z= °

8. OMIT

∠x= °

∠y= °

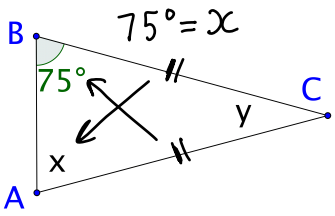
9. Given: ∠x = ∠y. Find ∠x & ∠y.



∠x= °

∠y= °

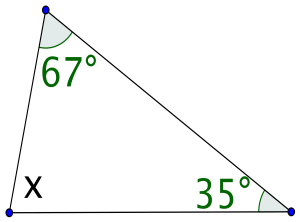
10. Given BC = AC. Find ∠x & ∠y.



∠x=75°, Isosceles Triangle property.

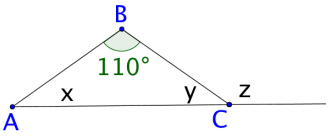
∠y=30°, ∠ in a Δ are supplementary.

11. Given: angles. Find ∠x.



∠x= °

12. Given AB = BC. Find ∠y & ∠z.

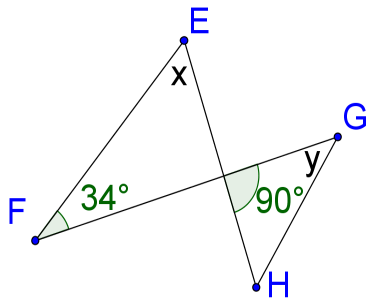


∠y= °

∠z= °

Determine the missing angles.

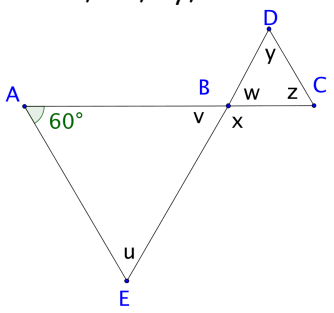
13. Given: DG = DH. (D is the intersection point) Find ∠x & ∠y.



∠x= °

∠y= °

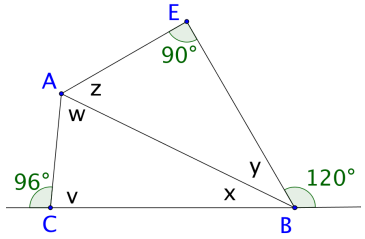
14. Given: AE=AB; BD=DC. Find ∠v, ∠w, ∠y, & ∠z.



∠v= °, ∠w= °

∠y= °, ∠z= °

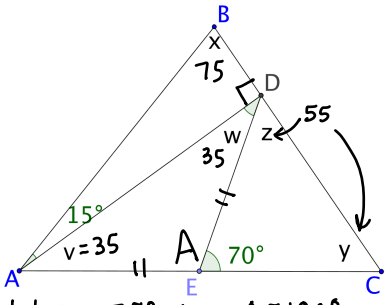
15. Given: AB = CB. Find ∠v, ∠x, ∠y, & ∠z.



∠v= °, ∠x= °

∠y= °, ∠z= °

16. Given: AE= ED; ∠BDA=90°. Find ∠v, ∠x, & ∠z. Name ΔDEC type.



1st ∠x=75°, ∠ in a △=180°

2nd ∠v=35, △AEW=△+∠A=110°

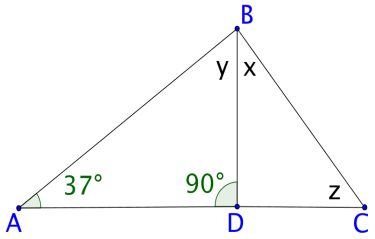
3rd ∠z=55°, ∠ on a line =180°

△DEC=△ 70,55,55△

∠v=35°, ∠x=75°

∠z=55°, ΔDEC: Isosceles

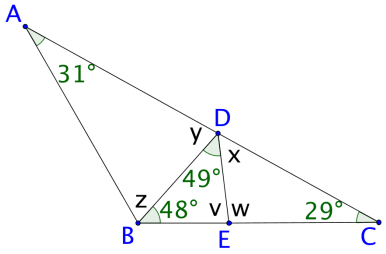
17. Given: ∠ABC = 90°.Find ∠x, ∠y, & ∠z.



∠x= °, ∠y= °

∠z= °

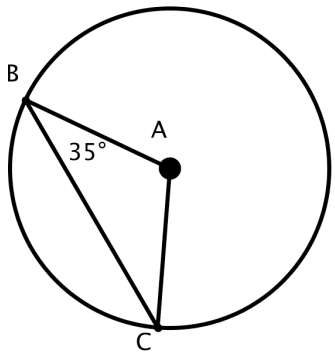
18. Find ∠x & ∠z.



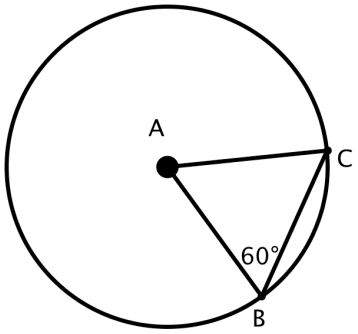
∠x= °, ∠z= °

Answers are on this page*.

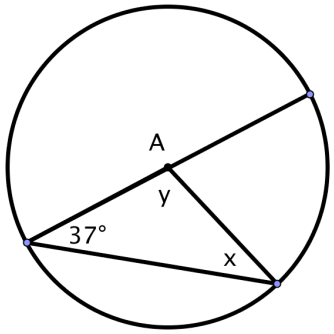
a. Why is triangle ABC an isosceles triangle?



b. What kind of triangle is triangle ABC?

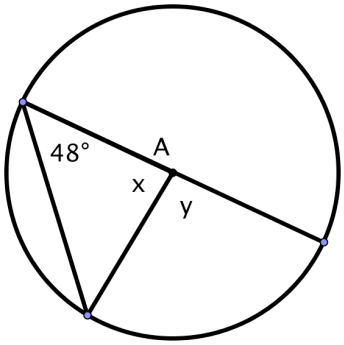


c. Determine angles x and y.



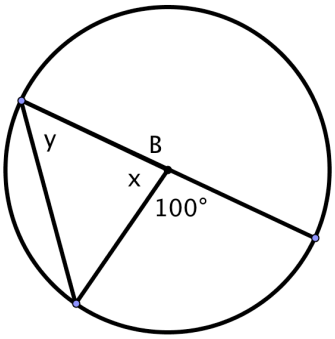
$\angle X =$ $\angle Y =$

d. Determine angles x and y.



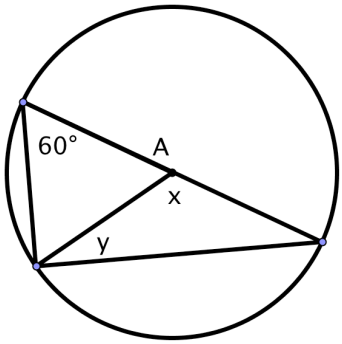
$\angle X =$ $\angle Y =$

e. Determine angles x and y.



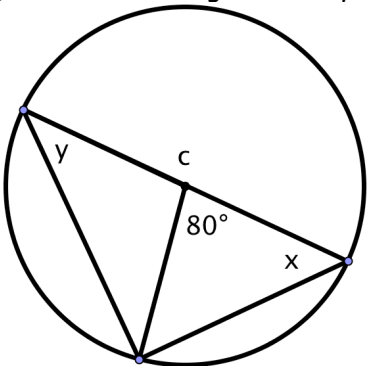
$\angle X =$ $\angle Y =$

f. Determine angles x and y.



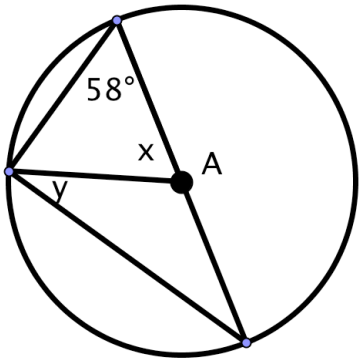
$\angle X =$ $\angle Y =$

g. Determine angles x and y.



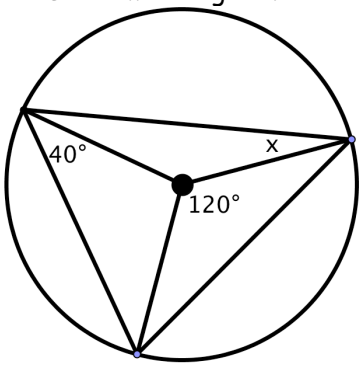
$\angle X =$ $\angle Y =$

h. Determine angles x and y.



$\angle X =$ $\angle Y =$

i. Determine angle x.



$\angle X =$

*Answers

a.	b.	c.	d.	e.	f.	g.	h.	i.
The side lengths AB and AC are both radii. Radii are equal so the triangle has 2 equal sides.	Equilateral	37, 106	84, 96	80, 50	120, 30	50, 40	64, 32	20

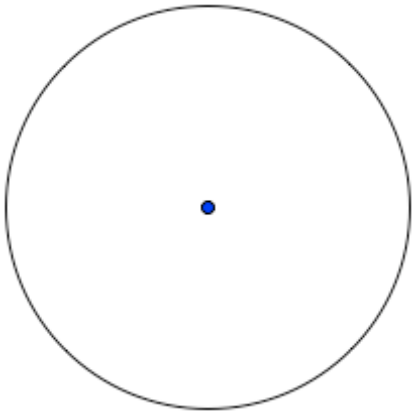
Circle Properties: Perpendicular Chord Theorem

Required equipment: Ruler and protractor.

Challenge #2:

A. Draw a chord: (A chord is a line with endpoints on the circle).
B. Bisect the chord: (Bisect means to cut in half).
C. Draw a radius through the bisection point.

D. Measure the angle. How big is it?_____
E. Draw another chord and repeat the process to confirm your finding.

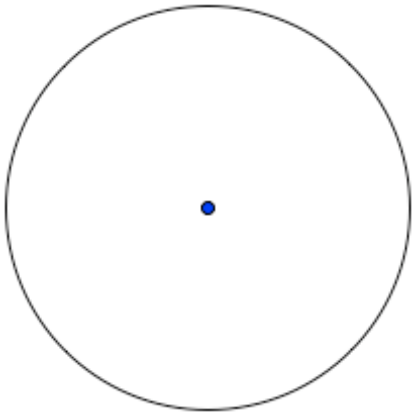


19. If a line goes through the center a circle and bisects a chord what angle do the chord and line meet at?

Challenge #3:

A. Draw a chord.
B. Draw a radius that is perpendicular to the chord.
C. Where does the radius intersect the chord?

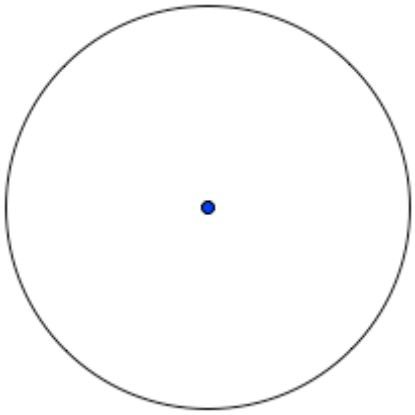
D. Repeat the process to confirm your finding.



20. If a line goes through the center of a circle and is perpendicular to a chord, what does the line do to the chord?

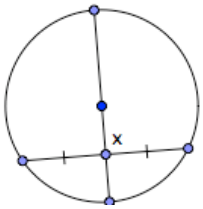
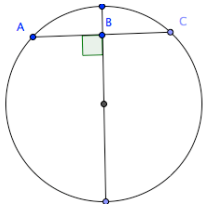
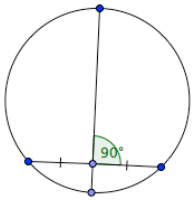
Challenge #4:

A. Draw two chords in the circle.
B. Bisect each chord.
C. Draw a perpendicular line through each chord and through the entire circle.
D. Where do the perpendicular chords intersect?

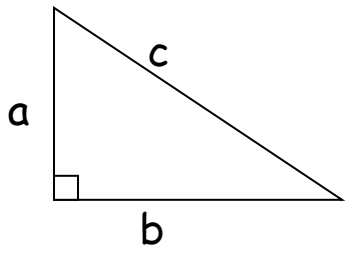


21. If a line is perpendicular to a chord and also bisects the chord, what else to you know about the line?

Circle Properties: Perpendicular Chord Theorem

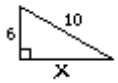
Perpendicular Chord Theorem		
If a line goes through the center and bisects a chord **then** the line is perpendicular to the chord.  x=90°	**If** a line goes through the center and is perpendicular to a chord **then** that chord is bisected.  AB=BC	The perpendicular bisector of any chord goes through center.  The center is somewhere on the vertical line.

The **Pythagorean Theorem**



$c^2 = a^2 + b^2$

22. Solve for x.



$c^2 = a^2 + b^2$

$10^2 = x^2 + 6^2$

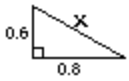
$100 = x^2 + 36$

$64 = x^2$

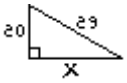
$\sqrt{64} = \sqrt{x^2}$

$8 = x$

23. Solve for x.

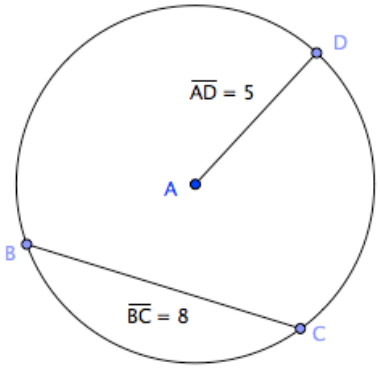


24. Solve for x.



Challenge #5:

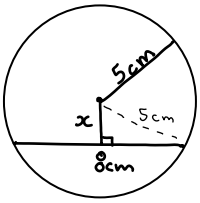
Determine the shortest distance between the center of the circle and the chord BC.



Notes:

Find the value of x.

25. Determine the length of x.



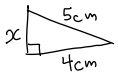
Possible solution strategy:
Move the radius so it forms a triangle with side lengths x, 5 &4(Since 8 is bisected by line x).
Solve using the Pythagorean theorem.

$x^2 + 4^2 = 5^2$

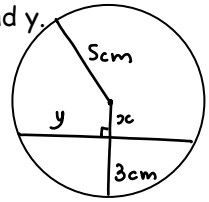
$x^2 + 16 = 25$

$x^2 = 9$

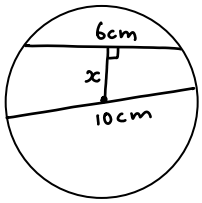
$x = 3$



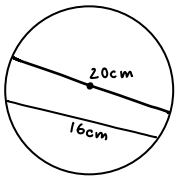
26. Determine the length of x and y.



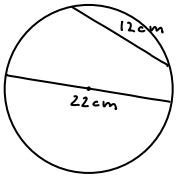
27. Determine the length of x.



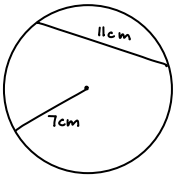
28. Find the shortest distance between the chord and the center of the circle.



29. Find the shortest distance between the chord and the center of the circle.

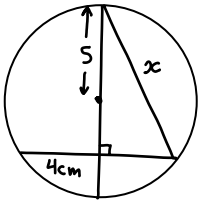


30. Find the shortest distance between the chord and the center of the circle.

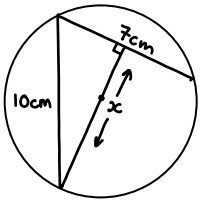


31. Omit. (

32. Find x.



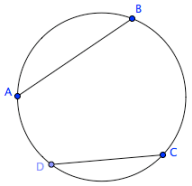
33. Find x.



34. A circle has diameter 20 cm. A chord is 8 cm long. How far from the center of the circle to the center of the chord?

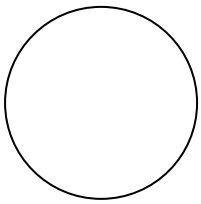
35. What is the diameter of a circle in which a chord 12 cm long is 7 cm from the center?

36. How could you use your knowledge of chords to find the center of the circle?



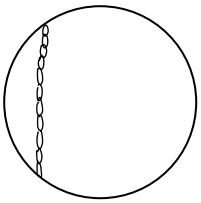
37. Let's just suppose you are walking through a large circular drainage pipe. The radius of the pipe is two meters. There is water running through the pipe. The width of the water line from side to side is 1.1m. How deep is the water if your head is not wet?

Draw a picture to help!

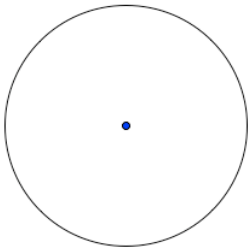


38. Vanillan is building a swimming pool in the shape of a circle with a 6m radius. The deep end will be separated from the shallow end by a 6m line. How wide will each section be if the shallow end is narrower than the deep end?

Draw a picture to help!
(The 6m line is drawn below.)

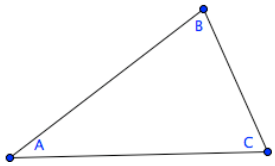


39. Explain the relationship between the center of a circle, a chord, and the perpendicular bisector of that chord. Draw a picture to support your answer.

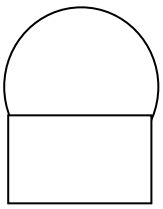


40. Look around your home or go online and provide an example that illustrates the perpendicular from the center of a circle to a chord bisects the chord.

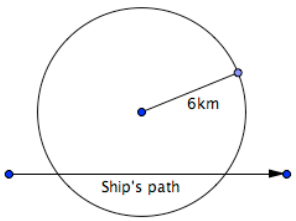
41. Explain how you could find the center of the circle that makes the triangle below an inscribed triangle. Draw a picture to support your answer.



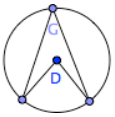
42. A ball with radius 10 cm rests on top of a box 16 cm wide and 12 cm tall. How far from the bottom of the box is the bottom of the ball?



43. A lighthouse keeper is able to see out into the sea about 6km with the help of light. A ship at its closest point was within 4km of the of the lighthouse. For how many kilometers was the ship within view of the lighthouse keeper? Round to one decimal.



Angles in a Circle: Inscribed & Central Angles

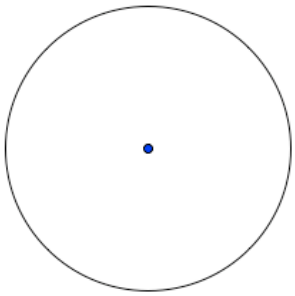


Basic definitions:
An **inscribed angle**(∠G), has a vertex on the circle and both endpoints on the circle.
A **Central angle**(∠D), has its vertex at the center of the circle.

Challenge #6:

A. Draw a single chord of any size.
B. Draw three inscribed angles coming off the same chord.
C. Measure each inscribed angle opposite the chord.
D. How big is each angle?

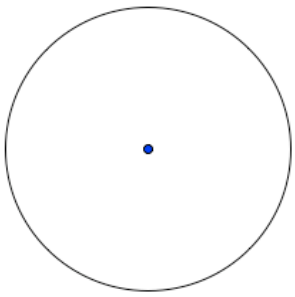
44. If two inscribed angles come from the same chord what can you say about the two angles?



Challenge #7:

A. Draw two equal chords.
B. Draw one inscribe angle off of each chord.
C. How big is each angle?

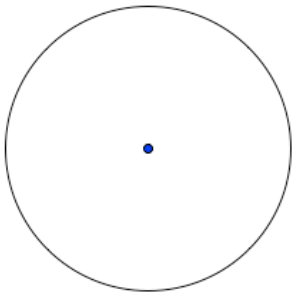
45. If two inscribed angles come from equal chords what can you say about the two angles?



Challenge #8:

A. Draw a single chord of any size.
B. Draw and measure one inscribed angle._____?
C. Draw and measure one central angle._____?

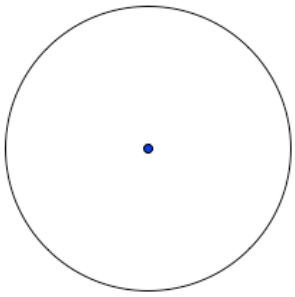
46. If the measure of a central angle is known, what do you know about an inscribed angle off of the same chord?



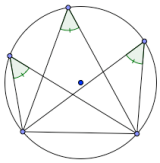
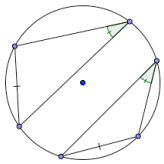
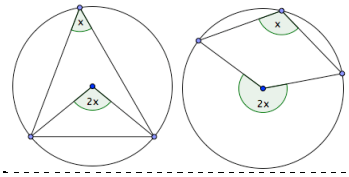
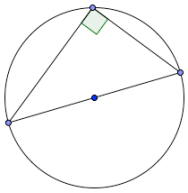
Challenge #9:

A. Draw a diameter.
B. Draw and measure three inscribed angles off of the diameter.
C. How big is each angle?

47. If an inscribed angle comes off the diameter, what do you know about the inscribed angle?

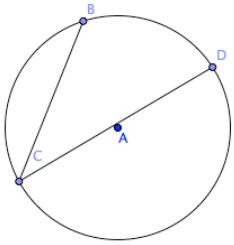


Angles in a Circle: Inscribed & Central Angles

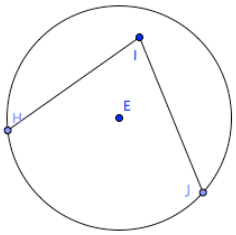
Incribed angles coming from the same chord or arc are equal.	Incribed angles coming from the same size chord or arc are equal.	Central angles coming from the same chord or equal chords are twice as big as inscribed angle.	Inscribed angles in a semi-circle equal 90°
			
Insc $\angle$'s from the same chord.	Insc $\angle$'s from equal chords.	Central angle = twice inscribed	Insc $\angle$ in semicircle.

State whether each angle is an inscribed angle, a central angle or neither.

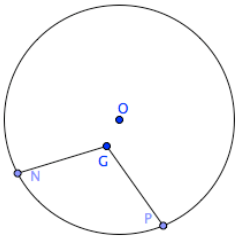
48. $\angle C$



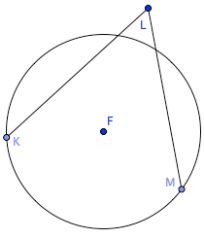
49. $\angle I$



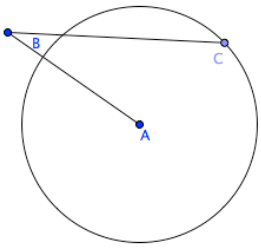
50. $\angle G$



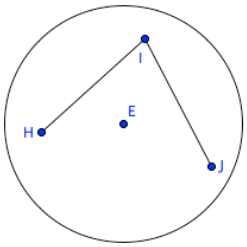
51. $\angle L$



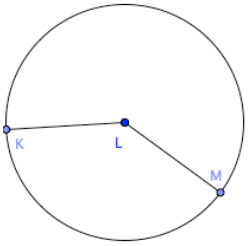
52. $\angle B$



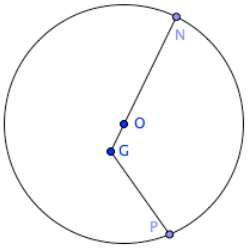
53. $\angle I$



54. $\angle L$

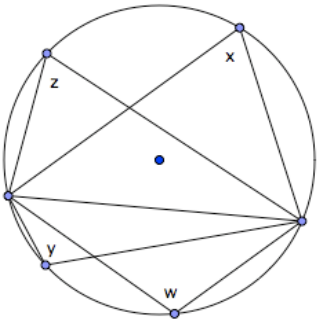


55. $\angle G$

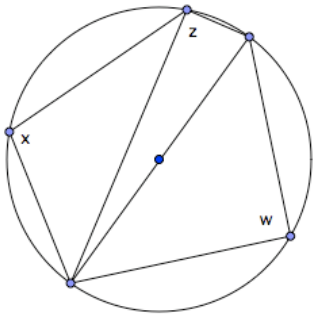


Answer each question

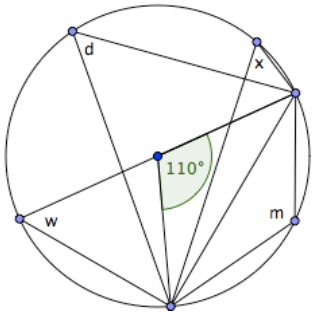
56. Which angles are equal?



57. Which angle(s) measure 90°?

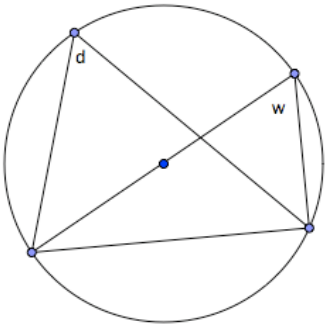


58. Which angle(s) measure 55°?

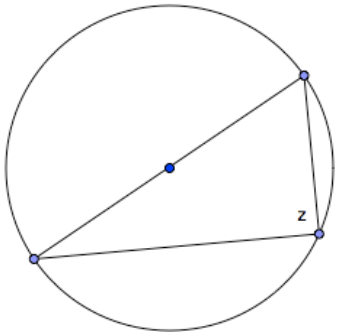


State the reason.

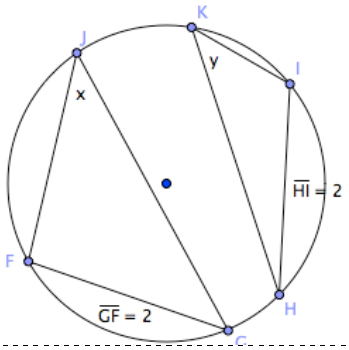
59. Why are angles d and w equal?



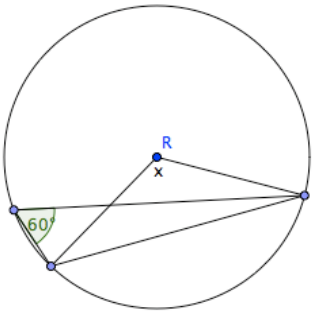
60. Why does angle z measure 90?



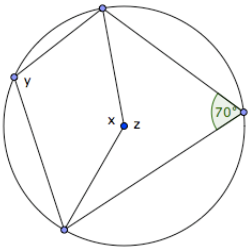
61. Why are angles x and y equal?



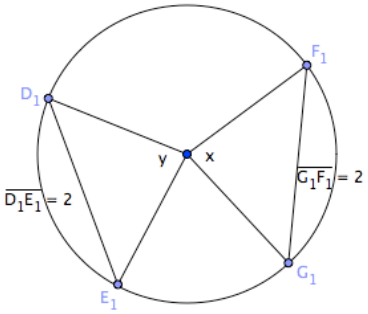
62. Why does angle x equal 120?



63. If y is the inscribed angle, which angle is the central angle off of the same chord?

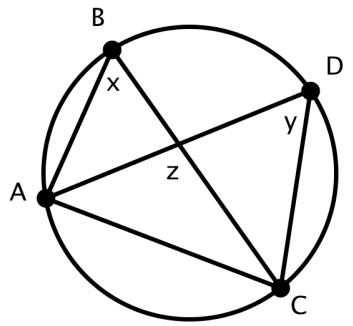


64. Why are angle x and angle y equal?

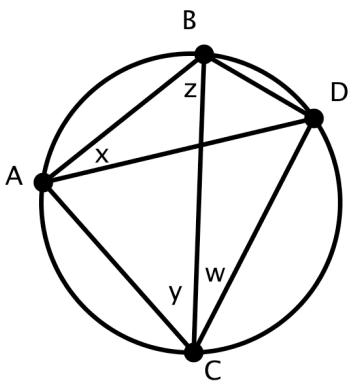


In each picture below, state the inscribed angles that are equal.* (Answers on this page.)

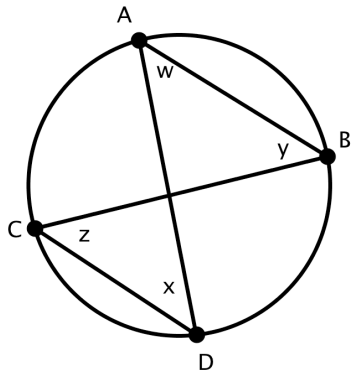
a.



b.

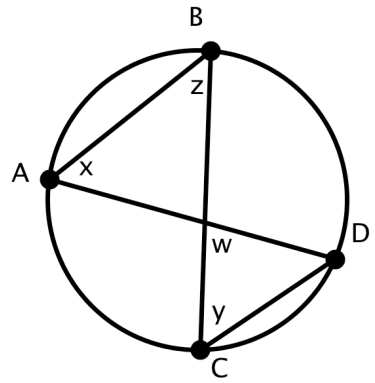


c.

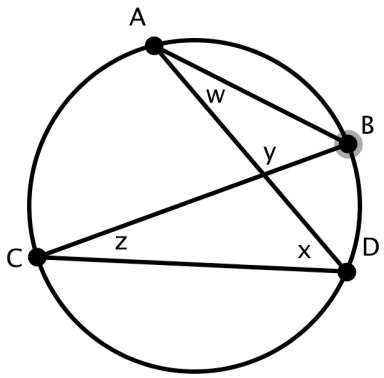


In each picture below, which two inscribed angles come from the same chord or arc?*

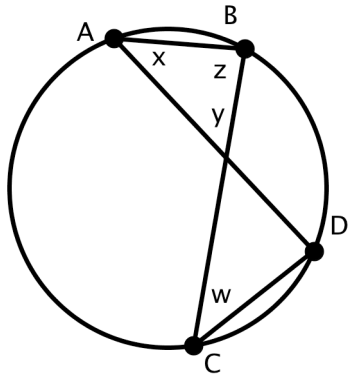
d.



e.

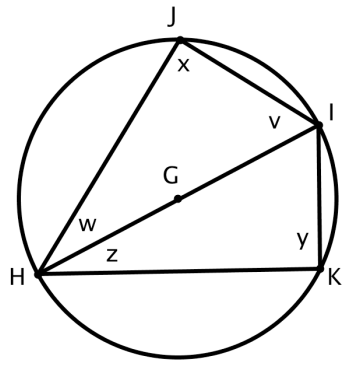


f.

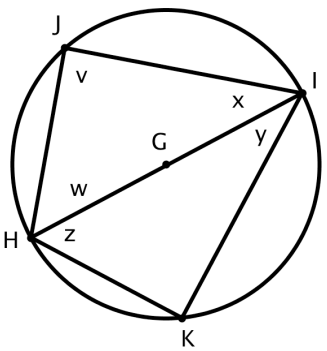


In each picture below, which angle(s) equal 90°?*

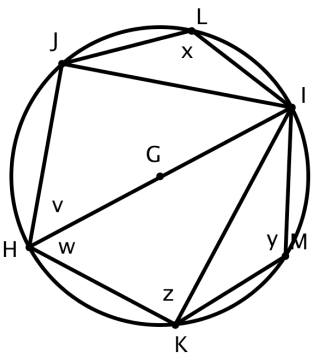
g.



h.



i.

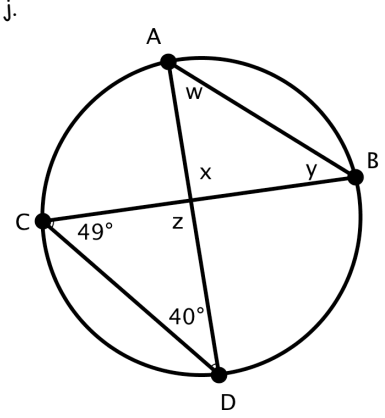


*Answers.

a.	b.	c.	d.	e.	f.	g.	h.	i.
X,Y	X,W	W,Z & YX	X,Y	W,Z	W,X	X,Y	V	Z

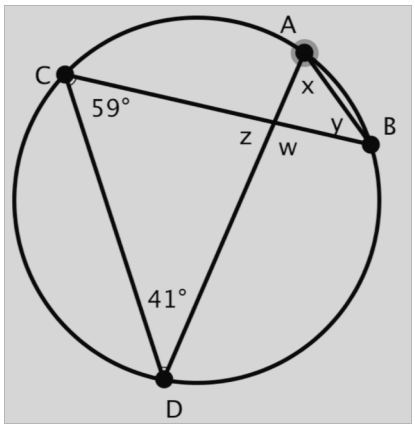
Determine the measure of each angle.* (Answers on this page.)

j.



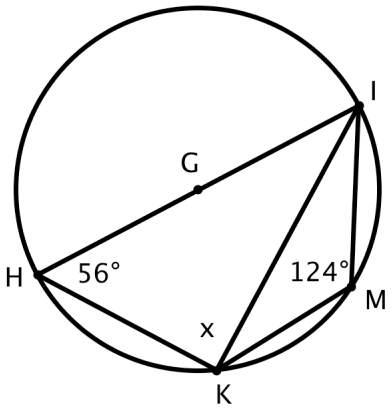
$\angle Y =$

k.



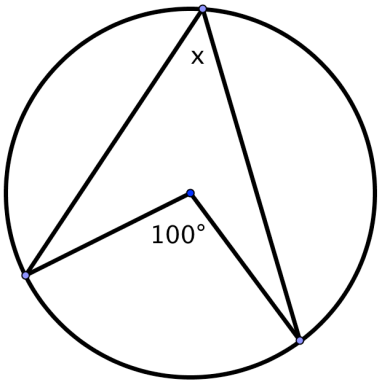
$\angle X =$

l.



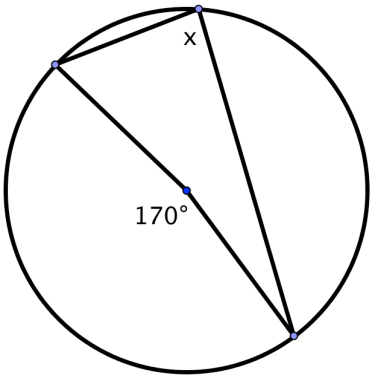
$\angle X =$

m.



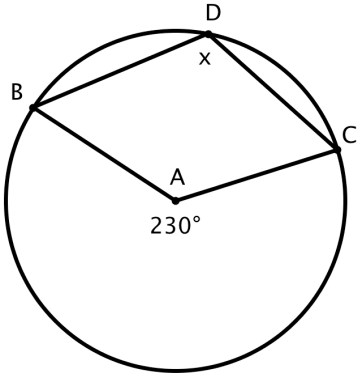
$\angle X =$

n.



$\angle X =$

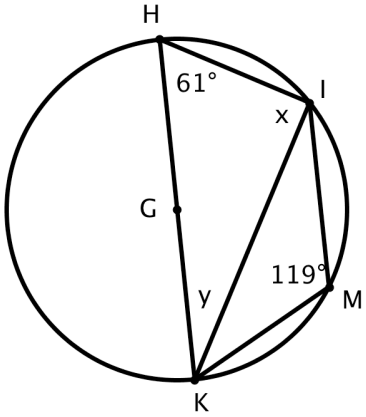
o.



$\angle X =$

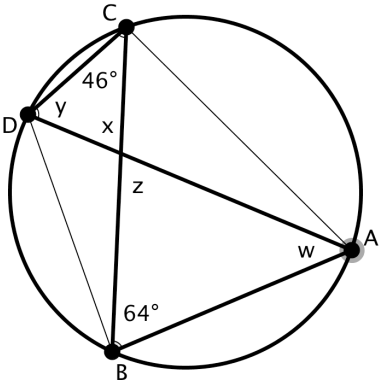
p. Omit

q.



$\angle X =$ $\angle Y =$

r.



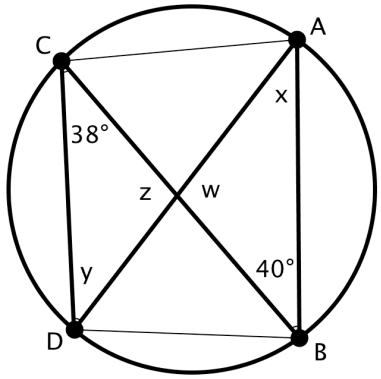
$\angle X =$ $\angle Y =$

*Answers.

j.	k.	l.	m.	n.	o.	p.	q.	r.
40	59	90	50	85	115		90, 29	70,64

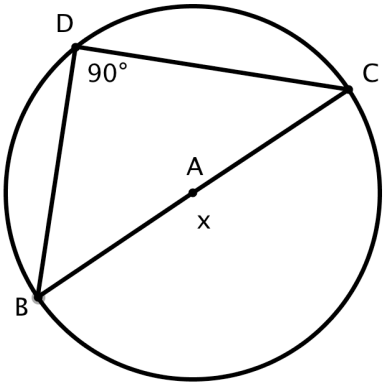
Determine the measure of each angle.* (Answers on this page.)

s.



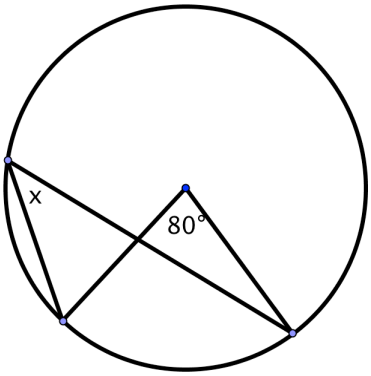
$\angle X =$ $\angle Y =$

t.



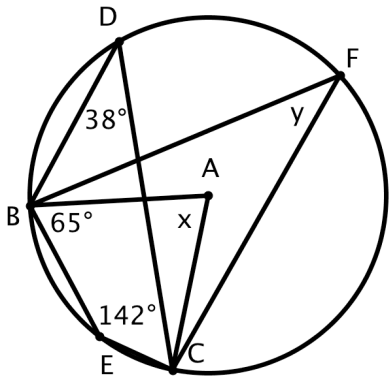
$\angle X =$

u.



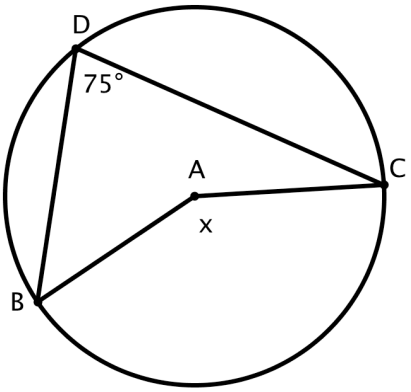
$\angle X =$

v.



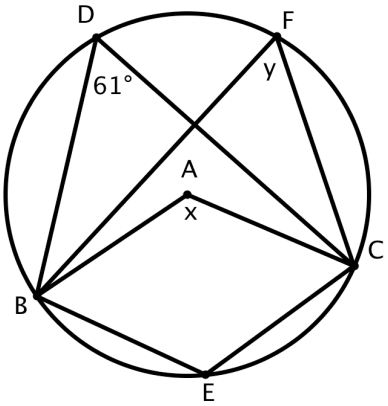
$\angle X =$ $\angle Y =$

w.



$\angle X =$

x.



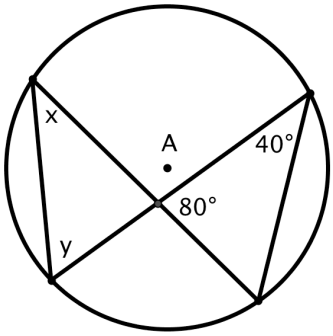
$\angle X =$ $\angle Y =$

*Answers.

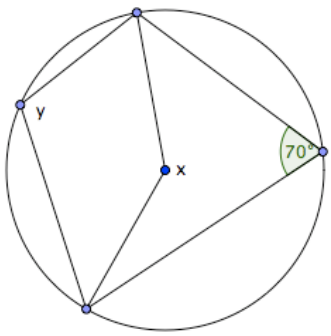
s.	t.	u.	v.	w.	x.
38, 40	180	40	76, 38	150	122,61

Challenge #10:

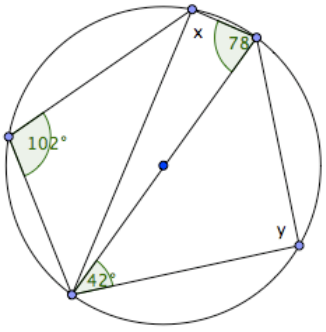
65. Find x and y.



66. Find x and y.

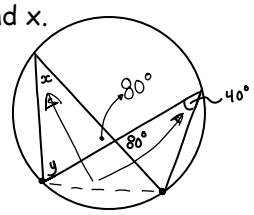


67. Find x and y.



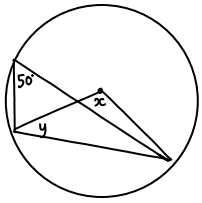
Find the missing value(s).

68. Find x.



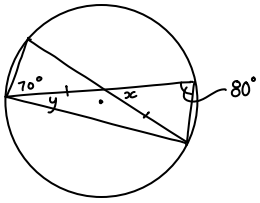
X=40° because ∠x is coming from the same chord as an ∠ measuring 40°.
Y=60° because ∠ in a triangle are supplementary.

69.



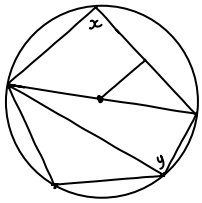
X= y=

70.



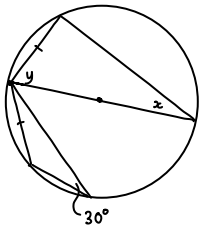
X= y=

71.



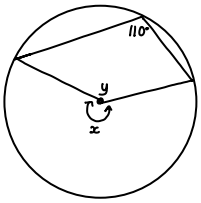
x= y=

72.



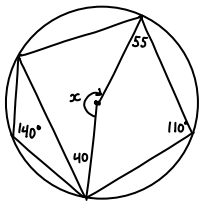
x= y=

73.



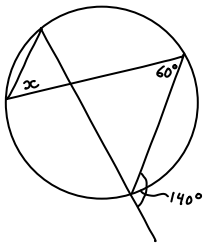
x= y=

74.



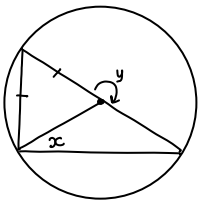
X=

75.



x=

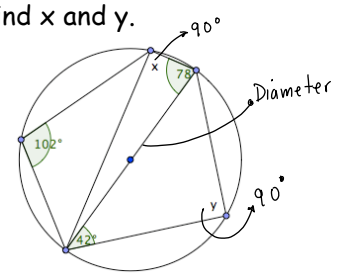
76.



x= y=

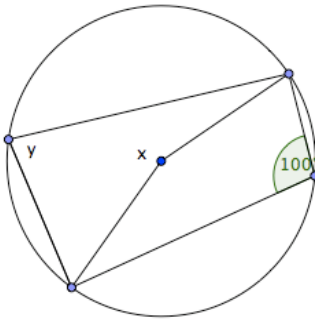
Find the angle measure of the missing angles.

77. Find x and y.



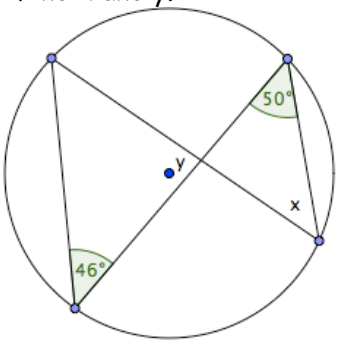
∠x=90° & ∠y=90° because inscribed ∠s in a semicircle =90°.

78. Find x and y.



X= y=

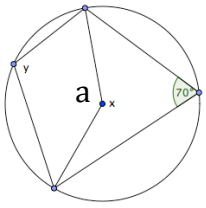
79. Find x and y.



X= y=

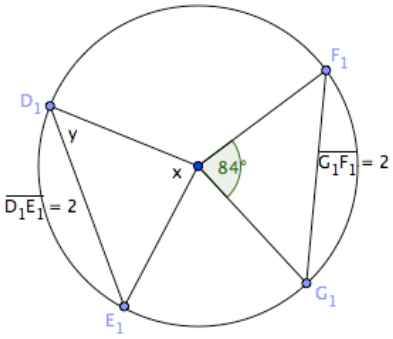
Find the angle measure of the missing angles.

80. Find x and y.



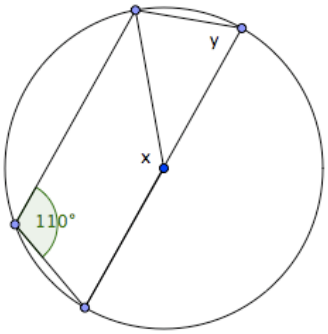
∠a =140° since central∠= twice insc∠.
∠x=220° since ∠a + ∠x = 360°.
∠y=110° since insc ∠ is half central∠.

81. Find x and y.



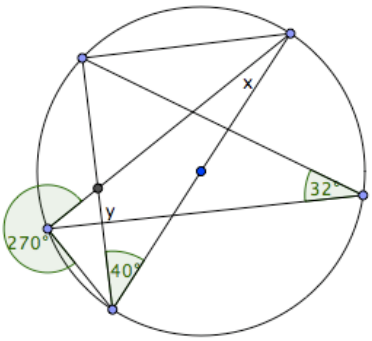
X= Y=

82. Find x and y.



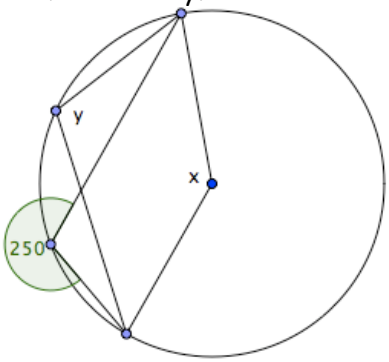
X= Y=

83. Find x.



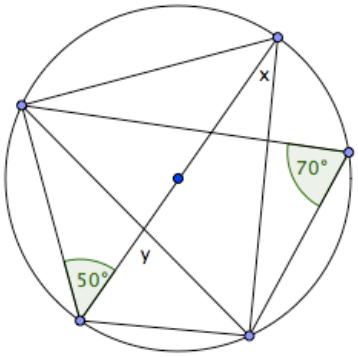
X=

84. Find x and y.



X= Y=

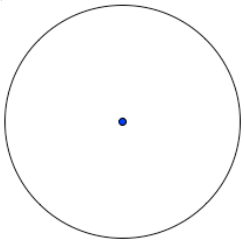
85. Find x and y.



X= Y=

Draw a picture to help explain your answer.

86. Describe how to draw a perfect rectangle without using a protractor. All angles must equal 90°.

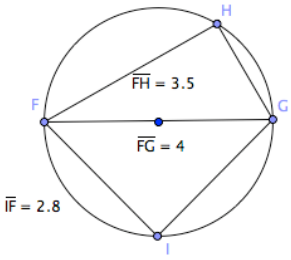


87. Go on line and provide an example that illustrates that inscribed angles coming from the same arc are congruent.

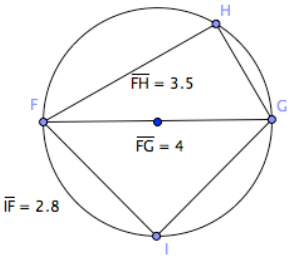
88. Look around your home or go online and provide an example that illustrates the measure of the central angle is equal to twice the measure of the inscribed angle coming from the same arc.

Determine the missing side lengths and round your answer to one decimal.

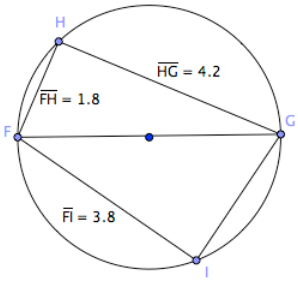
89. Determine the length of GI.



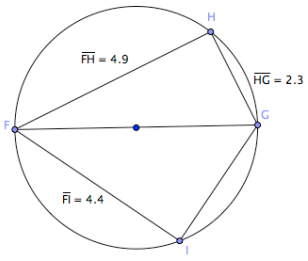
90. Determine the length of HG.



91. Determine the length of GI.

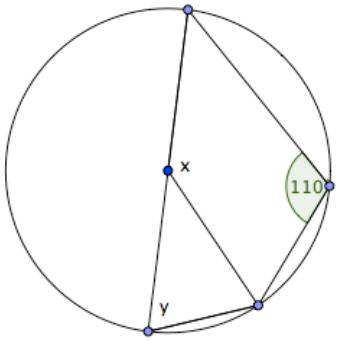


92. Determine the length of GI.



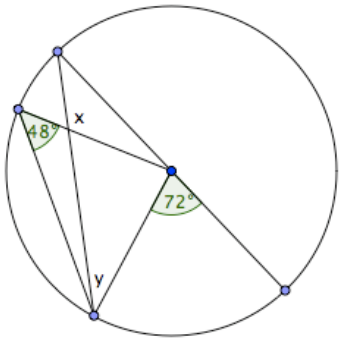
Find the angle measure of the missing angles.

93. Find x and y.



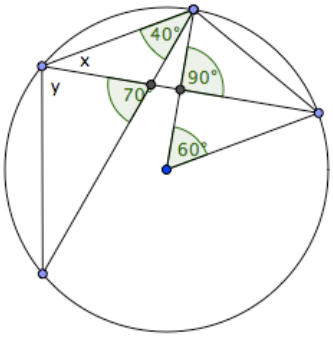
X= Y=

94. Find x and y.



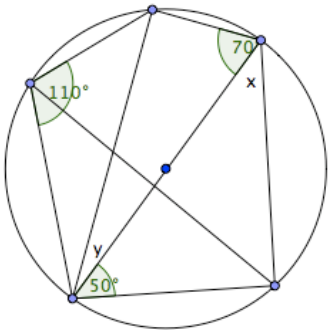
X= Y=

95. Find x and y.



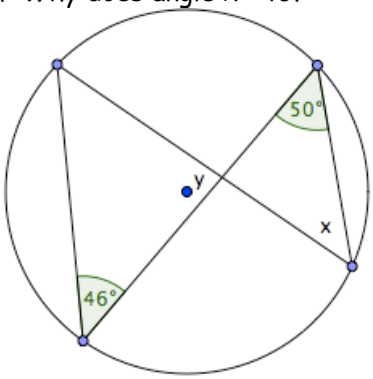
X= Y=

96. Find x and y.

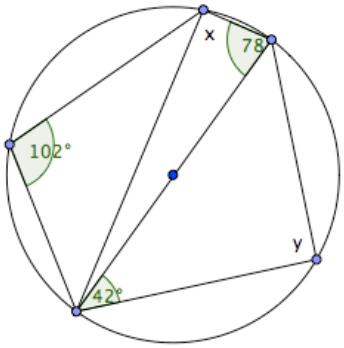


X= Y=

97. Why does angle x =46?



98. Why does angle x =90?

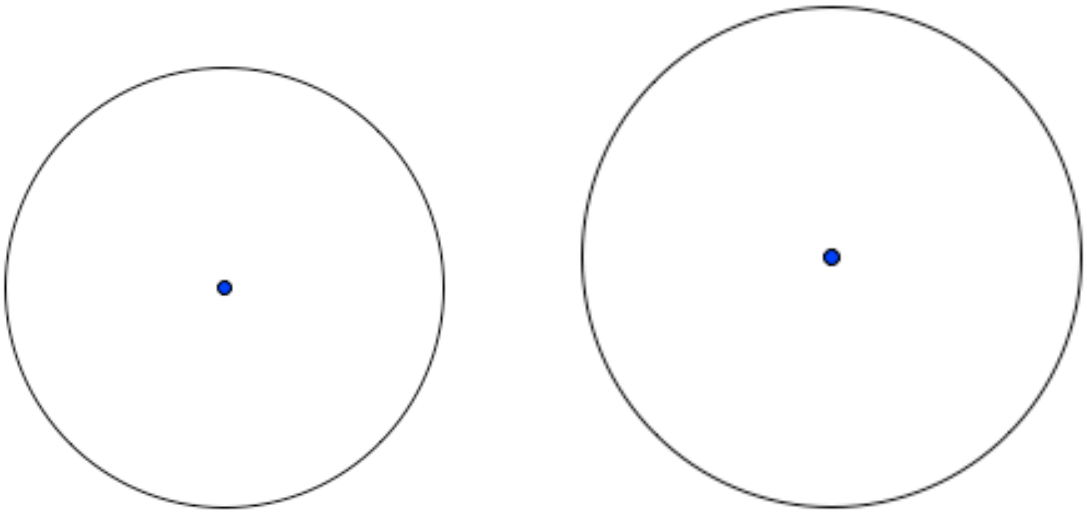


Inscribed Quadrilateral Properties

Basic definitions:
Inscribed Quadrilateral: A four-sided polygon whose vertices all touch the circumference of a circle.
Challenge #11:

- Draw four points on the circumference of each circle.
- Connect the four points to form two inscribed quadrilaterals.
- Record the angle measure of each angle.

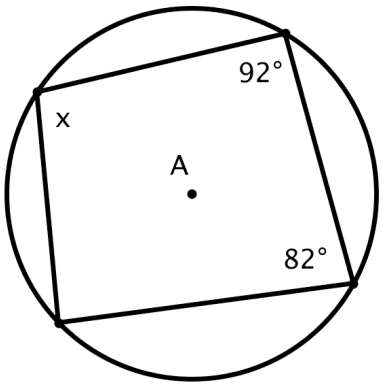
99. What relationship(s) exist between opposite angles in inscribed quadrilaterals?



100. If you know one angle in an inscribed Quadrilateral, what else do you know?

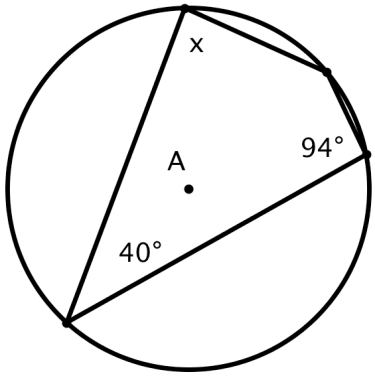
Answers are on the next page.

a.



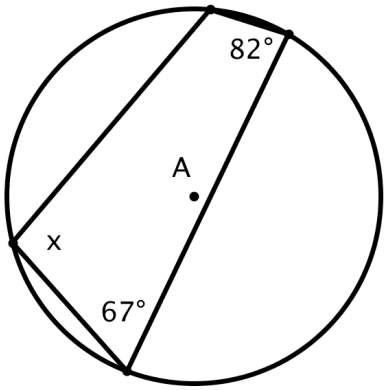
$\angle X =$

b.



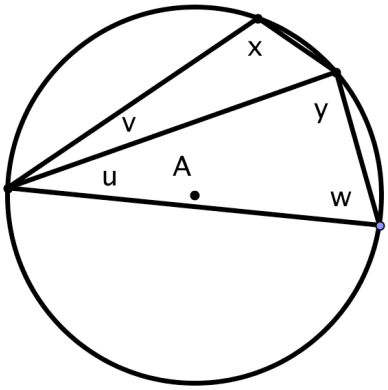
$\angle X =$

c.

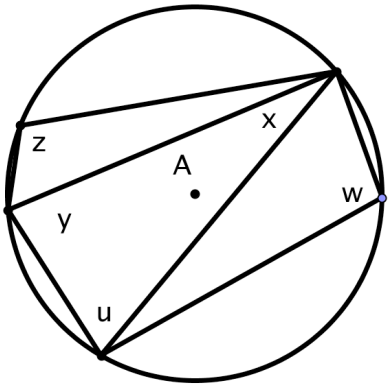


$\angle X =$

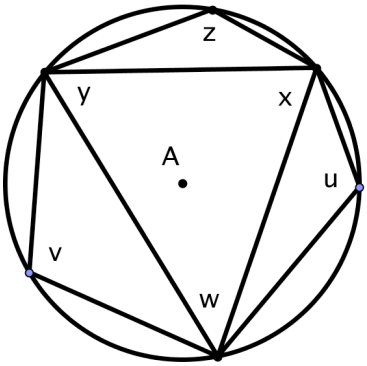
d. Which pairs of angles add to 180°



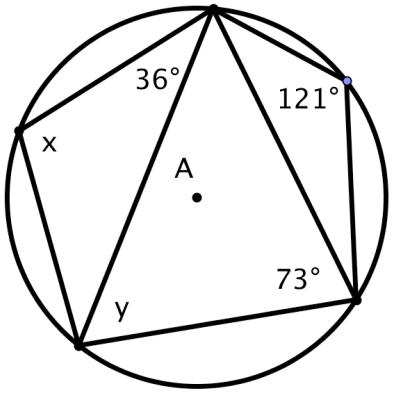
e. Which pairs of angles add to 180°



f. Which pairs of angles add to 180°

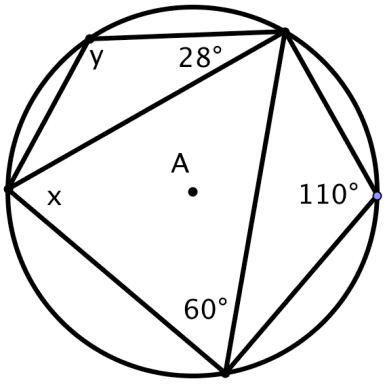


g.



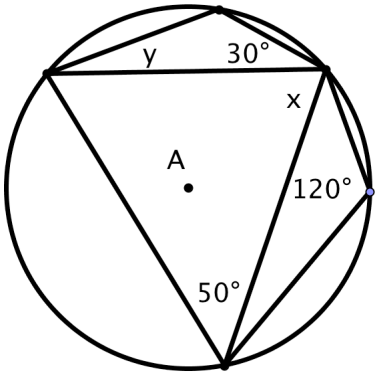
$\angle X =$ $\angle Y =$

h.



$\angle X =$ $\angle Y =$

i.



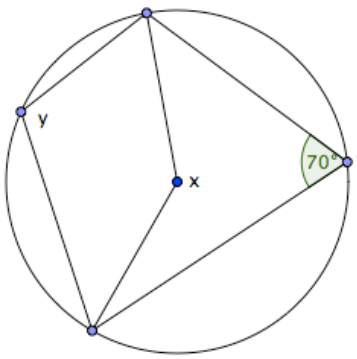
$\angle X =$ $\angle Y =$

*Answers to the questions on page 22 and 23.

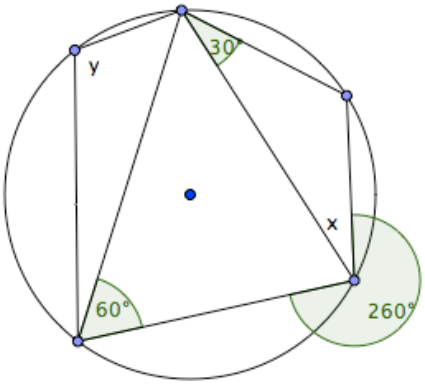
a.	b.	c.	d.	e.	f.	g.	h.	i.
98	86	98	W,X	W,Y & U,Z	X,V & W,Z & U,Y	107, 59	70, 120	70, 20

Challenge #12: Answers to 101 and 102 follow at #110 &#113

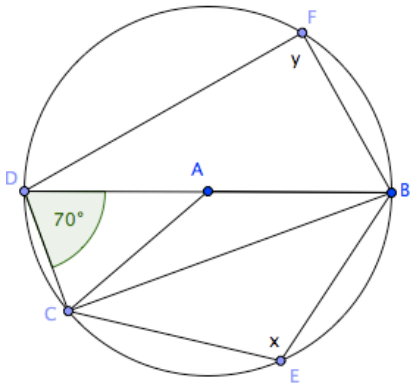
101. Find x and y.



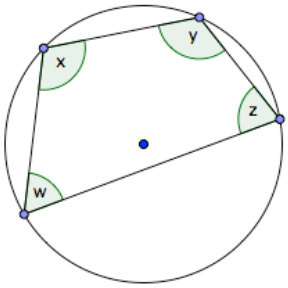
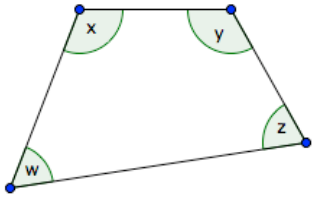
102. Find x and y.



103. Find x and y.

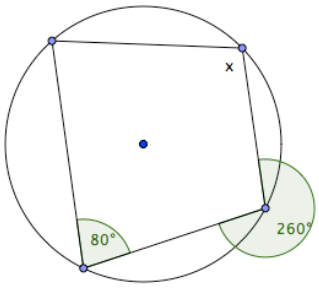


Inscribed Quadrilateral Properties

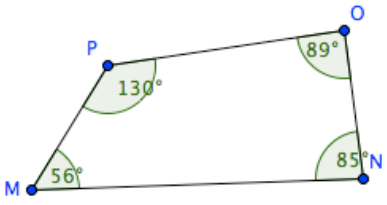
Opposite Angles in Inscribed Quadrilaterals are supplementary	**If** Opposite sides of a quad are supplementary **then** it is a inscribed Quadrilateral.
	
If WXYZ is an inscribed quadrilateral then $\angle X + \angle Z = 180°$ $\angle W + \angle Y = 180°$	If $\angle X + \angle Z = 180°$ then WXYZ is an inscribed quadrilateral. $\angle W + \angle Y = 180°$

State your reasons.

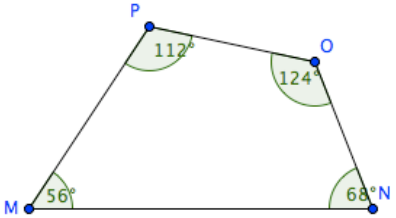
104. Why does angle x = 100?



105. Is MNOP an inscribed quadrilateral? Why?

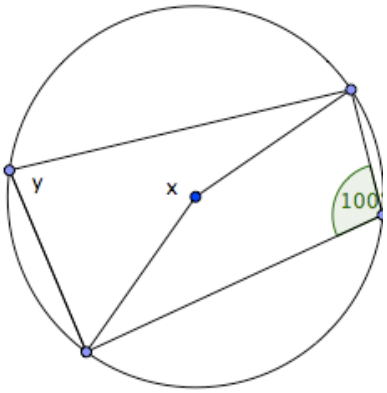


106. Is MNOP an inscribed quadrilateral? Why?



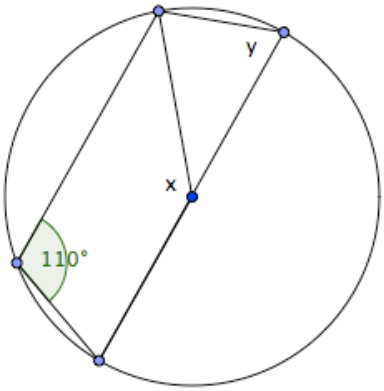
Determine the angle measure of the missing angle.

107. Find x and y.



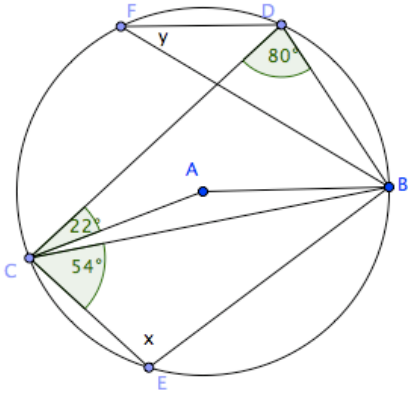
X= y=

108. Find x and y.



X= y=

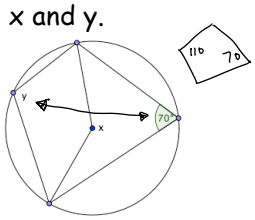
109. Find x and y.



X= y=

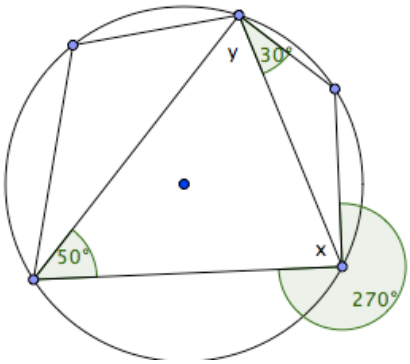
Determine the angle measure of the missing angle.

110. Find x and y.



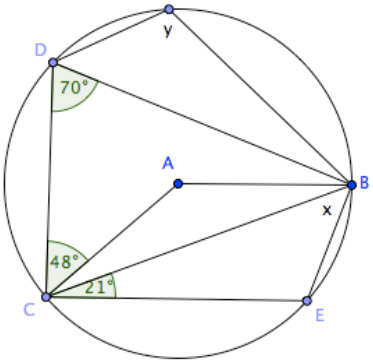
∠y=110° since opposite ∠s in an inscribed quadrilateral are supplementary.
∠x=220° since x is a central ∠ and therefore twice the insc∠y.

111. Find x and y.



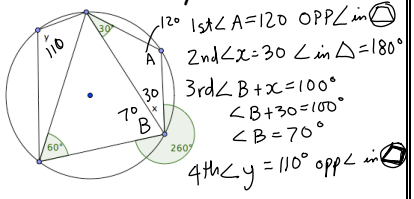
X= Y=

112. Find x and y.



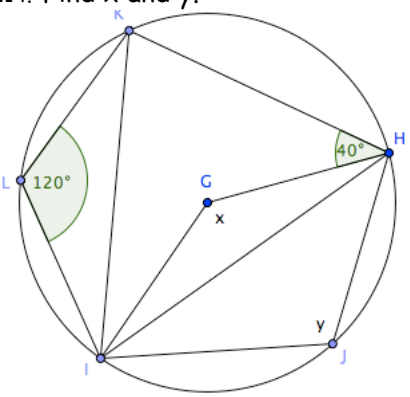
X= Y=

113. Find x and y.



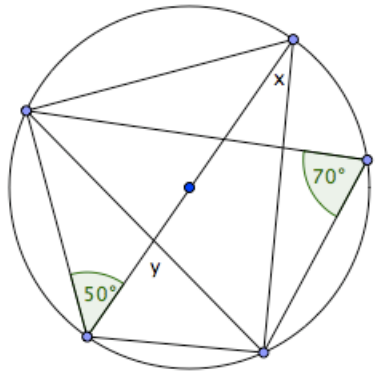
∠x=30° & ∠y=110°

114. Find x and y.



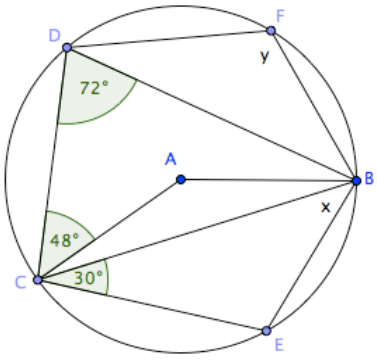
X= Y=

115. Find x and y.



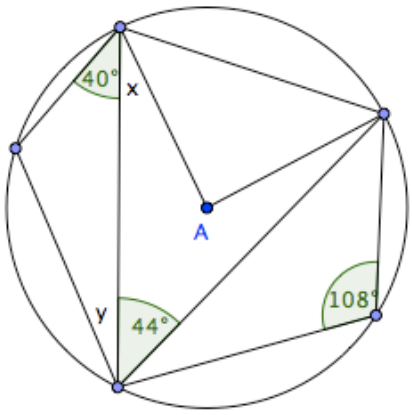
X= Y=

116. Find x and y.



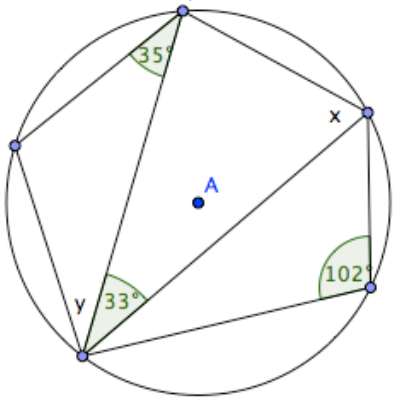
X= Y=

117. Find x and y.



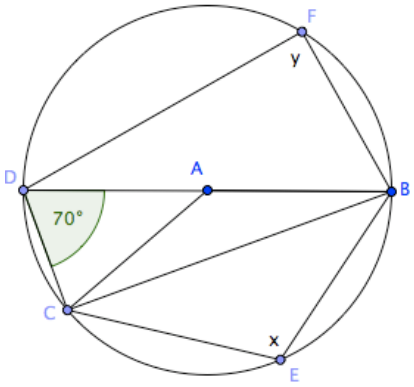
X= Y=

118. Find x and y.



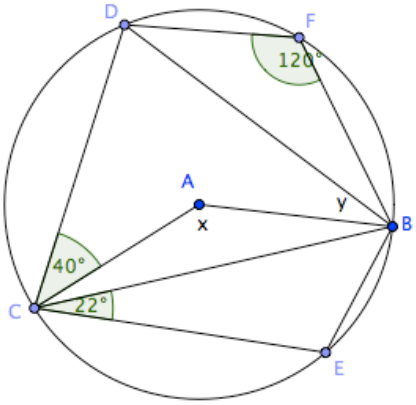
X= Y=

119. Find x and y.



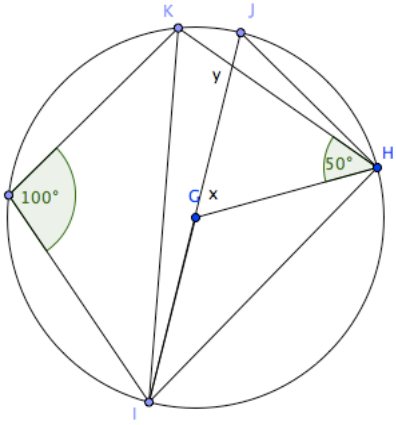
X= Y=

120. Find x and y.



X= Y=

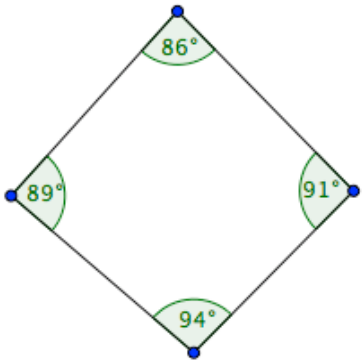
121. Find x and y.



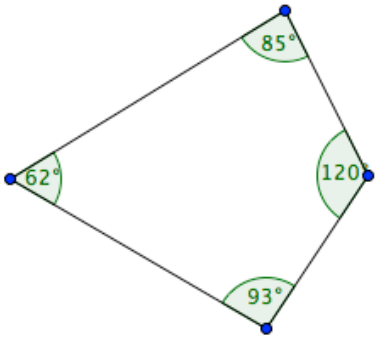
X= Y=

122. Which of the following quadrilaterals could be inscribed in a circle?

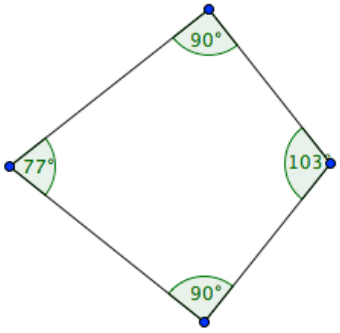
A.



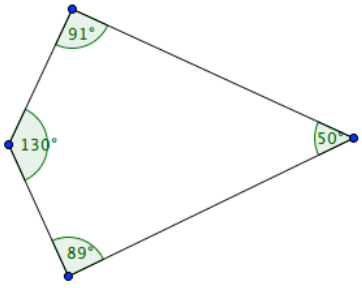
B.



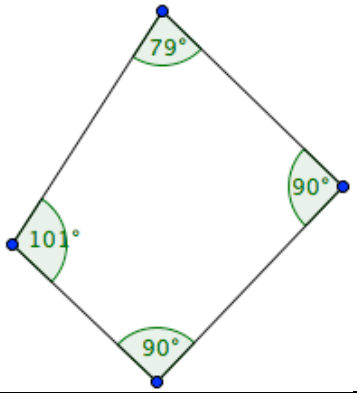
C.



D.



E.



123. For those that could be inscribed in a circle, explain how you could use circle properties to find the center of the circle that encloses the quadrilateral.

Tangent Properties

Definition:

A **tangent line** is a line that intersects a circle in just one place.

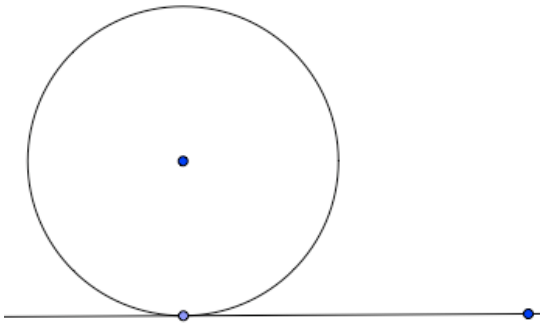
A **secant line** is a line that intersects a circle in exactly 2 places.

- Required equipment: Ruler and protractor.

Challenge #13:

A. Connect the radius and point of tangency.
B. Measure the angle.
C. How big is it?_____

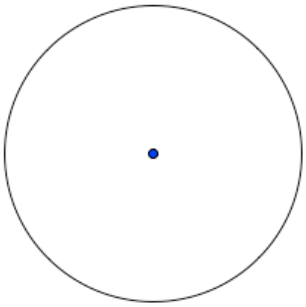
124. If a radius meets a tangent line at the point of tangency, what do you know about the angle created?



Challenge #14:

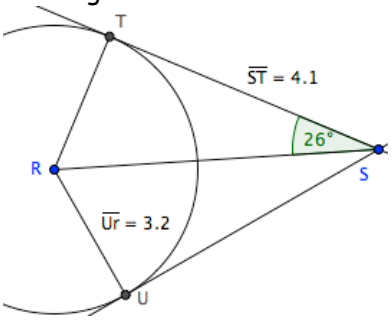
A. Draw a point outside the circle.
B. Draw two tangent lines from that point to the circle.
C. Measure each tangent line to the point of tangency. What do you notice?

125. If two tangent lines meet at an external point, what do you know?

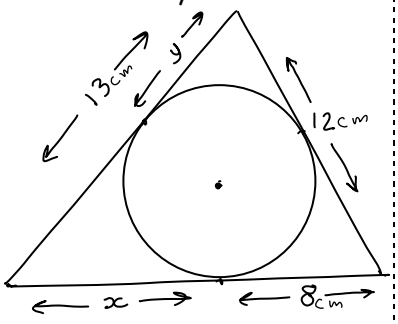


Challenge #15:

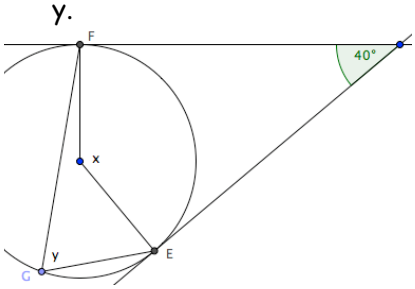
126. U and T are points of tangency. Determine the length of RS and ∠RTS.



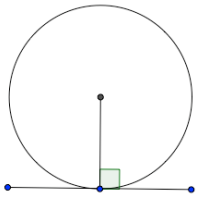
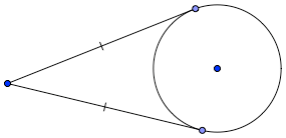
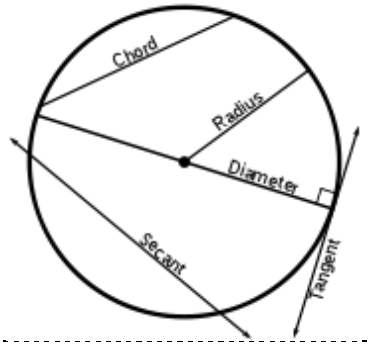
127. Find x and y



128. F and E are points of tangency. Determine x and y.

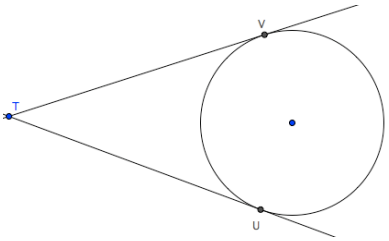


Tangent Properties

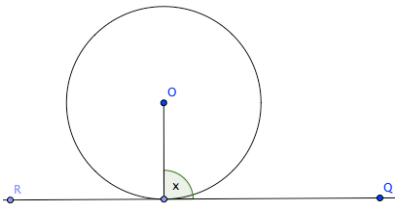
A tangent is perpendicular to the radius at the point of tangency 	Tangents from an external point are equal. 	
Tangent ⊥ to radius	Tangents from ext. point are =	

Complete the questions.

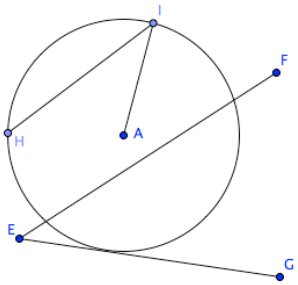
129. U and V are points of tangency. Why are line segment TV and line segment TU equal?



130. O is the center and line RQ is a tangent line. Why does angle x equal 90?



131. Match each line with its name.



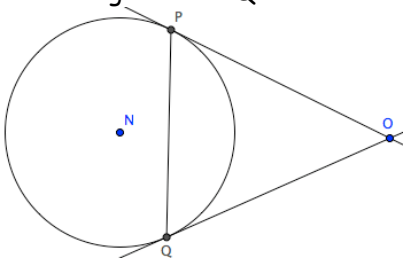
Secant line:

Radius:

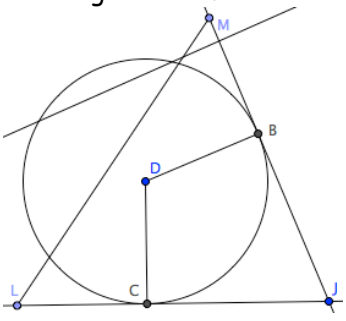
Chord:

Tangent line:

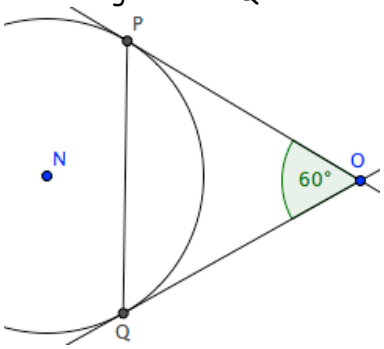
132. P and Q are points of tangency. What kind of triangle is ΔPOQ?



133. B and C are points of tangency. Name the tangent lines.

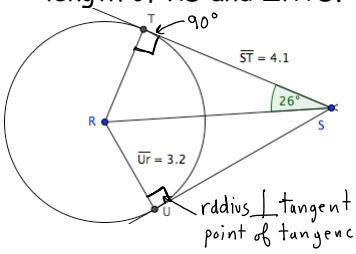


134. P and Q are points of tangency. What kind of triangle is ΔPOQ?



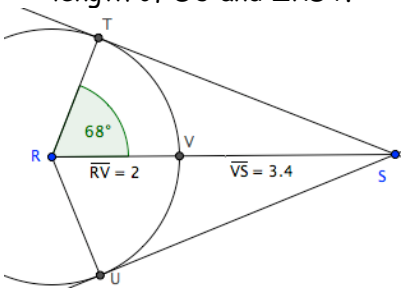
Determine the missing angles and lengths.

135. U and T are points of tangency. Determine the length of RS and ∠RTS.

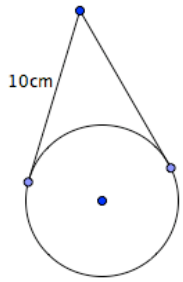


- ∠RTS = 90° radius ⊥ tangent at point of tangency.
- RS → 3.2, 4.1 $(3.2)^2+(4.1)^2=(RS)^2$ $27.05=(RS)^2$
- RS = 5.2

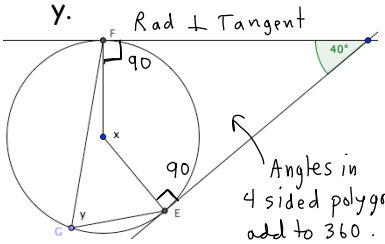
136. U and T are points of tangency. Determine the length of SU and ∠RST.



137. Determine the distance between the point and the center of the circle with radius 5cm.



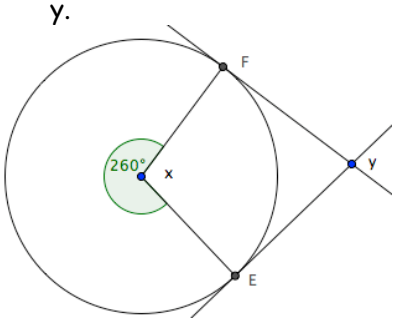
138. F and E are points of tangency. Determine x and y.



∠x = 360 − 90 − 90 − 40 = 140°
∠y = 70° Inscribed∠ = 2 × central∠.

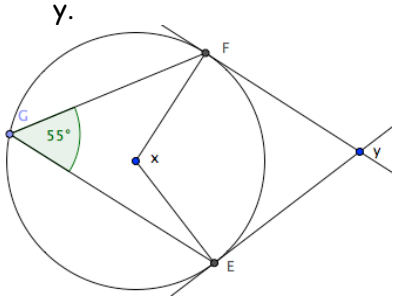
x= 140° y= 70°

139. F and E are points of tangency. Determine x and y.



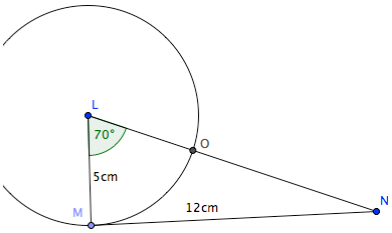
x= y=

140. F and E are points of tangency. Determine x and y.



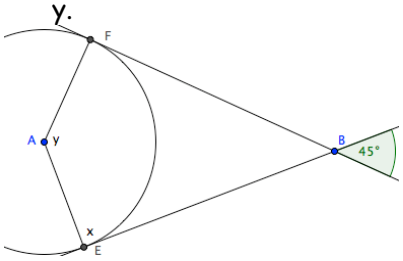
x= y=

141. M is the point of tangency. Determine the length of line segment ON and the size of ∠MNL.



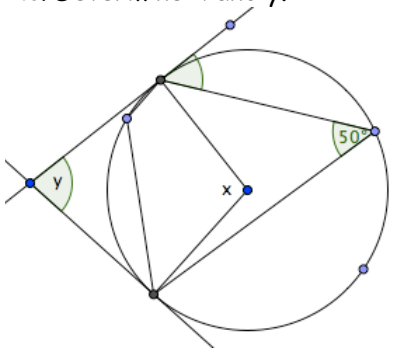
ON= ∠MNL =

142. E and F are points of tangency. Determine x and y.



x= y=

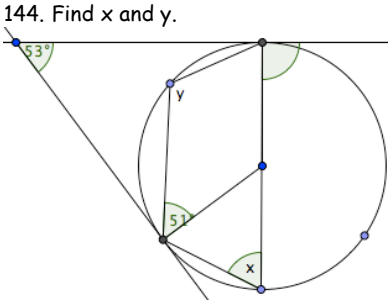
143. Determine x and y.



x= y=

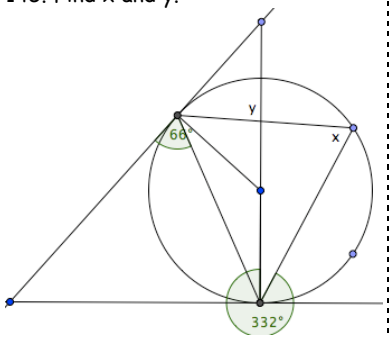
Determine the missing angles and lengths.

144. Find x and y.



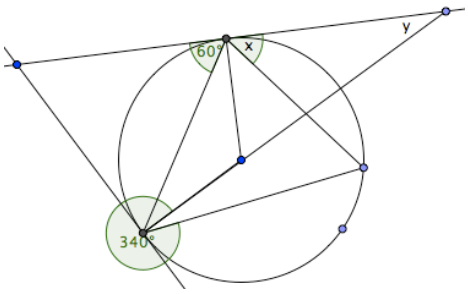
X= Y=

145. Find x and y.



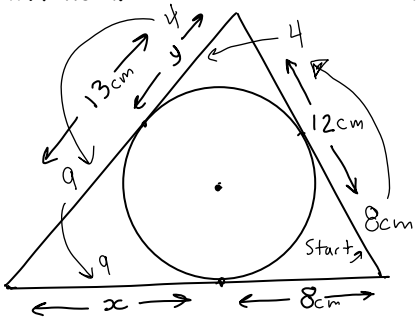
X= Y=

146. Find x and y.



X= Y=

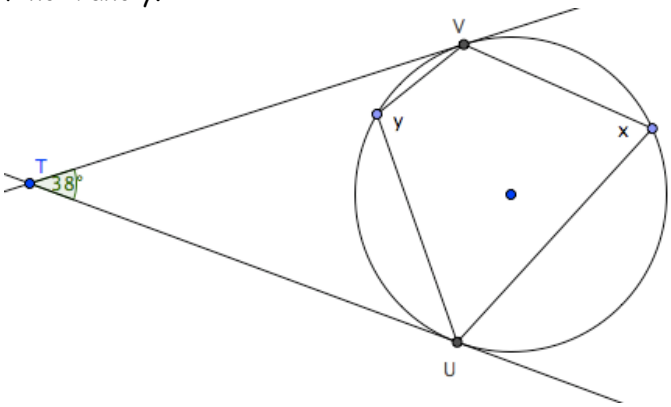
147. Find x.



Key Idea: Tangents to external point are equal in length.

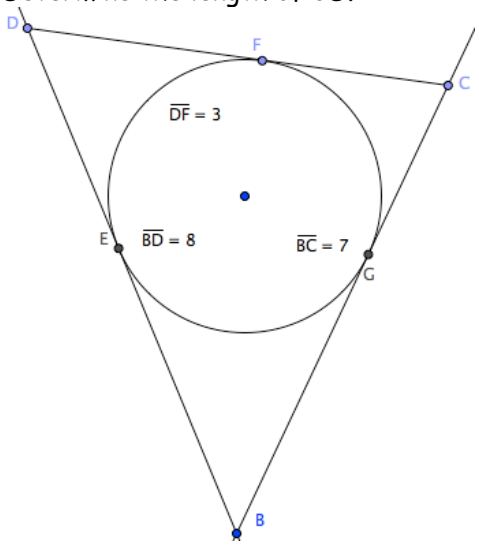
X= 9cm y=4cm

148. Find x and y.

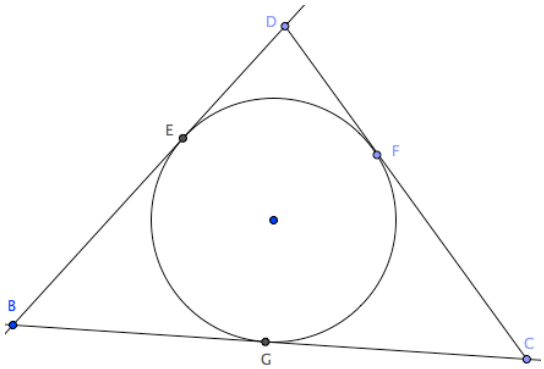


X= Y=

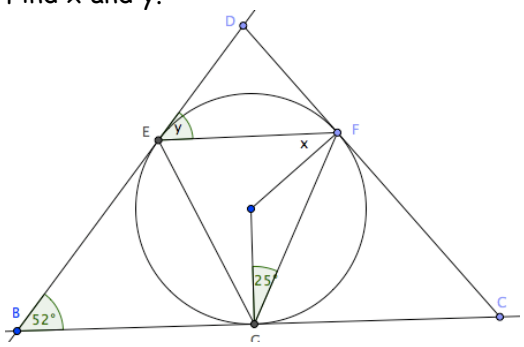
149. Determine the length of CD.



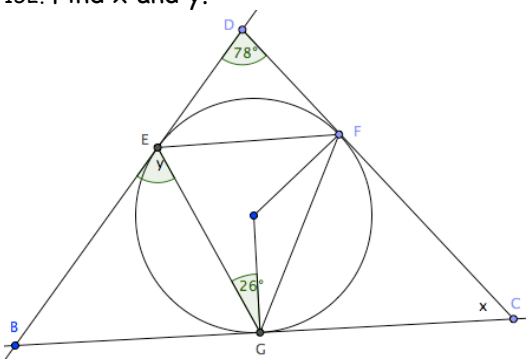
150. The length of BE is twice as long as DE. The length of CG is 2 less than BE. Determine the length of BD if the perimeter of triangle BCD is 26cm.



151. Find x and y.

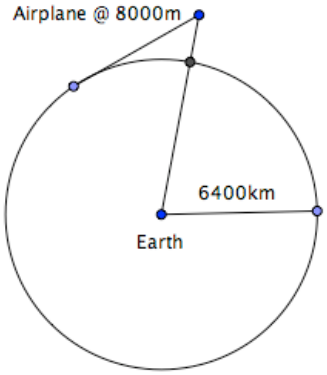


152. Find x and y.

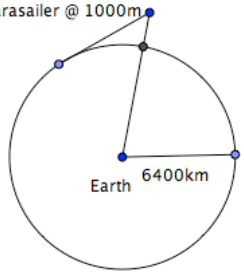


Solve these problems. (The drawings in #153 and #154 are not to scale.)

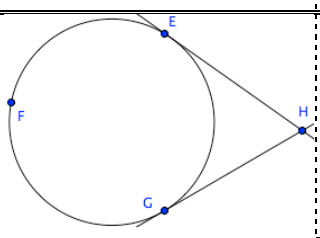
153. Flize Alut loves travelling. As he looks out through the airplane window, he wonders how far he can see out toward the horizon. The captain announces that they are flying at 8km above the earth. He knows that the approximate radius of the earth is 6400km. How far away is the horizon?



154. Kibben likes to parasail. This involves jumping off mountain cliffs with a large sail on your back and then flying. During his last flight he determined that he was 1km above the ground and wondered how far into the horizon he could see. Use the diagram to help him answer this question.



155. How could you use your knowledge of tangent lines to find the center of the circle.

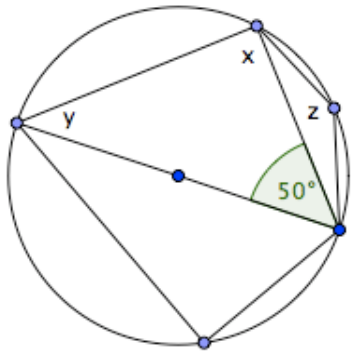


156. Go on line and find an application of the circle property, a tangent to a circle is perpendicular to the radius at the point of tangency.

Extra Practice

Find the missing angles in alphabetical order and state the geometric reasons.

157.

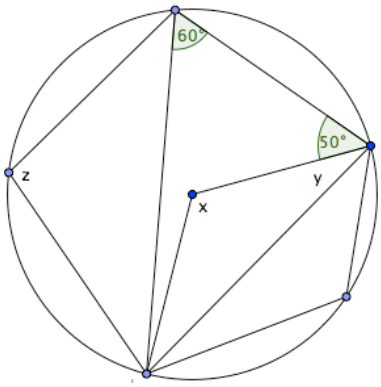


X=

Y=

Z=

158.

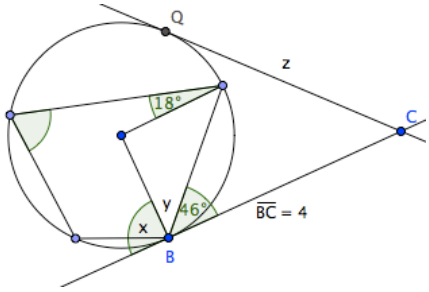


X=

Y=

Z=

159.



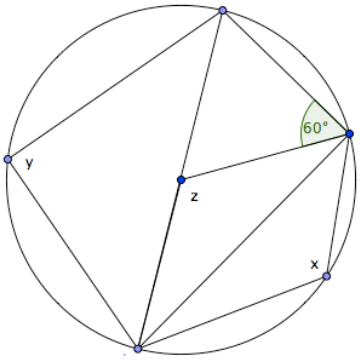
X=

Y=

Z=

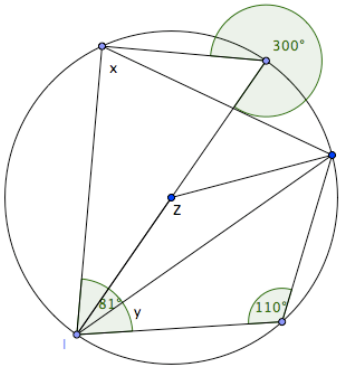
Determine the measure of the unknown angles.

160. Find x, y & z.



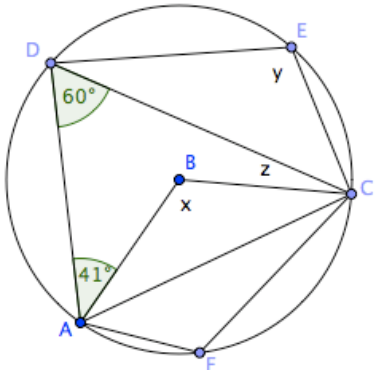
X= Y= Z=

161. Find x,y & z.



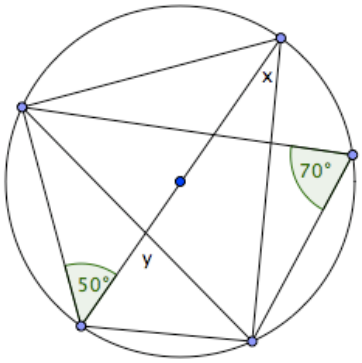
X= Y= Z=

162. Find x,y & z.



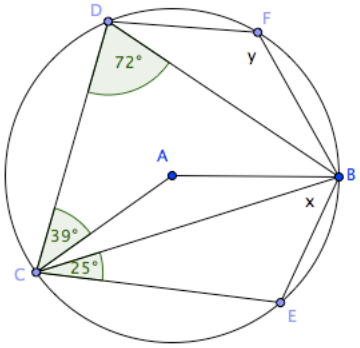
X= Y= Z=

163. Find x and y.



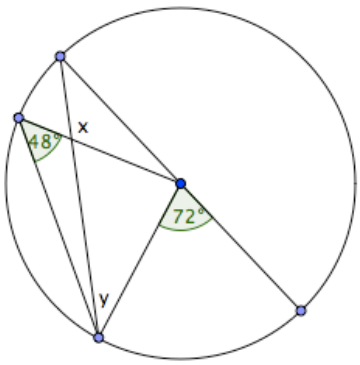
X= Y=

164. Find x and y.



X= Y=

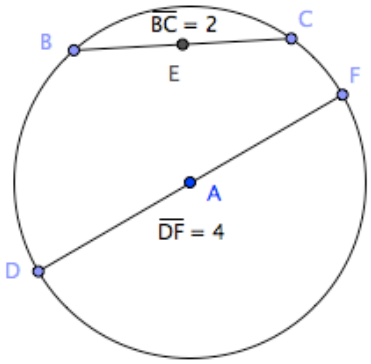
165. Find x and y.



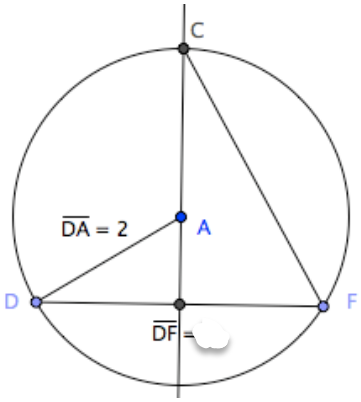
X= Y=

Determine the measure of the unknown angles and side lengths.

166. Determine the shortest distance between line BC and the center or the circle.

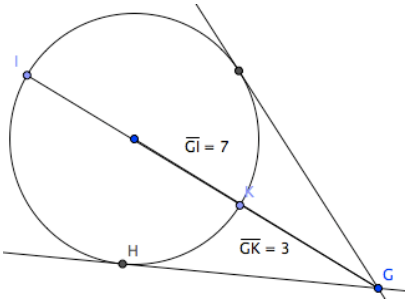


167. Determine the length of CF to the nearest tenth.

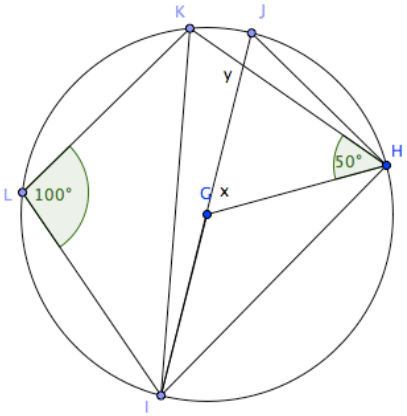


DF=3.4.

168. Determine the length of GH to the nearest tenth.

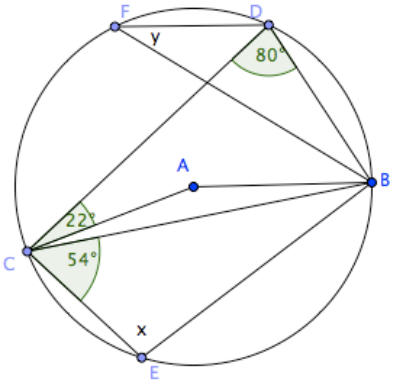


169. Find x and y.



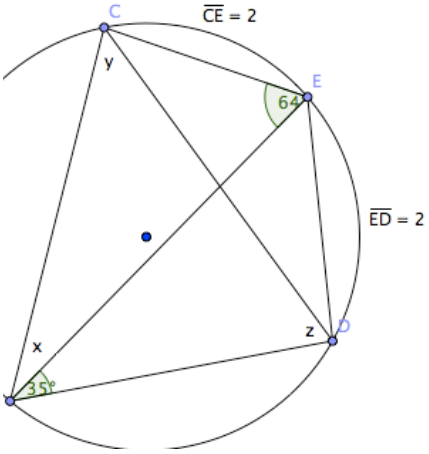
X= Y=

170. Find x and y.



X= Y=

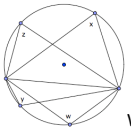
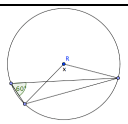
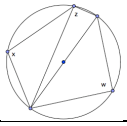
171. Find x, y & z.



X= Y= Z=

Review Check List

Definitions:		Pg #	Face it ☺☹*
Go to page 3/4 and write down any definitions that you are unsure of.	Define each word and be able to show your understanding with examples.	3,4	

Learning Targets	Example	Pg #	Face it ☺☹
Solve problems and justify the solution strategy using the perpendicular from the center of a circle to a chord bisects the chord.	A circle has diameter 20 cm. A chord is 8 cm long. How far from the center of the circle to the chord?	11	
Solve problems and justify the solution strategy using the measure of the central angle is equal to twice the measure of the inscribed angle coming from the same arc	 Which angles are equal?	14	
Solve problems and justify the solution strategy using the inscribed angles coming from the same arc are congruent	Why does angle x equal 120? 	15	
Solve problems and justify the solution strategy using a tangent to a circle is perpendicular to the radius at the point of tangency.	Flize Alut loves travelling. As he sits looking out the window an airplane, he wonders how far he can see out toward the horizon. The captain just announced that they are flying at 8km. He knows that the approximate radius of the earth is 6400km. How far can he see?	26	
Provide an example that illustrates the perpendicular from the center of a circle to a chord bisects the chord.	Look around your home or go online and provide an example that illustrates the perpendicular from the center of a circle to a chord bisects the chord.	12	
Provide an example that illustrates the measure of the central angle is equal to twice the measure of the inscribed angle coming from the same arc.	Go on line and provide an example that illustrates that inscribed angles coming from the same arc are congruent.	17	
Provide an example that illustrates the inscribed angles coming from the same arc are congruent.	Look around your home or go online and provide an example that illustrates the measure of the central angle is equal to twice the measure of the inscribed angle coming from the same arc.	17	
Provide an example that illustrates a tangent to a circle is perpendicular to the radius at the point of tangency.	Go on line and find an application of the circle property, a tangent to a circle is perpendicular to the radius at the point of tangency.	26	
Solve a given problem involving application of one or more of the circle properties.	Vanillan is building a swimming pool in the shape of a circle with a 6m radius. The deep end will be separated from the shallow end by a 6m line. How wide will each section be if the shallow end is narrower than the deep end?	11,12, 26	
Determine the measure of a given angle inscribed in a semicircle using the circle properties.	Which angle(s) measure 90°? 	14	
Explain the relationship among the center of a circle, a chord, and the perpendicular bisector of the chord.	Explain the relationship between the center of a circle, a chord, and the perpendicular bisector of the chord. Draw a picture to support your answer.	12	

*Face it. When you have mastered the content draw a ☺ OR if you are unsure, draw a ☹ and ask for help.

June 5th , 2015

Score:_____25

Practice Test

- Write this test and do not look at the answers until you have completed the entire test.
- Mark the test and decide whether or not you are happy with the result. FACE IT!
- Successful students will go back in the guidebook and review any questions they got wrong on this test.

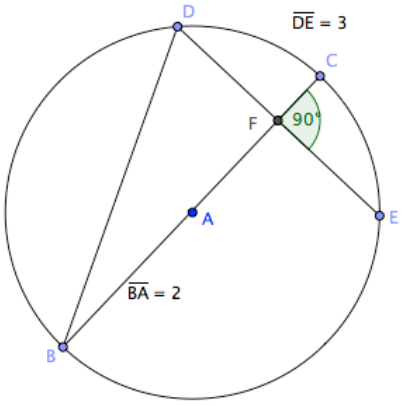
Draw a picture to represent each circle property.

1. **If** a line goes through the center and bisects a chord **then** the line is perpendicular to the chord.	2. Incribed angles coming from the same chord or arc are equal.	3. Incribed angles coming from the same size chord or arc are equal.	4. Central angles coming from the same chord or equal chords are twice as big as inscribed angle.
5. Inscribed angles in a semi-circle equal 90°.	6. Opposite Angles in Inscribed Quads add to 180°.	7. A tangent is perpendicular to the radius at the point of tangency	8. Tangents from an external point are equal.

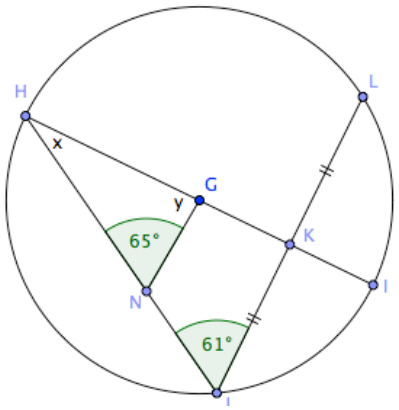
Determine the missing angles and sides lengths. Round your answer to 1 decimal where appropriate.

9. Find the shortest distance between the center of the circle and the chord if the diameter is of length 14m, and chord is of length 12m.

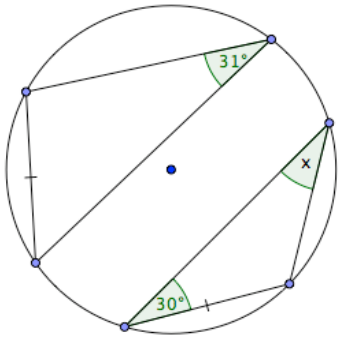
10. Determine the length of BD.



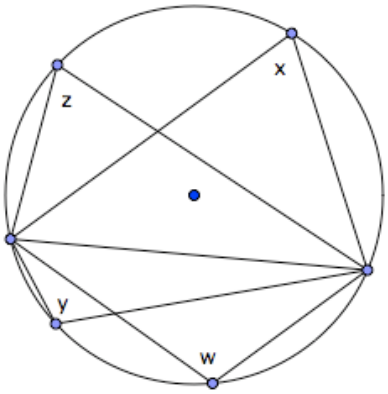
11. Determine the measure of angles x and y.



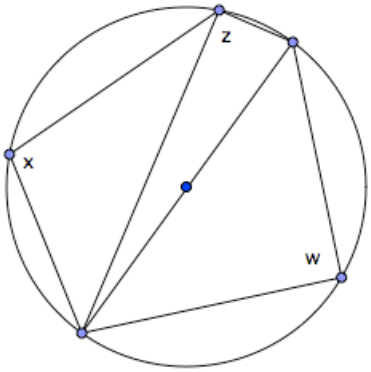
12. Determine the measure of angles x.



13. Which angles are equal?

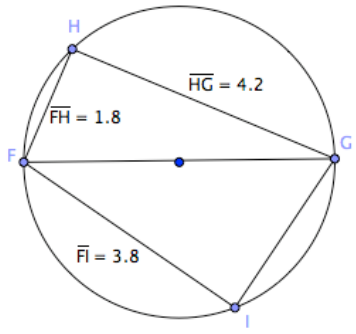


14. Which angle(s) measure 90°?

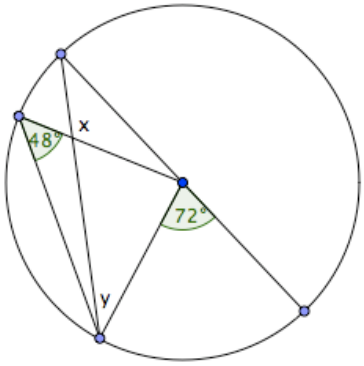


Determine the missing angles and sides lengths. Round your answer to 1 decimal where appropriate.

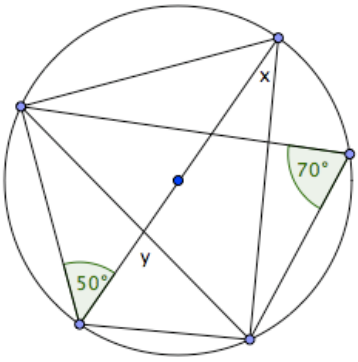
15. Determine the length of GI.



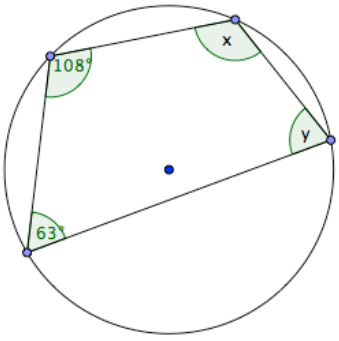
16. Find x and y.



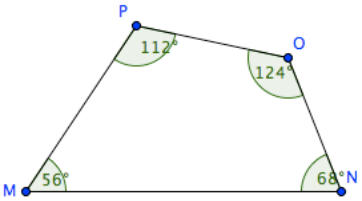
17. Find x and y.



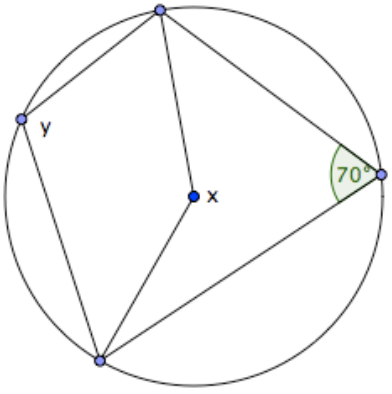
18. Determine the measure of angles x and y.



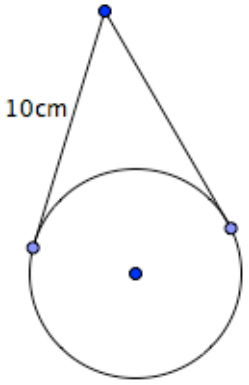
19. Could MNOP be an inscribed quadrilateral? Why?



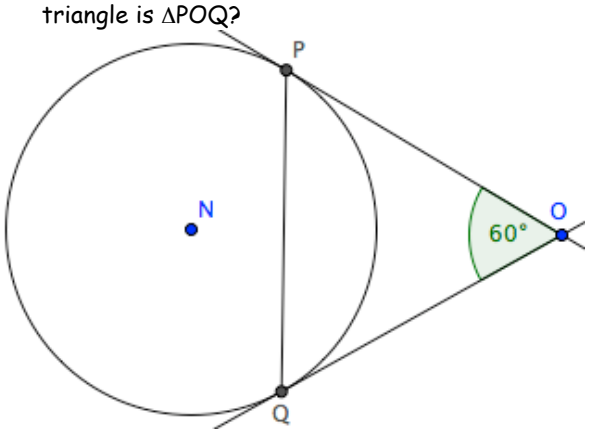
20. Find x and y.



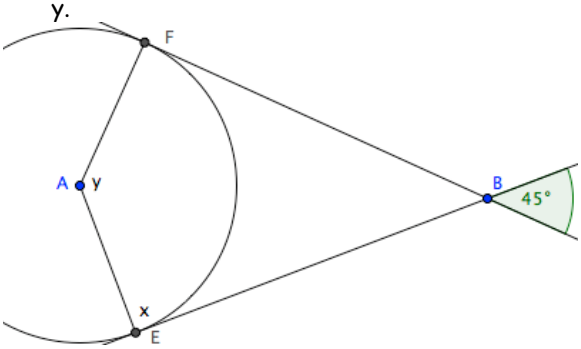
21. Determine the distance between the point and the top of the circle with radius 4cm.



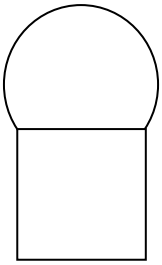
22. P and Q are points of tangency. What kind of triangle is ΔPOQ?



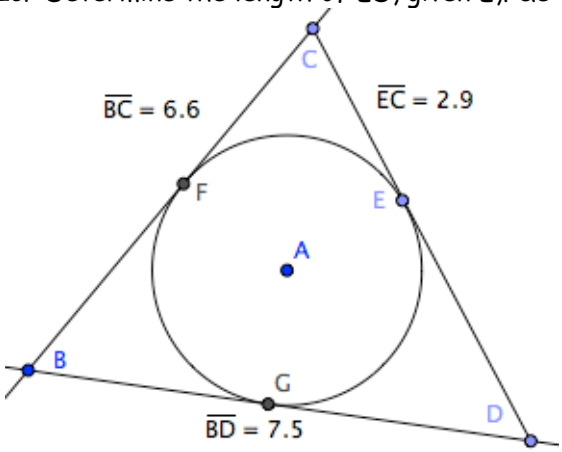
23. E and F are points of tangency. Determine x and y.



24. Billy the sneaky ice cream scooper is trying to make people think they are getting more ice cream than they really are. He makes his scoops with a radius of 5 cm and rests it on top of a cup that is 8 cm wide and 8 cm tall. How far from the bottom of the cup is the bottom of the ball?



25. Determine the length of ED, given E,F&G are all points of tangency.



Circle answers

Angles and Triangles

1. y = 137°– Vert Opp ∠'s
   z = 43° - Supplementary ∠'s
2. x = 75° - Isos Δ
   y = 30° - ∠ of Δ sum to 180°
3. v = 35° – Isos Δ
   x = 75° – ∠ of Δ sum to 180°
   z = 55° – Supp ∠'s
   ΔDEC = Isos Δ
4. y = 137°– Vert Opp ∠'s
   z = 43° - Supplementary ∠'s
5. x = 66° - Comp. ∠'s
6. y = 30° – Comp ∠'s
7. x = 52° – Comp ∠'s
   or Supp ∠'s
   z = 52°-Vert Opp ∠'s
8. x = 50° - Supp ∠'s
   y = 90° – Supp ∠'s
9. x = y = 135° – ∠'s at point add to 360°
10. x = 75° - Isos Δ
    y = 30° - ∠ of Δ sum to 180°
11. x = 78° - ∠ of Δ sum to 180°
12. y = 35° - Isos Δ
    z = 145° - Supp ∠'s
13. x = 56° - ∠ of Δ sum to 180°,
    y = 45° - Isos Δ

14. v = 60° – Equilateral Δ
    w = 60° – Vertically opposite
    y = 60° – Equilateral Δ
    z = 60° – Equilateral Δ
15. v = 84° – Supp ∠'s
    x = 12° – ∠ of Δ sum to 180°
    y = 48° – Supp ∠'s
    z = 42° – ∠ of Δ sum to 180°
16. v = 35° – Isos Δ
    x = 75° – ∠ of Δ sum to 180°
    z = 55° – Supp ∠'s
    ΔDEC = Isos Δ
17. x = 37° – Comp ∠'s
    y = 53° – ∠ of Δ sum to 180°
    z = 53° – ∠ of Δ sum to 180°

18. x = 54° – ∠ of Δ sum to 180°
    z = 72° – ∠ of Δ sum to 180°

Perpendicular chord theorem

19. Perpendicular
20. Bisect the chord
21. Passes through center
22. 8
23. 1
24. 21
25. 3
26. 2, y≈4.6
27. 4
28. 6
29. 9.2
30. 4.3
31. ≈2.9
32. ≈8.9
33. 9.4
34. 9.2cm
35. 18.4cm
36. Draw a perpendicular bisector through both chords. Both these lines will intersect at the center of the circle.
37. ≈0.1
38. S≈.8,D≈11.2
39. A line through the center that bisects a chord is perpendicular to the chord.
40.
41. Think of the sides of the triangle as chords of the circle. Draw perpendicular bisectors through the sides/chords. The bisectors will intersect at the center of the circle.
42. 8cm
43. 8.9km

Inscribed angles and central angles

44. Both equal
45. Both equal
46. Half as big
47. 90°
48. I
49. N
50. N
51. N
52. N
53. N
54. C
55. N
56. x=z & y=w
57. w & z
58. d, w, & x
59. Inscribed angles from the same chord.
60. Inscribed angles in a semicircle.
61. Inscribed angles from equal chords.
62. Central angle is twice inscribed angle.
63. Z is the central ∠ for y and x is the central ∠ for 70°
64. Central angles from equal chords.
65. X=40, y=60
66. Y=110, x=220
67. X=90, y=90
68. X=40, y=60
69. X=100 y=40
70. x=30, y=15
71. x=90, y=90
72. x=30, y=60
73. x=220, y=140
74. 220
75. 40
76. x=30, y=180
77. 90,90
78. 200,80
79. 46,84
80. 220,110
81. 84, 48
82. 140, 70
83. 18
84. 140, 110
85. 30, 80
86. Draw two diameters and connect the end points to each adjacent end point.
87. Answer will vary. Compare with a friend.
88. Answer will vary. Compare with a friend.
89. 2.857→2.9
90. 1.936→1.9
91. 2.538→2.5(Make sure you left at least 3 decimals all the way through this problem. You only round your answer.
92. 3.153→3.2
93. 140, 70
94. 120,36
95. 30,80
96. 40, 20
97. Inscribed angles from the same arc.
98. Inscribed angles in a semicircle.

Cyclic quads

99. Opposite angles =180°.
100. The opposite angle is supplementary.
101. 220, 110
102. 30, 110
103. 110, 90
104. Opposite angles in a cyclic quad are supplementary.
105. NO. Opposite angles do not equal 180°.
106. Yes. Opposite angles =180°.
107. 200, 80
108. 140, 70
109. 100, 32
110. 220, 110
111. 70,60
112. 49,112
113. 30, 110
114. 140, 110
115. 30, 80
116. 42, 114
117. 26, 24
118. 69, 34
119. 110, 90
120. 140, 30
121. 60, 110
122. ynyyn

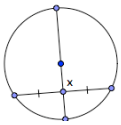
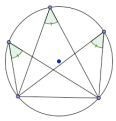
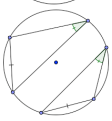
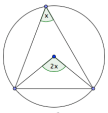
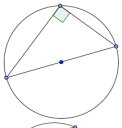
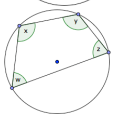
Tangent properties

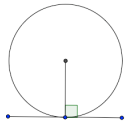
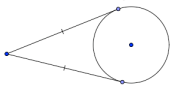
123. Think of each side length as a chord in the circle. Draw a perpendicular bisector through any two of the chords. The bisectors have to go through the center of the circle. Now that you know the center you could use a compass to draw the circle.
124. 90°
125. They both have the same length.
126. 90°, 5.2
127. x=9, y=4
128. 140,70
129. Tangents to an external point are equal.
130. Tangents meets radius at 90° at the point of tangency.
131. Secant line =EF, Radius=AI, Chord= HI, Tangent=EG
132. Isosceles since tangents to an external point are equal.
133. LJ and MJ.
134. Equilateral since it is an isosceles with 3 angles of 60° making it an equilateral.
135. 90°, 5.2
136. 22°, 5.0cm
137. 11.2cm
138. 140, y=70
139. x=100, y=80
140. 110, y=70
141. 20, ON=8
142. 90, y=135
143. 100, 80
144. 63.5, 116.5
145. 66, 86
146. 50, 30
147. 9, y=4
148. 71, 109
149. 5
150. 9
151. 39, 51
152. 50, 64

Extra Practice

153. about 320.1km
154. about 113.1km
155. Draw two perpendicular lines. One coming from E and the other G. Both these lines have to go through the center. Radii are perpendicular to the tangent line at the point of tangency.
156.
157. x=90→Ins angles in a semicircle, y=40→ angles in a triangle =180 z=140→ opposite angles in cyclic quad are supplementary.
158. x=120→Central angle is twice the inscribed angle. y=30→ Isosceles triangle because radii are equal. z=100→ opposite angles in cyclic quad are supplementary.
159. x=90→Tangent is perpendicular to the radius. y=44→ angles on a line are supplementary. z=4→ Tangents to an external point are equal.
160. 120, 90, 120
161. 70, 31, 140
162. 120,109, 19
163. 30,80
164. 47, 123
165. 120, 36
166. 1.732
167. 3.5
168. 4.6
169. 60, 110
170. 100, 32
171. 35, 46, 64

Practice Test

1. x=90° 
2. 
3. 
4. 
5. 
6. 

7. 
8. 
9. 3.606→3.6m
10. 3.646→3.6
11. x=29°, y=86°
12. x=31°
13. z=x &y=w
14. z & w=90°
15. 2.5377→2.5
16. x=120°, y=36°
17. x=30°, y=80°
18. x=117°, y=72°
19. yes because opposite angles add to 180°
20. x=220°, y=110°
21. 6.77→6.8cm
22. Equilateral→ all angles=60°
23. X=90°, 135°
24. 6cm from the bottom
25. 3.8

